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SYSTEMATIC PACKING OF SPHERES—WITH PARTICULAR RELATION TO POROSITY AND PERMEABILITY

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ABSTRACT

Geometrically systematic arrangements of uniform spheres are searchingly investigated in Part I. The relationships disclosed, important wherever orderly distribution of points or particles is involved, are here especially treated as the *underlying principles* of porosity and permeability. Besides packings defined respectively as "chaotic," "haphazard," and "chance," six cases of simple, systematic packing are recognized; two are orientation variants, but four are independent arrangements and include the two hitherto described: "loosest" and "tightest." Striking symmetries appear, and close analogies to crystal structure, including twinned and tripled arrangements.

The unit void of each case is thoroughly explored and illustrated, since in it lies the key to porosity and permeability. The stability of the several cases and their probabilities of being formed indicate that Case 6, tightest or rhombohedral, is most favored, and this is abundantly confirmed by experiment. The commonest natural packing, however, comprises colonies of Case 6 packing strewn in a surrounding mesh of haphazard, the whole constituting chance packing; the practical consequences of such arrangement are discussed. Translation from one packing to another is treated, and its bearing on dilatation outlined.

Part II considers permeability in all its relations to porosity, including the elements and degrees of dependence and independence. Geometry of the intersphere voids receives particular attention as affecting fluid flow through them; and plentiful graphs of the surprisingly complex void sections are shown, as well as of their integrated projections which somewhat influence rectilinear flow. Effect of assemblage orientation on flow is emphasized. Since permeability is of vectorial quality, every systematic assemblage of spheres is *anisotropic* with respect to permeability; therefore, if a single value is to be used for permeability, it must be the *mean* value.

It is concluded that experiment must still be relied on for determining permeability, since no existing means permit its *exact* computation. The Slichter formula and that by Fair and Hatch, both founded on the law of Poiseuille and aiming to state permeability in terms of porosity, are examined critically and found to yield only approximations, largely because of their deficient regard for details and effects of void geometry. However, comparative tabulation gives best available data on porosity and permeability of the several cases. This discloses the enormous differences in permeability caused by differences in packing, including effects of container walls and of included larger units (e.g., as in conglomerates). The varied effects of void geometry on capillary phenomena are discussed. Plea is made for further mathematical study of permeability, toward which the present paper may be helpful in contributing a geometrical basis.

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INTRODUCTION

SCOPE OF PAPER

For several years the authors have been examining, by various means and from diverse aspects, the highly important properties of porosity and permeability of rocks. A substantial portion of the results is set forth in the paper which immediately follows: "Experimental Study of the Porosity and Permeability of Clastic Sediments,"[1] by H. J. Fraser. The present discussion is introductory to the more strictly geological treatment of that paper; it investigates some of the fundamental space relationships that bring into being these particular properties of rocks and that must underlie a sound conception of these properties. But these same space relationships, surprisingly varied, and at the same time beautifully systematic, have wider application, as in elasticity, crystal structure, and similar fields that involve orderly arrangement of points or particles.

More specifically, this paper deals with the geometry of various ideal and near-ideal assemblages of discrete, ideal spheres; and it treats them primarily from the abstract geometrical standpoint, as if it made no difference whether the units were marbles, atoms, or planets, or whether they were assembled by man, machine, or Nature. The ideal abstractions here developed are, in the paper that follows, used and interpreted with due regard to the geological materials and processes actually concerned, and therefore with appropriate consideration for all those departures from the ideal which may or must be involved in natural rocks of clastic origin.

The conditions under which this investigation of porosity and permeability were conducted and acknowledgment of the support which made it possible are given in the following paper. With re-

[1] *Jour. Geol.*, Vol. XLIII (1935), pp. 910–1010.

spect to that portion of the entire subject embraced within the limits of this introductory article, we wish to express our indebtedness and thanks to several of our Harvard colleagues, especially the following: the late Professor O. D. Kellogg, for assistance in the mathematical study of packing; Professor G. M. Fair, for valuable criticism in regard to flow; Dr. H. H. Lester, for certain ideas regarding analogies to the crystal space-lattice; Dr. M. A. Peacock, for crystallographic guidance; and Mr. E. B. Dane, Jr., for many helpful suggestions and for collaboration with the illustrations.

HISTORICAL PERSPECTIVE

Systematic arrangements in space have long been subjects of scholarly inquiry. Since the classic contribution of Haüy in 1784, mineralogists and crystallographers have given much consideration to the space arrangement of atoms and molecules in crystals; and especially since the revelations of internal structure by means of X-rays and the attendant defining and improvement of the conception of the crystal space-lattice, a great deal has been learned by crystallographers and physicists regarding systematic atomic arrangements. Analogous, though more restricted, considerations have been pursued by the chemists in their efforts to understand the space-constitution of chemical compounds, the meaning of isomerism, the cause of polarization by liquids, etc. The engineers, physicists, and mathematicians have followed somewhat similar approaches in their investigations of physical and optical elasticity and related considerations regarding the solid state. The mathematicians have pursued this general line of inquiry into the realm of pure geometry, in dealing with the homogeneous partition of space and the ideal conceptions of point arrangement. Finally, the geologists, the hydrologists, the students of soils, and the civil and the sanitary engineers have, with some help from physicists and mathematicians, attacked the problem of arrangement or packing of solid particles with particular reference to the topics of major interest in this paper, viz., porosity and permeability of natural materials, as affecting water supply, migration of oil and gas, plant growth, foundations, filtration, and drainage.

From these varied sources an imposing bibliography could be as-

sembled relating more or less directly to this question of the packing of solid units, and embracing, among countless others, the names of such outstanding investigators as Haüy, Frankenheim, Delafosse, Bravais, Sohncke, Federow, Laue, the Braggs, Pasteur, Kékulé, William Barlow, Lord Kelvin, C. S. Slichter, and many others.

It might be expected, therefore, that the subject of arrangement of units in space has by this time been thoroughly explored and that a sound and complete lot of principles would now be available for use in any specific applications that might arise, in the same way that the multiplication table or the list of atomic weights is at hand, ready-made for whoever has use thereof. Strangely enough, this does not appear to be the case. It was not until an even hundred years after Haüy's first incursion into this general field that William Barlow[2] pointed out that analysis of the manner of arranging solid units to form component aggregates may best be approached by dealing with the most ideal case possible, viz., that in which the units consist of spheres of uniform size. Such delay in recognition of a fact that now seems self-evident can be explained only on the presumption that prior investigators had envisaged the problem only in a narrow, piecemeal fashion. The same seems to be true to some extent of Barlow himself and of those who have followed him.

In consequence, for one reason or another, preceding treatments do not seem to be at the same time directly applicable and wholly satisfactory with respect to the present problem, which, as elaborated later, is concerned with those simplest assemblages of uniform spheres so arranged that the spheres mutually support one another. Some of these earlier analyses do not fit our present problem because the arrangement concerned is due to "forces" or "bonds" which give inherently different spacings from those that result from the mere condition of mutual physical support afforded by tangency of sphere surfaces. To a similar category must go those cases that envisage interpenetration instead of rigid mutual exclusion of adjacent units. In other cases, special heterogeneities of size and shape of the particular units involved bring departures from the fruitful pathway of consideration of our particular subject. Other presentations give only restricted or incomplete consideration of the possibilities perti-

[2] *Nature*, Vol. XXIX (1884), pp. 186–88, 205–7.

nent to our problem and are especially deficient with respect to the matter of highest importance in the present connection, viz., the shape and arrangement of the intersphere voids. Still other analyses appear to contain outright errors.

Therefore, because of this apparent absence of a direct, but adequate, exposition of the systematic packing of spheres and because of the great light shed on the nature and variations of porosity and permeability by such means, it is here undertaken to set forth an analysis of those cases of systematic packing of uniform spheres which seem to have direct and practical relation to the porosity-permeability problem. This analysis is furthermore believed by us to be of more general nature and more complete from the geometrical standpoints than prior treatments have been. It may therefore possibly be found of wider interest and use than in specific relation to porosity and permeability.

Part I. Considerations Especially Affecting Porosity

Essential definitions

For present purposes, certain definitions are required, as below.

The term *packing* will here be applied to any manner of arrangement of solid units in which each constituent unit is supported and held in place in the earth's gravitational field by tangent contact with its neighbors. From what has been said shortly above, it will be evident that this is a more restricted sense of the term "packing" than that in which it is often employed in such geometrical analyses.

The *unit solid* is an ideal sphere of radius R.

The packing of uniform spheres may be either *disorderly* or constantly repetitive, i.e., *geometrically systematic*. It will be advantageous first to consider the several simplest types of systematic packing and then to examine the deviations that become more and more complex and chaotic.

The *row* is an aggregate of uniform spheres arranged with their centers along a straight line and successively spaced at the distance of $2R$, so that each sphere along this line is tangent to its neighbors on either side.

A *set of rows* is an assemblage of rows in parallel direction. The rows in a set may be spaced closely or widely with respect to one

another; and they may lie in a single plane or in more than one plane, but in the latter case the planes must intersect in lines parallel to the rows.

The *layer* (closely corresponding to the *plane net* of space-lattice terminology) is a parallel arrangement of rows in the same plane and so spaced that the spheres in each row are tangent to the corresponding spheres in adjacent parallel rows. As a result of this condition of tangency, it follows that a second (and in some cases a third) set or direction of rows is established, so that each layer comprises two (or three) sets or directions of rows.

There are various types of layers, depending upon the angle of intersection of the sets of rows in the layer. But it will be seen that

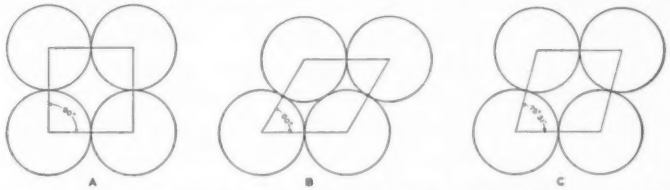


FIG. 1.—Types of layers. *A*, square layer; *B*, "simple" rhombic layer; *C*, special rhombic layer.

under the condition of mutual tangency of spheres in a row, the angle at which two sets of rows intersect must lie between values of 60° and 90°. The type of layer that seems most natural and familiar (like the layout of trees in an orchard) is that in which two sets of rows intersect at 90°. This may be called the *square layer*. See Figure 1, *A*.

The next most familiar and simple type of layer is that in which three sets of rows intersect at 60°. This may be called the *simple rhombic layer*.[3] See Figure 1, *B*.

Layers are possible in which the sets of rows have any angle of intersection between these limiting values of 60° and 90°; but only the square layer and the simple rhombic layer will be given general consideration[4] here, since it appears that these two are adequate to

[3] The "simple rhomb" is here defined as one in which the short diagonal is of the same length as a side, and the ratio of diagonals is 1:1.73 (i.e., $1:\sqrt{3}$).

[4] One other kind of layer that results under certain circumstances from the stacking of simple rhombic layers is discussed later and is shown in Fig. 1, *C*.

represent the limiting types of systematic packing actually encountered. The layer is here emphasized particularly because of its fundamental influence on the resulting packing.

The *body* is a stack of parallel layers so arranged that the spheres in one layer are tangent to corresponding spheres in adjacent layers. See page 828 for definition of *colony*.

The *unit cell* is that smallest portion of a body (as just defined) which gives complete representation of the manner of packing and of the distribution of voids throughout that body. Its analogy to the unit cell of crystallography is evident. More explicit definition is given on page 797.

The *unit void* is the void contained within the unit cell. It is further specified on pages 807–8.

By giving consideration to the centers of the spheres involved, the unit solid may be visualized as a *point*, the row as a *linear* arrangement, the layer as a *plane* arrangement, and the body as a solid or *three-dimensional* arrangement. It is only to the three-dimensional arrangement that *packing* applies in the sense here entertained. Systematic packing of the kind here considered is achieved by various simple arrangements of either the square layer or the simple rhombic layer; such packing may be designated *simple systematic packing*. There could be systematic arrangements of layers intermediate between the square and the simple rhombic; but these would give intermediate values of porosity and permeability and therefore need not be considered in detail. See pages 827–28 for definitions of *chaotic* packing, *haphazard* packing, and *chance* packing.

GEOMETRY OF THE SIX PACKINGS

GENERAL RELATIONSHIPS

It has been generally customary hitherto—at least in connection with porosity and permeability—to recognize and consider only two kinds of systematic packing, often called, respectively, "loosest" and "tightest" packing. But while these two represent the opposite limits possible in simple systematic packing, consideration of only these two extreme cases involves an unjustifiable neglect of certain equally systematic but intermediate modes of packing.

As a matter of fact, it proves that there are three geometrically simple ways of stacking either square layers or simple rhombic layers upon one another. Therefore, there are three geometrically simple schemes of systematic packing for each of the two simple types of layers making six schemes in all. But it develops that two of the three ways of stacking the square layers are respectively identical in packing (though of different orientation in space) with two of the three ways of stacking the simple rhombic layers; so there are, in fact, only *four different*[5] simple and systematic ways of packing within the limitations already mentioned.

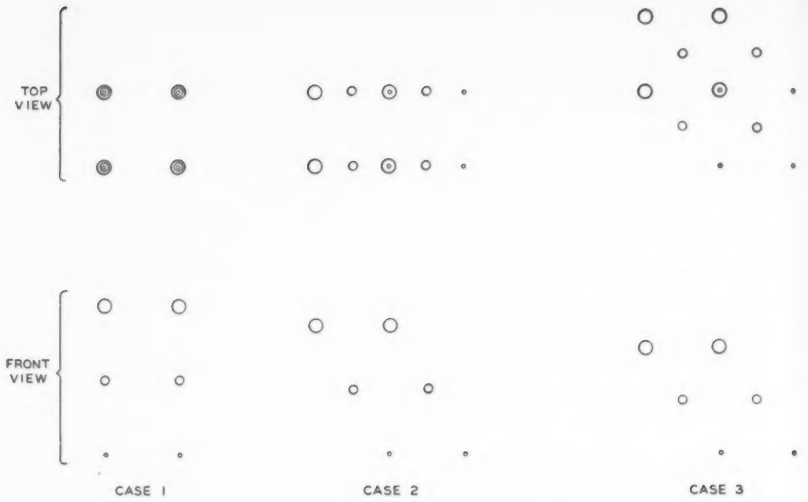
The manner in which these several packings are attained is shown in the following key diagram, Figure 2. In each of the six cases there shown, the smallest circles locate the centers of spheres of the lowest, or first, layer; the intermediate circles locate similarly the centers of corresponding spheres of the next-overlying, or second, layer; and the largest circles locate the centers of spheres of the third layer. Comparison of the top and the front views in each case will indicate the three-dimensional relations of the layers to one another. In the diagram, only four spheres are indicated in each layer; but the layers may actually be extended indefinitely, if holding to the square or simple rhombic pattern, respectively. Also, only three layers are indicated in each case, but additional layers may be added indefinitely by stacking each successive layer on the one next below in the same manner that the second layer is stacked on the first. For convenience in description, the conventional assumption is made that in all cases the *stacked* layers are horizontal; but newly produced layers resulting from the stacking may have different attitudes with respect to the horizontal basement.[6]

The diagram of Figure 2 shows that for each of the two simple types of layers the three simple arrangements are:

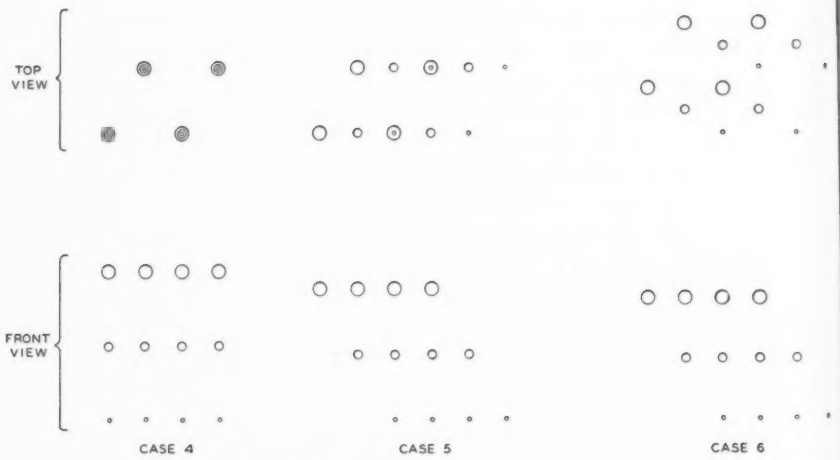
I. Cases 1 and 4—spheres in second layer vertically over those in first layer.

[5] It will appear later, in connection with the discussion of twinned and tripled packings, why it is desirable to carry through the discussion on the basis of the *six* cases instead of on the basis of the fundamentally different *four*.

[6] For example, in Case 1, formed by stacking horizontal square layers vertically over one another, two sets of square layers in vertical attitude and mutually at right angles are produced.



A. ARRANGEMENT OF SQUARE LAYERS



B. ARRANGEMENT OF SIMPLE RHOMBIC LAYERS

FIG. 2.—The six schemes of stacking simple layers

II. Cases 2 and 5—spheres in second layer horizontally offset with respect to those of the first layer, by a distance R along the direction of one of the sets of rows.

III. Cases 3 and 6—spheres in second layer horizontally offset with respect to those of the first layer, in a direction bisecting the angle between two sets of rows and by a distance of $R\sqrt{2}$ in Case 3 and of $2R\sqrt{1/3}$ in Case 6.

In Figure 3 is illustrated the essence of each of the six cases, represented in each instance by an assemblage of eight spheres, four in each of two layers that are stacked one on top of the other. The groupings in Cases 1, 2, and 3, shown in perspective, are depicted in the attitude of resting on the square layer, and Cases 4, 5, and 6 on the simple rhombic layer.

These several modes of packing have certain simple geometrical relationships to one another. These relationships involve internal arrangements and external shapes that are analogous to various crystal forms. A tabulation of these relationships is given in Table I; each case is given the name of its general crystal analogue.

In Table I the line headed "Spacing of layers" represents the vertical distance from the horizontal plane passing through the sphere-centers of one layer to the corresponding plane of the next layer. It is evident that, although there is no interpenetration of spheres, there is interpenetration of layers in all of the arrangements except Cases 1 and 4, as a consequence of the placing of spheres of one layer into the cuspate or hopper-like depressions between spheres of the adjacent layer. The line headed "Tangent neighbors" relates to the number of spheres which each sphere touches. In the square layer, each sphere touches four neighbors in that layer; and, depending on the arrangement of the layers, it is touched also by one, two, or three spheres of the layer next above and of the layer next below. In the simple rhombic layer each sphere touches six neighbors in the same layer, and likewise one, two or three spheres of the layer next above and of the layer next below. (In the special rhombic layer mentioned on page 803, which develops from a certain method of piling simple rhombic layers, each sphere touches four neighbors in its own layer and one or two—but never three—

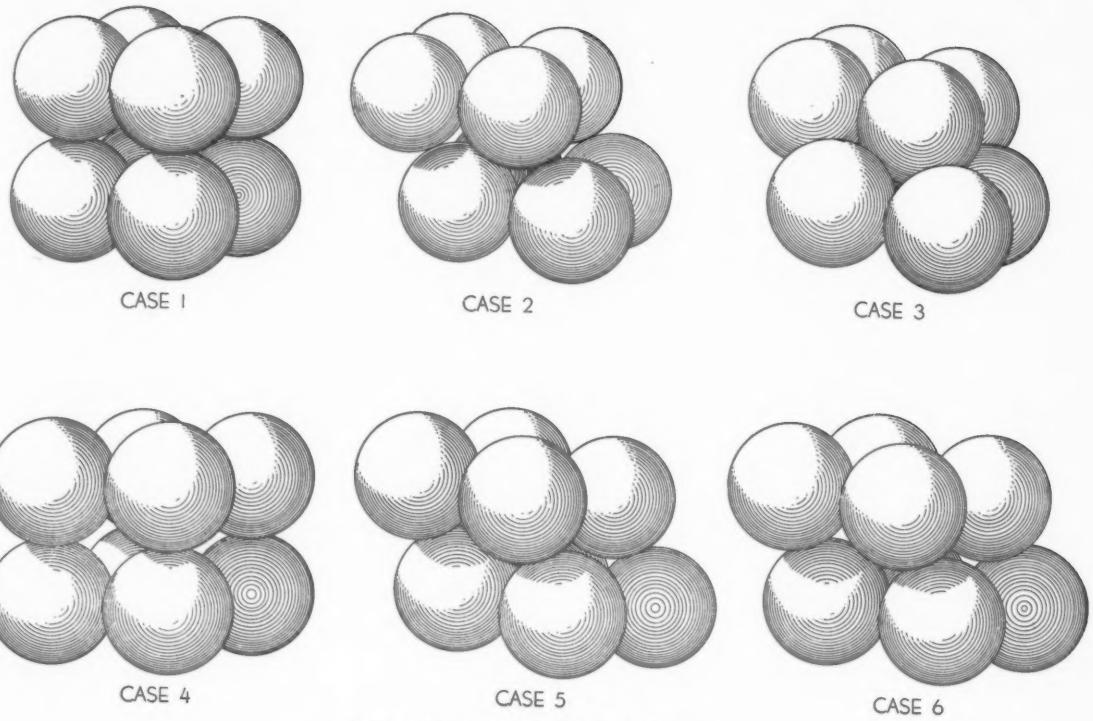


FIG. 3.—Type sphere groups of the six cases

spheres in each of the two adjacent layers.) It thus follows that the minimum number of tangent neighbors in any simple systematic scheme of packing is six (Case 1) and the maximum possible number is twelve (Cases 3 and 6).

TABLE I

GEOMETRICAL RELATIONSHIPS OF THE VARIOUS PACKINGS

ARRANGEMENTS OF SQUARE LAYERS

	Case 1	Case 2	Case 3
Name (crystal analogue)	Cubic	Orthorhombic	Rhombohedral
Spacing of layers	$R\sqrt{4}$	$R\sqrt{3}$	$R\sqrt{2}$
Tangent neighbors	6	8	12
Face angles	90°	90°, 60°–120°	90°, 60°–120°
Interfacial angles	90°	90°, 60°–120°	54°44′–125°16′

ARRANGEMENTS OF SIMPLE RHOMBIC LAYERS

	Case 4	Case 5	Case 6
Name	Orthorhombic	{Tetragonal-Sphenoidal}	Rhombohedral
Spacing of layers	$R\sqrt{4}$	$R\sqrt{3}$	$2R\sqrt{2/3}$
Tangent neighbors	8	10	12
Face angles	90°, 60°–120°	{60°–120° / 75°31′–104°29′}	60°–120°
Interfacial angles	90°, 60°–120°	{90° / 63°26′–116°34′}	70°32′–109°28′

NATURE OF THE UNIT CELL

Before individual consideration of the six cases of packing, further description is desirable and is now possible for the unit cell, beyond that already given in the list of definitions on page 792. The unit cell may be defined from two standpoints: (1) it is bounded by planes so passed through the centers of adjacent spheres that the resulting polyhedral form meets the requirement of being the smallest possible sample that will completely represent the manner of packing and of void distribution throughout the body of which it is a part; or (2), it may meet the foregoing requirement plus the added stipulation that it also completely represent the *orientation* of void distribution

throughout the body.[7] The cell as considered from the first of these standpoints may be called the *minimum cell;* that considered from the second standpoint may be designated the *complete cell*, and this latter is the unit cell in the general sense.

As a matter of fact, the minimum cell, is identical with the complete cell in three of the schemes of packing, i.e., Cases 1, 3, and 6, since in these, when shape and distribution are determined, orientation is fixed also. But this is not true in the remaining cases. The effect of adding the requirement regarding orientation, as under (2) above, may be explained as follows. In the packing of Case 2, for example, as will shortly appear, the essence of sphere arrangement and of void distribution is typified by a five-sided cell having the

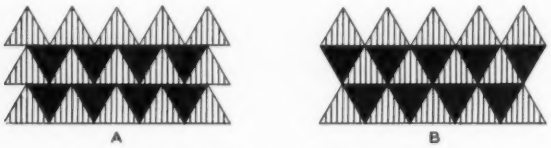


FIG. 4.—Relation of minimum cell to complete unit cell of Case 2

shape of a right trigonal prism, i.e., three square sides and two equilateral triangular ends. But if cells of such shape were assembled together all in the same orientation to make a multi-celled body, they would *fill only half the space* within which they are assembled. In order that the space be completely filled, an equal additional number of identical cells would be required, in an attitude appropriately rotated with respect to the first lot and arranged alternately therewith. This is indicated (in two different patterns) by Figure 4, in which the hatched areas represent the ends of the trigonal prisms in one attitude and the solid black areas represent the ends of the trigonal prisms in the rotated attitude. Such a right trigonal cell, therefore, does not fully comprehend and represent the factor of orientation. In order to have, for Case 2, a cell which will embrace both the shape factor and the orientation factor, it must consist of

[7] In discussing a closely analogous relationship, Lord Kelvin expressed the two requirements here involved as "the same shape and sameways oriented" (*Proc. Roy. Soc. London*, Vol. LV [1894], p. 1).

two right trigonal prisms so rotated with respect to one another as to be joined along a prism face. As so joined, the two right trigonal prisms would now collectively constitute a six-sided form or cell, an orthorhombic prism having four square sides and two simple rhombic ends. Such cells, when packed close together in parallel orientation, will completely fill the space and will thus produce a solid body (cf. Fig. 4, *B*). The right trigonal prism is thus the minimum cell of Case 2; and the orthorhombic prism is the complete cell of Case 2. The same holds true for the identical packing of Case 4. In Case 5 an analogous situation arises, where a minimum cell and a complete cell are both possible.

It proves that, for every one of the cases, the complete unit cell is a six-sided parallelopiped, each edge of which has a length of $2R$. These are illustrated in Figure 5, which shows the portions of the spheres involved (stippled areas) and discloses on the cell faces the trace of the enclosed void or "throat-plane" (white areas).[8] In this figure the viewpoint or orientation for each cell is identical with that for the sphere assemblage of the corresponding case in Figure 3. These cells range from the cube in Case 1 with all interfacial angles of 90° to the simple rhombohedron of Cases 3 and 6 with no interfacial angles of 90°. In each case, the complete cell contains parts[9] of eight adjacent spheres; and if these eight parts were appropriately rearranged, they would form exactly one complete sphere. In other words, each complete cell contains the equivalent of exactly one sphere; the remainder of the volume of the cell is void. The minimum cell of each of Cases 2, 4, and 5 contains the equivalent of one-half sphere. Each face of the complete unit cell contains one sector of each of four different great circles; the aggregate area of these four sectors equals one great circle; and the area of the throat-plane is the area of the parallelogram face *minus* πR^2.

[8] See definition of "throat-plane" on p. 808.

[9] The authors have failed to find an established geometrical name for this portion of a sphere bounded by three or more intersecting great circles and the sphere surface. It is like a pyramid except that it has a base that is curved instead of plane. For convenience, we shall call the form a "sphero-pyramid"; and since in the present examples three great circles are always involved, the form may be called a "trigonal spheropyramid."

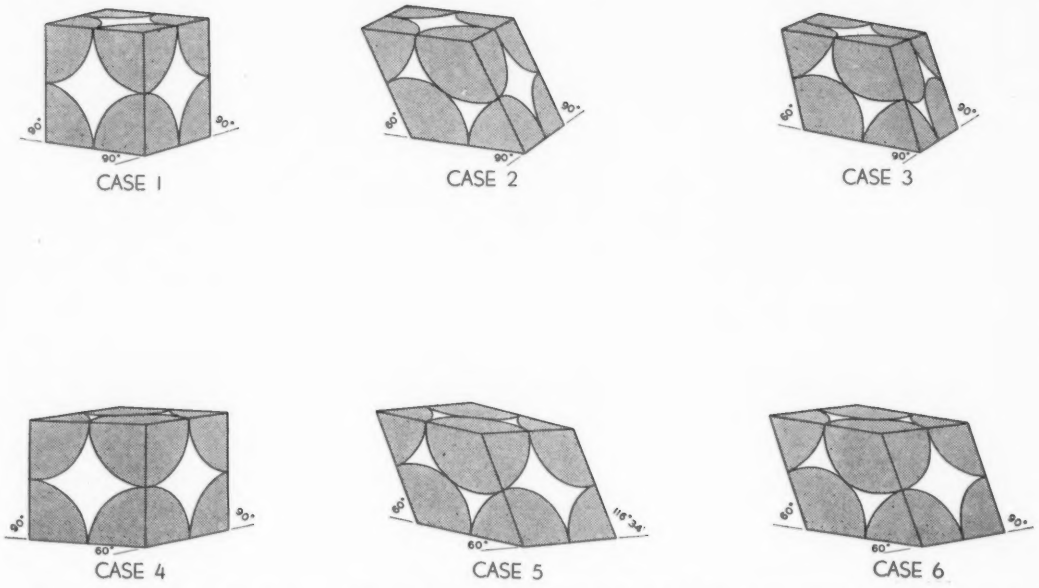


FIG. 5.—Unit cells of the six cases. The supplements of the face-angles are shown. Same orientations and scale as in Figure 3.

DESCRIPTION OF THE INDIVIDUAL PACKINGS

Case 1—cubic packing.[10]—In this type of packing, the "square layer" arrangement is present not only horizontally but as well in two vertical directions at right angles, so that three systems of square layers are present at angles of 90° to one another; they correspond to the cube faces. There are also present six identical systems of plane arrangement (not to be called "layers," as herein defined, because some of the rows are separated rather than contiguous or tangent); they are of rectangular pattern with sides respectively $2R$ and $2.83R$; and they pass through diagonally opposite edges of the cube, so that they have the same orientation to one another as do the faces of a rhombic dodecahedron. The unit cell of this scheme of packing is formed by passing three pairs of parallel planes through the sphere-centers located at the corners of a cube of edge $2R$; it has a volume of $8R^3$ and is shown under Case 1 in Figure 5. This is the only one of the six schemes of arrangement in

[10] There seems to be an unfortunate tendency in writings on the space-lattice to develop a terminology without regard, in all instances, to already established usages. This presumably can only lead to confusion. For example, the authors of an interesting paper entitled "The Structure of Topaz" (*Zeit. Krist.*, Vol. LXIX [1929]) show on their p. 164 diagrams illustrating two different types of closest packing; although both of these are based on the simple rhombic layer, they call one the "hexagonal" type and the other the "cubic" type. As a matter of fact, the first is a twinned *rhombohedral* packing of Case 6 which, because twinned, has lost the square layer and, therefore, lost some of its isometric characteristics (cf. herein p. 819); and their "cubic" packing is a normal rhombohedral packing of Case 6 with the square layer present in three sets of planes, and therefore possessing a considerable degree of isometric symmetry (cf. footnote herein on p. 802). But if this packing is to be called "cubic" packing, what is to become of the long-known "loosest" packing of Case 1, which is based directly on the cube? Still more disconcerting is the following concluding statement on p. 165 of the same paper:

". . . . there are an infinite number of ways in which an arrangement of atoms in closest packing can be formed by stacking in different sequence, one on top of the other, identical single layers of atoms in contact. The hexagonal and cubic types may be regarded as the simplest example of such stacking. . . . ."

The writers of the present article find no confirmation of the infinite number of ways of attaining truly "closest" packing (with a porosity of 25.95 per cent), and surmise that confusion in terminology has led to the erroneous conclusion. In order to avoid possible misunderstanding, we are avoiding the long-used "closest" packing (with a porosity of 25.95 per cent), and substitute for it the term "tightest" packing. See also W. P. Davey, *A Study of Crystal Structure and Its Applications* (New York, 1934), pp. 359–62.

which the simple rhombic layer is absent. It is, in fact, the "loosest" or the most open-textured packing that can be produced under the simple systematic conditions previously prescribed.

Case 2—orthorhombic packing.—Here, as in Case 1, the "square layer" arrangement is present in three planes; but in this case they lie at angles of 60° to one another. There is also present one plane or system of "simple rhombic layer" arrangement, lying at right angles to the three "square layer" planes. Also present is a plane rectangular pattern (not "layer") of sides $2R \times 3.46\ R$; this occurs in three planes or systems at 60° to each other and at 90° to the simple rhombic layer. The minimum cell is a trigonal[11] right pyramid all of whose edges are $2R$; the volume of this minimum cell is $3.46R^3$. The complete cell is a six-sided form, four of whose faces are squares of side $2R$ and two of whose faces are "simple" rhombs of side $2R$; the volume of such a cell is $6.93R^3$. (See Fig. 5.)

Case 3—rhombohedral packing.—The "square layer" arrangement is again present in three different planes at 90°, in this instance corresponding to the three diagonal planes of a regular octahedron. There are also present four planes or systems of "simple rhombic layer" arrangement, parallel to the respective faces of the octahedron (or of the tetrahedron). The unit cell is formed of six planes passed through eight sphere-centers situated at the corners of a regular or "simple" rhombohedron each of whose edges measures $2R$; the volume of this unit cell is $5.66R^3$. (See Fig. 5.) This is the "tightest" or most compact possible arrangement of solid spheres. This manner of packing may very appropriately be called rhombohedral, as Slichter[12] named it.[13]

[11] It is obvious that there are elements of hexagonal symmetry of a low order in this packing; indeed, if it were not for the factor of orientation, the case might properly be named "trigonal" instead of "orthorhombic."

[12] C. S. Slichter, *U.S. Geol. Surv. 19th Ann. Rept.*, Part II, (1899) pp. 306 ff.

[13] But while the hexagonal (rhombohedral) symmetry dominates, this form possesses strong isometric affiliations also. This is evidenced in part by the presence of three systems of the square layer arranged mutually at 90°. Indeed, in 1893, Lord Kelvin (*Proc. Roy. Soc. London*, Vol. LIV, p. 61), who called this manner of packing the "equilateral homogeneous assemblage," noted its *tetrahedral* characteristics. It is, moreover, the "face-centered cubic" arrangement of space-lattice terminology (cf. Bragg, *The Crystalline State* [1933], p. 144). And, as shown on p. 813, its complete unit cell may be

Case 4—orthorhombic packing.—In internal arrangement, this packing is identical with Case 2. If a body built by the stacking of square layers as in Case 2 is rotated 90° about the edge formed by the junction of the basal square layer with the simple rhombic layer, the attitude of the body in space will become that of Case 4. In Figure 5 the unit cell of this case is shown; it is the same cell as shown in the same figure for Case 2 but seen from a different angle corresponding to the different orientation in space.

Case 5—tetragonal-sphenoidal packing.—This type of packing has the "simple rhombic layer" present in two planes or systems at right angles. This is the only one of the six schemes of packing which possesses no "square layer" arrangement. But instead it possesses four planes or systems of a layer just off "square"; namely, a layer determined by a unit rhomb of side $2R$ and having face angles of 75°31′–104°29′ and diagonals with a ratio of 1:1.29 (instead of 1:1 for the square and 1:1.73 for the "simple" rhomb). These four planes have the same positions relative to one another as do the faces of a first-order tetragonal sphenoid; and with respect to such planes the two systems of simple rhombic layers are disposed as the faces of the tetragonal prism of the second order. There are also present two systems of rectangular pattern (not "layers" in the strict sense) of sides $2R$ and $2.45R$, lying at right angles to each other and parallel to the faces of a first-order tetragonal prism.

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regarded as composed of two tetrahedra and an octahedron. Slichter briefly noted (*op. cit.*, p. 307) the presence of the three directions of "square layer," but made no reference to the propagation of this *cubic* element throughout the entire assemblage in rhombohedral packing and gave no evident recognition of the marked effect produced on porosity and permeability by the presence everywhere of this open-textured (square) component in so-called "tightest" packing. Kelvin, on the other hand, while clearly appreciating the factor of orientation, apparently failed, nevertheless, to recognize that the tetrahedron is not the complete unit cell of this packing, for while four spheres located at the respective corners of a tetrahedron give the essence of this scheme of "equilateral" or rhombohedral packing, as Kelvin pointed out, and while two such tetrahedral groups of four spheres each, when assembled base to base and one rotated 60° with respect to the other, will give the complete rhombohedral assembly, it is nevertheless true that the angular and distance relations between the spheres in each tetrahedral group are *not* identical with the relations of the spheres of one tetrahedral group to those of the other similar group. In short, the tetrahedron does not embody the complete expression of this rhombohedral packing. The surprisingly complex geometry of this tightest packing is more fully explained on pp. 810–15.

The sphenoidal or low-symmetry character of this arrangement gives it some analogies to the monoclinic system, but there is nothing in its makeup that violates tetragonal symmetry. The unit cell of this packing lies within six planes passed through eight sphere-centers located at the corners of a rhombohedron-like figure of which four faces are the "simple" rhomb of edge $2R$ and two faces are of the special rhomb described just above; the volume of this cell is $6R^3$ (see Fig. 5). As a matter of fact, this is double the minimum cell for this case, since, as in Cases 2 and 4, the essence of the packing—except for orientation—is given by a cell bounded by five planes as follows: one of "simple" rhombic shape of side $2R$, two of equilateral triangular shape of side $2R$, and two of the special rhomb already specified above. Such a minimum cell has a volume of $3R^3$.

Case 6—rhombohedral packing.—Internally, this packing is identical with that of Case 3; it therefore has the same unit cell. If the body built upon the square layer of Case 3 be rotated 109°28′ about an edge of the basal square layer, its various layers will take up the same attitude in space as that produced by building Case 6 on the simple rhombic layer. In Figure 5 is shown this unit cell in its proper orientation for this case; it is the same cell as is shown for Case 3 in the same figure but seen from a different angle.

GEOMETRY OF THE VOIDS

General Considerations

While porosity and permeability are determined, obviously, by the *voids* between the spheres as packed, attention in most earlier discussions of packing has been too much focused on the spheres themselves and too little concern has been given to the openings which the various modes of packing leave between the spheres. Not only has the treatment of the voids themselves been inadequate, but, because inadequate, it has permitted in some instances the entrance of outright errors. In the following discussion, the shape, volume, and arrangement of the intersphere voids are given specific consideration as they are determined by the several packings.

Each independent kind of packing is associated with a characteristic geometry of voids. In each type of packing, the ratio of the

total volume of the voids to the total volume of the body is the porosity for that particular mode of packing; this is the same ratio as that of the complete unit void[14] to the complete unit cell. Thus, for each of the different kinds of packing there is a given characteristic porosity, which of course holds constant regardless of the size of the unit sphere. Table II shows the volume of the unit cell, the volume of the unit void, and the porosity value in per cent for each of the several modes of packing.

TABLE II

POROSITIES OF THE VARIOUS PACKINGS

ARRANGEMENTS OF SQUARE LAYERS

	Case 1	Case 2	Case 3
Volume of unit cell*	$8.00R^3$	$6.93R^3$	$5.66R^3$
Volume of unit void*	$3.81R^3$	$2.74R^3$	$1.47R^3$†
Porosity	47.64%	39.54%	25.95%

ARRANGEMENTS OF SIMPLE RHOMBIC LAYERS

	Case 4	Case 5	Case 6
Volume of unit cell*	$6.93R^3$	$6.00R^3$	$5.66R^3$
Volume of unit void*	$2.74R^3$	$1.81R^3$	$1.47R^3$†
Porosity	39.54%	30.19%	25.95%

* The volume of the complete cell and of the corresponding void is given for each case. The volume of the unit void is in each case derived by subtracting the volume of the unit sphere, $4.19R^3$, from the volume of the unit cell.

† As shown on pp. 812–13, the void of Cases 3 and 6 is composite, and it is the volume of the composite void that is given here.

It will be noted that, just as the square layer is a looser or more open arrangement than the "simple" rhombic layer, so the packings built on the square layer are in general more open or of higher porosity than those built on the "simple" rhombic layer. It will further be seen, by reference to Table I, that, in general, as would be expected, the porosity decreases with decreased spacing of the layers and with increased number of tangent neighbors. As Dr. Peacock has suggested, it might be expected, also, that the porosity would vary in some orderly way with the degree of symmetry of the

[14] See definition on pp. 792 and 808.

several packings. But it becomes evident on inspection of the porosity percentages that such a relationship does not consistently hold; Case 1 has the highest porosity and it also obviously has the highest symmetry of any of the six cases, but corresponding condition does not prevail steadily as one passes to packings of progressively lower porosity. The reason for this failure of direct relationship between symmetry and porosity appears to be that symmetry, after all, is entirely determined by *angular* relationships and not primarily by *distances;* for example, spheres of radius R placed at the corners of a cube of edge $3R$ or $4R$ or any larger multiple of R would produce an assemblage having the same symmetry as if the cube had the edge of $2R$ (as in Case 1), but the porosity of assemblages built on the larger cubes would be greater than of that built on the smaller cube. Thus, porosity and symmetry cannot be expected to vary proportionally. The identity of the porosities of Cases 2 and 4 and of Cases 3 and 6 as shown in Table II is a consequence of the corresponding identity of these schemes of packing.

It has already been pointed out that the six simple schemes of packing herein considered are only special cases of an infinite variety of possible packings lying between the extremes of Case 1 and Case 6. Slichter has treated this situation by regarding the cube and the "simple" rhombohedron[15] as special limiting cases of a "general rhombohedron" capable, by variation of its face angles to any intermediate value between the 90° and the 60°–120° of these limiting cases, of representing the equivalent of every possible variety of systematic packing between the cubic, or loosest, and the rhombohedral, or tightest. He has then computed the porosities[16] for several of these intermediate rhombohedral arrangements; thus a rhombohedron whose voids represent a porosity of 30 per cent he finds to have face angles 62°36′–117°24′, and one with a 40 per cent porosity has face angles of 71°28′–108°32′. These examples of Slichter's may be contrasted with the closely similar porosities of 30.19 per cent and 39.54 per cent afforded, respectively, by Cases 5 and 2, in neither of which is the unit cell a true regular rhombohedron of equal faces.

[15] A rhombohedron each of whose faces is the "simple" rhomb of edge $2R$ already defined; it is the unit element or unit cell of Cases 3 and 6.

[16] Slichter, *op. cit.*, p. 312.

This conception of an infinite series of possible "intermediate" rhombohedrons, each with a given porosity, as entertained by Slichter, helps undoubtedly to emphasize the inescapable effect of packing on porosity. But it would seem that such intermediate rhombohedral arrangements are, in the first place, less likely actually to occur in a natural assemblage of solid units; moreover, they seem more arbitrary, less varied, and therefore less general, less easy of visualization, and less enlightening than are the six cases here presented, which are based on the simple geometrical relationships shown in Figure 2 and in Table I, and are analogous to well-known simple crystal forms; and finally, complete and exclusive commitment to this conception of a "general rhombohedron" leads to various real errors.[17] Therefore, Slichter's intermediate packings and his "general rhombohedron" will not be considered further in connection with porosity. The simplicity and symmetry of the six cases herein entertained becomes especially evident in connection with the analysis of permeability on later pages; on the other hand, the range of variation embraced by these six schemes is evidenced by the strikingly different shapes of the unit voids, now to be discussed.

The Unit Voids

The voids in a mass of material may be of two kinds: isolate or conjugate. The individual vesicles in an otherwise dense volcanic glass and the space occupied by the gas- or liquid-inclusions in a single crystal afford good examples of the isolate void. The intercommunicating and ramifying openings between the fragments of a

[17] For example, this conception provides only for unit cells (or "unit elements of volume," as Slichter called them) all of whose faces are identical. This not only contradicts the relationship shown for tightest rhombohedral packing in Slichter's Fig. 55, p. 307, where square and rhombohedral arrangements occur together, but also would exclude the very real and simple packings of Cases 2-and-4 and 5 here described. Undue adherence to this conception of a universal *rhombohedral* packing must also evidently be accountable for Slichter's assertion (*op. cit.*, p. 307) that if the spheres are not in tightest packing each will touch only *six* neighbors (cf. Cases 2 and 4 where eight neighbors, and Case 5 where ten neighbors actually touch); and may possibly be partly responsible for his repeated reference to the triangular or approximately triangular cross section of the ducts (or "pores," as he called them) that pass through the various assemblages and determine their permeability—whereas in fact, as shown hereafter, the triangle by no means typifies the cross section of these ducts.

talus heap or between the grains of unconsolidated sand are conjugate voids.

It is obvious that all the voids present in any assemblage of discrete or uncemented spheres must be of the interconnecting or conjugate type. The anastomotic or sponge-like composite void present in any considerable aggregation of spheres is therefore likely to be too complex for easy analysis. But such a composite entity may conveniently be visualized in its elemental units in consequence of the fact that the curvature of the sphere surfaces occasions alternate constrictions and enlargements of the intersphere openings; at the narrowest or most constricted places, which may be designated *throats,* imaginary transverse planes may be passed so that they will separate adjacent enlargements—these planes may be called *throat-planes;* and when each enlargement has thus been cut off by throat-planes from all its several immediately neighboring enlargements, we shall have produced what may be termed the *unit void.* In a chance aggregation of spheres, the unit voids are of variable shape, size, attitude, and distribution. But in simple systematic packing, of whatever particular type, the unit void is a three-dimensional shape bounded partly by (eight) spherical and partly by (six) plane surfaces. The orderly repetition of identical spheres brings about the orderly repetition of identical unit voids (or identical combinations of unit voids); and the (imaginary) throat-planes that separate the unit voids from one another have a fixed and simple geometrical relationship to themselves and to the body as a whole. For example, in systematic packing, the throat-planes invariably stand perpendicular to the sphere surfaces that limit them—i.e., if extended, the planes pass through the centers of these spheres and thus become continuous with the throat-planes for all other unit voids lying between spheres of the same pair of adjacent layers. Stated otherwise, planes passed through the centers of neighboring spheres are invariably throat-planes for the intersphere voids. These throat-planes bound not only the unit voids but the unit cells also; that is, as has already been stated on an earlier page, the unit void in systematic packing is the void contained within the unit cell.

The unit void, when considered as a geometrical entity, must obviously be closed in on all sides; that is to say, in every direction it must be limited either by the converging cuspoid surfaces of tangent spheres or by the converging walls of a throat and finally a throat-plane. It is therefore impossible to secure such a closed-in or surrounded unit void in a single layer of spheres; the spheres of two adjacent layers are required. It is for the same reason that the unit cell always includes parts of spheres of two layers.

In Figure 6 are shown the unit voids for each of the six cases; the graduated shading of the curved areas readily distinguishes them from the white or uniformly stippled throat-planes. The void for each case is shown in perspective from exactly the same angle of view and to the same scale as that for the corresponding unit cells of Figure 5 and the corresponding sphere assemblages of Figure 3. The voids shown in Figure 6, in other words, are what would be revealed if the portions of spheres (trigonal sphero-pyramids) shown in the unit cells of Figure 5 were removed, just as the unit cells shown in Figure 5 represent what would result from Figure 3 by passing planes through the adjacent sphere-centers and removing the outer portions of spheres so cut away.

The unit void of Case 1, lying between eight spheres in cubic arrangement, shows eight spherical triangular surfaces and six throat-plane surfaces that are concave-squares. It may be called a *concave-octahedron*. Its volume is 3.81 R^3, which is 47.64 per cent of the volume $8R^3$ of the cubic body that contains it.

In Case 2 and the identical Case 4, the unit void is bounded by eight spherical triangular surfaces, four concave-square throat-planes and two throat-planes each having the shape of two concave-triangles point to point. The void is shown in Figure 6, Case 4, oriented as if standing on the orthorhombic base, and in Fig. 6, Case 2, as if lying on one face of an orthorhombic unit prism. Its volume accounts for the 39.54 per cent porosity of this orthorhombic packing.

The unit void of tetragonal-sphenoidal packing, Case 5, is a peculiar double-wedge-shaped form bounded by four large and four small spherical triangles, two concave-rectangle throat-planes, and four throat-planes of the double-concave-triangle shape. Its volume

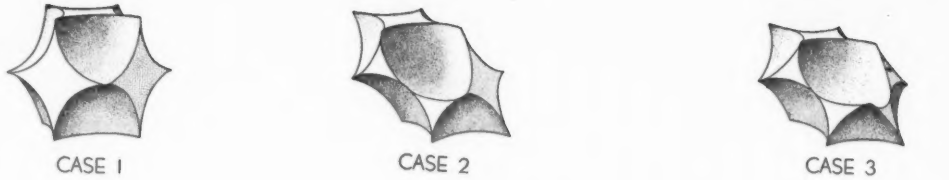

FIG. 6.—Unit voids of the six cases. Same orientation and scale as in Figures 3 and 5

accounts for the 30.19 per cent porosity of tetragonal-sphenoidal packing.

The unit void of Case 6 presents rather unexpected complexity. The simplest and the smallest void that can be formed among uniform spheres is that lying between four spheres in tetrahedral arrangement. Such a void is bounded by four concave-spherical surfaces having the shape of spherical triangles and by four throat-planes of concave-triangular outline. Collectively, these eight bounding surfaces have the angular relationships suggestive of an octahedron. But since the spherical surfaces are the dominant ones while the throat-planes are incidental, this type of void may be conveniently designated a *concave-tetrahedron;* this name is suggestive also of its four bounding spheres. A true regular tetrahedron formed by passing planes through the centers of the four spheres so involved has a volume of approximately $0.94R^3$; the portions of the four spheres included in such a tetrahedron have an aggregate volume of approximately $0.73R^3$. Therefore the void in this tetrahedron has a volume of about $0.21R^3$, and represents a porosity for the tetrahedral body of 22.00 per cent.

Now, since the building-up of such a tetrahedral arrangement of four spheres starts with three spheres of one layer in the simple rhombic arrangement and in the second layer a fourth sphere placed in the three-sided hopper between the other three, this arrangement would appear to be identical with that by which Case 6 is produced (cf. Fig. 2). Therefore, one would, at first thought, conclude that the unit void of Case 6 would be the concave-tetrahedron just described and that the porosity in Case 6 would be the tetrahedral porosity of 22.00 per cent just mentioned. But one here comes upon a somewhat unexpected relationship. If one looks down upon a layer of spheres in the simple rhombic arrangement (see Fig. 7), one sees the three-sided hopper lying between each three spheres in equilateral-triangular arrangement, and the eye instinctively conveys the expectation that if a sphere be placed in each such hopper, the second layer of spheres will be thus filled out. But one finds that it is impossible (because there is not enough room) to place spheres of the second layer in *every* such first-layer hopper; each alternate

hopper must be left empty. Those hoppers of the first layer marked "1" in Figure 7 can be occupied by spheres of the second layer, but thereby the hoppers marked "2" will be so covered that spheres will not fit into them and they must be skipped; when spheres are put into the hoppers marked "3," these spheres will just fit (in simple rhombic arrangement) with those occupying the "1" hoppers and the empty "2" hoppers will then be completely surrounded. As a matter of fact, in a layer of simple rhombic arrangement, there are

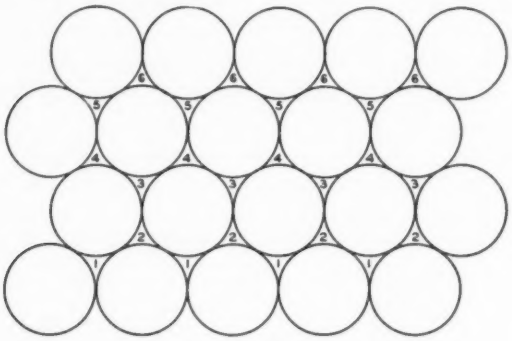

FIG. 7.—Space relations of the simple rhombic layer

twice as many hoppers as spheres, therefore only half these hoppers (each alternate one) can be occupied by spheres of the next-overlying layer.[18]

When each hopper that has had to be left unoccupied becomes fully surrounded by three spheres of the second layer, it becomes a closed-in void, but proves to have larger size and different shape from the concave-tetrahedron discussed shortly above. As a matter of fact, it is a void bounded by parts of six spheres lying at the corners of an octahedron and by eight throat-planes of concave-triangular shape and coincident with the octahedron faces; the six dominant curved surfaces of this void are so arranged as to justify for it the name *concave-cube*. It has a volume of about $1.05R^3$, or

[18] That is, if all the odd-numbered hoppers of Fig. 7 are occupied, all the even-numbered hoppers will remain empty; and, similarly, if the even-numbered are occupied, there will be no chance to fill the odd-numbered.

slightly more than five times that of the concave-tetrahedron void; and with respect to its containing octahedral cell (which has a volume of $3.77R^3$), it represents a porosity of 27.93 per cent.

In short, although Case 6 might seem to be the simplest, as well as the "tightest," mode of packing, it is in a sense the most complex, for it actually contains two kinds of voids: the concave-tetrahedron and the concave-cube. That this must be true is shown by the fact that in the identical packing of Case 3, built by starting with the square layer, four-sided hoppers are present; these, when closed in by the second layer, become the concave-cube voids, while alternating between these are voids of the concave-tetrahedron shape. It proves that, in both Case 3 and Case 6, for each concave-cube void there are two concave-tetrahedron voids. These relationships are presented in Figure 8, where *B* shows the unit cell of Case 6 (when assembled according to the arrangement indicated in *A*[19]) and *D* shows the corresponding unit void. In *C* of Figure 8 the unit cell is shown separated into two tetrahedra and an octahedron, and in *E* are shown the corresponding concave-tetrahedra and the concave-cube which collectively are equivalent to the full unit void shown in *D*. The concave-cube of *E* is the void element that occupies the alternate unfilled three-sided hoppers of Case 6 and occupies the square hoppers of Case 3. In *C* and *E* of Figure 8, the oblique parallel

[19] Comparison of Fig. 8, *B*, with Case 6 of Fig. 5 indicates that the two unit cells are not identical. This difference does not arise from mere difference in angle from which they are viewed (as is true for the apparent difference between Case 3 and Case 6 in Fig. 5), for these are actually different cells. But each is equally expressive of rhombohedral packing. The difference between the two cells is this: The sphere assemblage of Case 6 as shown in Fig. 3 is built by such stacking of the second layer on the first as puts each sphere of the second layer obliquely upward *to the "northwest"* from the corresponding sphere of the first layer, and the unit cell shown for Case 6 in Fig. 5 corresponds to that arrangement; on the other hand, the unit cell of Fig. 8, *B*, corresponds to a sphere assemblage built by such stacking as puts each sphere of the second layer obliquely upward *to the "southwest"* from the corresponding sphere of the first layer. This difference may be graphically seen by comparing the top-view key to arrangement of Case 6 in Figure 1, *B*, with that in Fig. 8, *A*. Actually, different unit cells result. The cell of Case 6 in Fig. 5 contains two concave-tetrahedrons, the half of one concave-cube and the half of another concave-cube; the cell of Case 6 in Fig. 8, *B*, contains two concave-tetrahedrons and a single whole concave-cube. But both cells come from identically the same packing. The unit cell and the unit void illustrated by Slichter (*op. cit.*, pp. 309 and 310) correspond exactly to those shown here in Figs. 8, *B*, and 8, *D*, respectively.

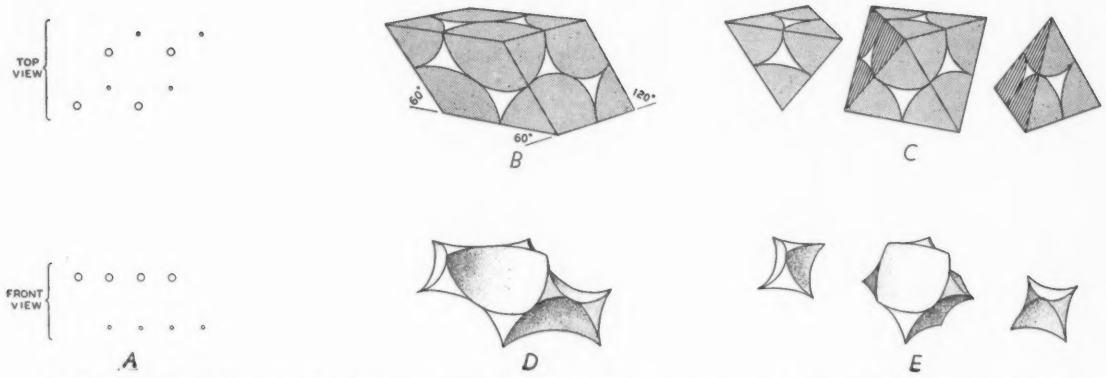

FIG. 8.—Elements of the unit cell and the unit void of Case 6. *A*, key to arrangement; *B*, unit cell; *C*, elements of unit cell; *D*, unit void; *E*, elements of unit void. Same scale as in Figure 3.

hatching indicates the planes along which the unit cell of *B* and the unit void of *D* have been cut and separated into their elements.

The value 25.95 per cent given as the porosity of "tightest" packing of Case 3 and 6 is thus, in fact, a weighted average of the porosity of 22.00 per cent in the two tetrahedral elements and of 27.93 per cent in the octahedral element, which three elements collectively make up the complete rhombohedral unit cell. Each of the two concave-tetrahedron voids accounts for 3.68 per cent and the concave-cube for the balance, or 18.58 per cent of the total of 25.95 per cent porosity of this rhombohedral packing.[20]

This composite nature of voids in "tightest" packing is of especial importance in connection with the factor of permeability as discussed later; and failure, apparently, to recognize this composite character in earlier discussions of the geometry of permeability may account for some of the errors that have crept therein.

TWINNED AND TRIPLED PACKINGS

By reference to Figure 2, it will be seen that in Cases 1, 3, and 4, no other manner of stacking the successive layers is possible than that indicated, if the conditions stipulated in Table I are to be met. But in the other three cases, alternative stackings are possible while still adhering to the conditions imposed. In Case 2, for example, the second layer may be offset to the right with respect to the first layer, as is shown in Figure 2, or it could equally well be offset away from the observer. These alternatives are better shown in Figure 9, *A*, where the projections of the four spheres typical of the first or basal layer are shown in faint circles and are numbered, and the corresponding projection of the No. 1 sphere for the second layer is shown with heavy circle; the relation of the No. 1 faint circle to the heavy circle gives the key to the manner of stacking the layers. Figure 9, *A*, shows the two alternative methods, *I* and *II*, of stacking in Case 2. Figure 9, *B*, shows, analogously, the two alternatives possible in Case 6; and Fig. 9, *C*, shows the three alternative ways of stacking the layers of Case 5.

[20] Stated in another way, the concave-cube void accounts for 71.60 per cent of the total porosity of "tightest" packing, and the two concave-tetrahedrons account for 28.40 per cent, or 14.20 per cent each.

Now, in Case 2, for example, systematic stacking of layers always in accord with alternative *I* will give the same identical internal packing[21] as by stacking always in accord with alternative *II*. But if the second layer is stacked on the first layer with the offset according to alternative *I*, and the third layer stacked on the second with the offset according to alternative *II*, and this offsetting first one

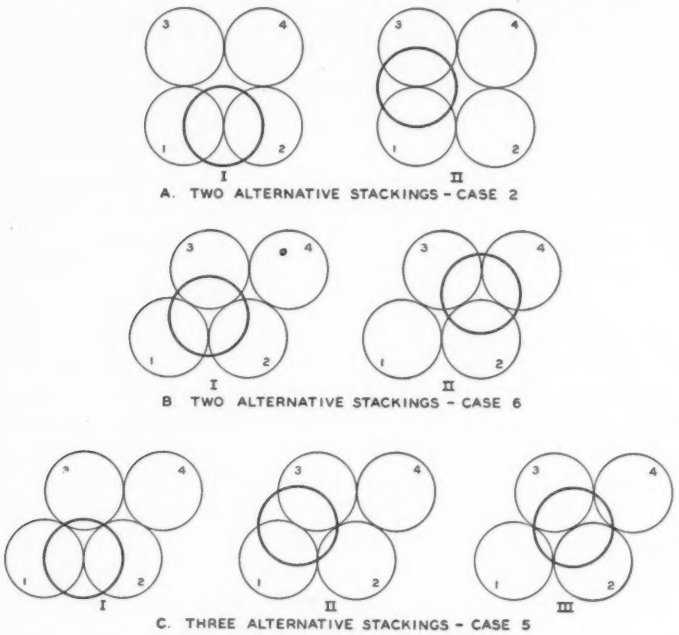

FIG. 9.—Alternative methods of stacking

way and then the other is systematically repeated, then an actually different packing will result. Under such circumstances, the voids between the first and second layers, the third and fourth layers, etc., will be oriented identically; but the voids between the second and third layers, the fourth and fifth layers, etc., will be rotated

[21] The orientation in space, however, of the resulting body or assemblage will differ: that in the one case will be rotated in space 90° in the plane of the base with respect to that in the other body.

90° in the horizontal plane with respect to the voids first mentioned. Except for this difference in orientation, however, all the voids are identical. Such systematic and repetitive alternation of stacking produces a packing and a structure entirely analogous to multiple twinning in crystallography; and the packing thus resulting may therefore properly be designated a *twinned packing*. Since no change in number, shape, or size of the voids results form the twinning, the porosity of the twinned packing is identical with that of the normal or untwinned packing of Case 2; but, as will later be seen, twinning has a marked effect on permeability.

The key to the twinning in Case 2 is shown in Figure 10, *A*, where the position of each tiny circle represents the downward-projected position of the center of the No. 1 sphere for the layer whose number is indicated; thus circle 1 represents the position of a given sphere of the first layer, circle 2 represents the projected position of the corresponding sphere of the second layer, circle 3 of the third layer, etc. The two alternative ways, *I* and *II*, of untwinned stacking are there indicated, and also the twinned stacking that results from stacking first according to *I*, then according to *II*, then according to *I* again, and so on. Similarly in Figure 10, *B*, are shown the two untwinned stackings and the one twinned stacking of Case 6. And in Figure 10, *C*, are shown the key to three untwinned packings—the two options of twinned packing and the one tripled packing of Case 5.

It is noteworthy that although Case 2 and Case 4 give identical normal or untwinned packings, Case 2 permits twinning while Case 4 does not; that is to say, Case 4 cannot be produced in a twinned arrangement by the stacking of *plane layers*, which are the conventional elements of stacking adopted in the present paper. The explanation is that, although the normal packing of Case 2 contains the simple rhombic layer upon which Case 4 is built, the twinned arrangement of Case 2 breaks up the simple rhombic layer into a series of staggered offsets or steps. Similarly, although Case 3 and Case 6 give identical normal packings, Case 6 permits twinning, but Case 3 does not. The reason is analogous to the previous instance: while the normal packing of Case 6 contains the square layer upon which Case 3 is built, the twinned arrangement of Case 6 breaks

up the square layer. From this standpoint of twinning, therefore, there is a difference between Cases 2 and 4 and between Cases 3 and 6, occasioned by their respective differences in space-orientation; it is for this reason, as earlier noted, that the discussion has been con-

FIG. 10.—Key to untwinned, twinned, and tripled packings

sistently carried through on the basis of all six cases of packing, instead of combining Cases 2 and 4 and Cases 3 and 6 and thus treating only four packings.

It is the presence of *two* alternative ways of untwinned stacking in Case 2 and in Case 6 that permits the one manner of twinned

packing in each of those cases; it is the presence of *three* alternative ways of untwinned stacking in Case 5 that gives two different ways of twinned stacking and one[22] way of tripled stacking in that case.

It is obvious that these alternatives involving twinned or tripled packing may be adopted rhythmically and systematically, or, on the contrary, quite erratically. Only the very simplest systematic examples have been considered in the preceding paragraphs. With an option as between alternative *I* and alternative *II*, there is probably no more reason why the selection in natural stacking must always be in the order *I*, *II*, *I*, *II*, *I*, *II*, than why it must always be *I*, *I*, *I*. Therefore, it may be wholly natural for the stacking to involve such chance order as *I*, *II*, *II*, *I*, *II*, *II*, *II*, *I*, *I*, etc. But in this matter, as in all other phases of the present geometrical analysis, it is only the more simple or the limiting cases that can be advantageously considered in any detail. Systematic twinning would, of course, result if the options were alternated every second layer, or every third layer, etc.; but the very simplest arrangement would be alternation with *every* layer, and the only systematic twinnings that are entertained in this paper involve that simplest condition. In Figure 11 are illustrated, as samples of such twinning and tripling, the skeleton outlines of a multiple-twinned unit cell of Case 2 and of a multiple-tripled unit cell of Case 5; the heavy lines designate the twinning planes, and the Roman numeral on each layer indicates, by reference to Figures 9 and 10, the particular alternative manner of stacking adopted for that layer with respect to the layer below.

It is again to be emphasized that, regardless of whether the various alternations that result in twinning are selected rhythmically or irregularly, the *porosity* remains constant. The question, therefore, naturally arises as to why there is anything to be gained in this consideration of twinned and tripled packings if they have no effect upon the porosity. The answer is that, while the number and shape of the voids is not changed by twinning or tripling, their angular relation to one another in space is very definitely altered; and this has an

[22] Stacking in accordance with the order *I*, *II*, *III*, *I*, *II*, *III*, etc., gives exactly the same tripled packing internally as if the order adopted were *I*, *III*, *II*, *I*, *III*, *II*, or were *III*, *II*, *I*, *III*, *II*, *I*.

influence on the stability of the body, as shown just below, and it also has an important effect upon the permeability of the body, as will be graphically seen from subsequent illustrations.

RELATIVE STABILITY OF THE VARIOUS PACKINGS

It is evident that in some of the packings complete stability of arrangement is inherent in the manner of stacking, while in others a lower degree of stability is attained. Any solid mass (e.g., a

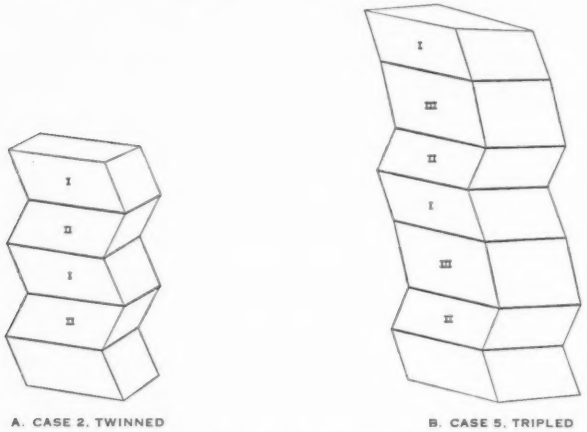

FIG. 11.—Examples of twinned and tripled packings. Orientation of lowest layer of *A* as in Figure 5, Case 2. Orientation of lowest layer of *B* as in Figure 5, Case 5. The thickness of all the layers is equal; the apparent disparity in thickness is due to the different degrees of fore-shortening on the planes lying at different angles to the line of sight.

sphere) acted upon by gravity and supported from below requires at least three points of support in order to attain equilibrium, i.e., acquire a fixed and stable position. When this requirement is applied to spheres in an assemblage, it is found that of the six schemes of packing here entertained, only Cases 3 and 6 are completely stable arrangements (cf. Fig. 2), since only in these two cases do the individual spheres of any layer have three or more points of supporting contact with the spheres of the layer next below (four such points of support in Case 3 and three in Case 6).

In Cases 2 and 5, each sphere receives support from the underlying layer at only two points; with respect to this underlying layer, each sphere is thus "balanced" in the cusp between the two underlying spheres, and except for lateral support afforded by its neighbors in its own layer (which may themselves be no better supported) it can be very easily toppled, to one side or the other of the plane of this cusp, thus falling or rolling down into the nearest hopper-like depression of the underlying layer, whereupon it acquires adequate support but, in so doing, changes the packing from Case 2 to Case 3 in the one instance and from Case 5 to Case 6 in the other instance. Cases 2 and 5 may therefore be regarded as possessing partial but

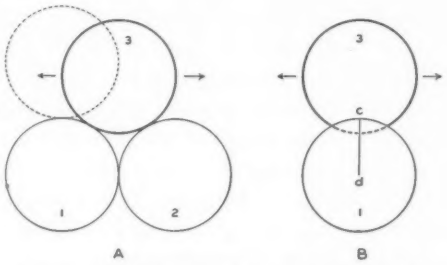

FIG. 12.—Stability relations in "balanced" packings

not complete stability: such packings can easily be toppled in one direction or its opposite (i.e., to either side of the plane of the cusp in which each sphere rests) (see Fig. 12, *B*) but are stable with respect to lateral impulses acting in the plane of the cusp (see Fig. 12, *A*).

In Cases 1 and 4 each sphere is perched even more precariously on the very pinnacle of a single underlying sphere; therefore these packings are still more acutely unstable except as they acquire some fixity and stability through support afforded by their lateral neighbors. When such a perched sphere of Case 1 or Case 4 topples and rolls down on the curved surface of the underlying sphere, it is most likely to roll all the way down into one of the adjacent hoppers that surround the underlying sphere; the toppled sphere thus reaches a position of complete support and stability, but the packing thereby

changes from Case 1 to Case 3 in the one instance or from Case 4 to Case 6 in the other instance. But if, in the course of thus rolling downhill, the toppled sphere falls in any one of four special directions (spaced successively at 90°) in Case 1, or in any one of six special directions (spaced successively at 60°) in Case 4, it will find at least a temporary resting-place of partial stability (i.e., two-point support) in the cusp between the sphere down which it is rolling and the adjacent neighbor of the latter (see Fig. 12, *A*); if each sphere of a given layer of Case 1 were to topple in the same one of these four alternative directions and come to rest in such a cusp, Case 2 packing would result; and if each sphere of a given layer of Case 4 were to topple in the same one of these six alternative directions and come to rest in the appropriate cusp, Case 5 packing would result.

It is thus seen that Case 2 and Case 5 are, respectively, special intermediate packings of partial stability that may be produced in the course of the translation from the unstable Case 1 to the stable Case 3 in the one instance and from the unstable Case 4 to the stable Case 6 in the other instance. These intermediate Cases 2 and 5, while admittedly highly special, have a degree, even if a limited degree, of stability which the "general rhombohedral" arrangement visualized by Slichter would not possess at all. His "general rhombohedron" is, as it were, only a *snapshot* of the spheres at the positions they hold at some instant in the act of rolling down from the one extreme of Case 1 (cubic) packing to the other extreme of Case 3 (rhombohedral) packing; successive such snapshots would represent various of the infinite possible examples of the "general rhombohedron" as the face-angle changes between the limiting values from 90° to 60°; and there would be no quality of stability (not even the precarious perching of Case 1) in any of these general intermediate positions—save in those special instances of partial stability (support in cusps) that constitute Case 2 and Case 5.

In all these cases of unstable or partially stable packing, a given sphere located in the midst of its own layer receives from neighboring spheres in the layer a semblance and quality of support, and in its turn reciprocates the same manner of support to these neighbors; but this mutual "support" is of a spurious kind, and unless at

the eventual margins of the layer there is some fixed and stable limit or container wall, there will be everywhere (except in the bottommost layer) a tendency to topple, and therefore no true stability to the body.

There are other ways of expressing the relative stability of the various packings besides this mere consideration of the tendency to topple. For instance, a lone sphere in a given layer (except the first layer) of Case 1 may start to topple in *any* direction from its perched position and will roll down a considerable distance before coming to stable rest in any one of the four equally near hoppers of the underlying layer. A similar lone sphere in a given layer of Case 4, although likewise perched on a pinnacle and capable of toppling in any direction, will roll down *less far* before finding complete support and will have *six* equally near hoppers from which to choose its final position of stable support; moreover, the lateral stability afforded by the four-point contact in the horizontal square layer of Case 1 is less than the lateral stability contributed by six-point contact in the horizontal simple rhombic layer of Case 4. For these reasons, therefore, the stability of Case 4 is higher than that of Case 1. By corresponding analysis it will be found that the stability of the various packings[23] increases in the following order: Case 1, Case 4, Case 2, Case 5, Case 6, Case 3.

Still other ways of indicating or measuring the relative stabilities of the several packings are these: Of any two packings, that one is the more stable

a) Which has the lower porosity, and therefore the smaller volume and the higher density;

b) In which each sphere touches the larger number of neighbors;

c) In which a larger number of the tangent neighbors are in the underlying layer;

d) Which (for a given number of spheres) has, as a body, the lower center of gravity; i.e., the vertical spacing between layers is less;

e) In which each sphere occupies a position having lesser potential energy, and thus the potential energy of the body as a whole is less.

[23] For discussion of the stability of haphazard packing, see pp. 842–43.

The interdependence of these various factors is too obvious to need elaboration. But the reasons why identical packings, e.g., Cases 2 and 4, and Cases 3 and 6, have different stabilities need a word of explanation. With respect to Cases 2 and 4, reference to Table I shows that the spacing between layers is greater for Case 4 than for Case 2. Therefore, although these two Cases are on an equal footing with respect to the factors porosity and number of tangent neighbors, the arrangement in Case 2 has two of these tangent neighbors in the underlying layer (as against only one in Case 4), it yields a lower center of gravity, and its spheres have a smaller potential energy than the arrangement of Case 4. The greater stability of Case 2 than of Case 4, as already cited, is thus confirmed. For the same basic reasons, i.e., (*c*), (*d*) and (*e*) of the foregoing list, Case 3 is more stable than the identical packing differently oriented in Case 6.[24]

When a packing of one degree of stability is subjected to jarring or to jigging, there is a tendency for it to change or translate into some packing of higher degree of stability. If the lateral support is adequate, this tendency is nullified. But the lateral support acts not only to prevent each individual from toppling and rolling downhill—it operates to the same end also in another way, as follows: Attainment of a more stable packing means attainment of a lower center of gravity of the body, and this can be achieved only by a decrease in height and an appropriate increase in lateral dimensions of the body; adequate support from the sides prevents this lateral spreading of the body and therefore maintains the initial packing, no matter how unstable it may otherwise have been. Obviously, the degree of rigidity and adequacy of the lateral support will determine the extent to which the tendency of the body to "settle" into a closer, more stable packing will find realization.

With respect to Cases 3 and 6, the facts that Case 3 is of higher stability than Case 6 and that a packing of lower stability tends to translate into one of higher stability should imply that Case 6

[24] This difference of stability of the same packing in different orientations may be likened to the difference in stability of a box, depending on whether it is laid on its broad side or stood on its small end.

should try to become reoriented by internal readjustment into Case 3. But in this particular pair of cases, an additional factor discourages such translation besides the factor of lateral support already mentioned. In Case 6 as well as in Case 3, each sphere is already "stably fixed" in position by at least three-point support in a hopper of the underlying layer. In order that the orientation of Case 6 may change so as to attain the lower center of gravity and the four-point support from below that characterize the more stable Case 3, it would be necessary for the assemblage to pass through intermediate arrangements of greater porosity than the common least porosity of Cases 3 and 6; this would require a temporary *lifting* of each sphere from the Case 6 hopper in which it lay, and a temporary raising of the center of gravity of the body as a whole—this certainly would not result from a simple jarring wherein the downward pull of gravity is the maximum force acting, and it would probably only rarely happen as a consequence of jigging (alternate up and down forces), namely, when the amplitude of the jigging is exactly right to bring about this special outcome and not instead embarrass the whole scheme by causing local regions of disordered packing.

The probability of translation of Case 6 into the more stable orientation but identical packing of Case 3 therefore seems slight. Since, as will later appear, stacking is more likely to start on the simple rhombic than on the square layer as a base, Case 6 is more likely to form initially than is Case 3; and, when formed it seems probable that, although inherently less stable than Case 3, Case 6 will remain permanent, save under strong deforming forces or under just the right rhythm and intensity of jigging.

The effect of twinning or tripling is, in Cases 2 and 5, to increase the stability of the resulting assemblage. It will be recalled (see Fig. 12) that the stability of these two cases is low with respect to any disturbing impetus *across* the plane of the supporting cusp in which each sphere lies, while the stability is high with respect to impetus in the plane of the cusp. Twinning in these two cases brings about change from layer to layer in the orientation of the supporting cusps; and this has the effect of "locking" the spheres

in place and thus stiffening[25] the entire structure. This increased degree of stability comes by the addition of steadying from the layer above to the support from the layer below; but the gain in stability thus attained is not equal to that afforded by complete hopper support from below, as in Cases 3 and 6. Obviously, the increased stability due to twinning is greatest when *every* layer boundary is a twinning plane, and decreases as the interval between twinning planes widens. The twinning in Case 2 with 90° rotation is more effective in imparting stability than that in Case 5, where the rotation is 60°. Twinning in Case 6 produces no steadying not already present, and therefore no increase in the already high degree of stability.

It is to be noted that this entire discussion of stability is postulated on the assumptions that the downward pull of gravity is the only force of consequential importance at work on the system after the assemblage has been initially attained, and that the body remains (as a unit) untilted from the attitude it assumed when horizontal layers were piled to produce it. Lateral transporting or deforming forces, of any appreciable magnitude, or tilting of the base upon which the body rests, could bring about important modifications of stability relationships.

It is the difference in stability of different packings (whether systematic or otherwise) and the consequent tendency toward translation from a less stable to a more stable and less porous packing that give rise to the important and interesting phenomenon of dilatation[26] as exemplified in sands, slimes, and other similar assemblages.

The indicated great instability in the packings of Cases 2 and 5

[25] The situation may be likened to the piling of round sticks of firewood: it is impossible to build up a pile with vertical ends if the sticks are all laid parallel; but if at the ends an alternately crisscross arrangement or cobwork is built, a stable rectangular pile can be formed. Such alternate crisscrossing corresponds to twinning.

[26] Cf. W. J. Mead, "The Geologic Rôle of Dilatency," *Jour. Geol.*, Vol. XXXIII (1925), pp. 685–98. It may be noted here in passing that the customary definition of "dilatation" as that class of strain which involves volume changes but no angular deformation does not, after all, strictly apply to these translations from one packing to another packing of particles or grains, for, whenever such translation involves a change of volume for the body so affected, there is also a change in the angular relations between the grains.

and the even greater instability in those of Cases 1 and 4, and the related fact that when these sensitively perched or balanced packings topple they invariably *tend* to translate into the one really stable rhombohedral packing of Case 3 or 6, collectively afford some measure of justification for Slichter's virtual confinement of consideration to the "tightest" rhombohedral packing and for his development of a theory of permeability based directly on that simplest rhombohedral arrangement. But such extreme limitation and simplification of arrangement is not vindicated by natural packings. Instead, there is adequate justification for giving consideration to each of the four independent modes (six cases) of simple systematic packing treated in this paper.

But in addition, there are, of course, various other schemes of arrangement which, while yet geometrically systematic and orderly, are more arbitrary and less simple than the six cases principally treated herein. The most readily visualized examples are arrangements in which the angle and spacing of the sphere-centers differ in some systematic manner and degree from the angle and spacing in the six simplest cases. Among such are those instances typified by the "general rhombohedron" of Slichter. Furthermore, there are those countless varieties of assembly in which no orderly or systematic repetitive arrangement can be discovered; all these may be grouped together under the general term *chaotic* packing. All packings that do not correspond exactly to some one or other of the six simple systematic cases are here arbitrarily classed together as *haphazard* packing; since in these packings there may be present lower degrees or more complicated kinds of orderliness than characterize the six simple cases, they cannot be grouped as a whole under such a term as disorderly or chaotic packings; nevertheless, disorderly arrangement generally dominates in haphazard packing. The stability of haphazard packing is more effectively discussed later (see pp. 842–43). Finally, a given considerable assemblage of spheres is likely to vary from place to place as to manner of arrangement of the spheres—at certain places one of the six simple cases may be developed, at other places another case, while elsewhere various phases of haphazard arrangement exist, some of them entirely chaotic while others show some degree of orderliness. Any such as-

semblage which is not arranged throughout according to a single one of the six simple cases will be designated herein as *chance* packing. The stability of chance packing is the resultant of the stability of each of its component varieties of packing.

In any sphere assemblage, that portion which is arranged according to some one of the six simple cases is called a *colony* of that case. The minimum colony embraces the eight spheres necessary to enclose the unit void; but there is no maximum limit, since the entire assemblage—no matter how large—may consist of a single colony. Most commonly, however, the average natural aggregation or assemblage is composite, containing one or more colonies separated or surrounded by regions of haphazard packing; when more than one colony is present, all the colonies may represent a single case or, on the other hand, more than one case may be represented. There is a tendency for a given colony to be of compact chunky shape and more nearly equidimensional than strikingly long or thin; this probably is an expression of the magnitude of the tendency for a given simple case of packing to continue growing upward once it gets started.

PROBABILITY FACTORS FOR THE VARIOUS PACKINGS

Building of the Layer

Under the ideal conditions postulated in the present geometrical analysis, the chance that a given case of packing will be initially produced, instead of any of the other cases, will depend on a variety of factors. One of the most obvious and important of these factors is the relative stability of packing of that particular case. Other things being equal, that case is most likely to result which has the highest degree of stability. As shown in the preceding section, this would point to Case 3 as the most probable packing to be formed. But the question of stability of the completed body concerns primarily the manner in which the layers are stacked upon one another; or, stated in another way, stability is more concerned with the vertical than with the horizontal relationships between the spheres. If adequate enlightenment is to be had regarding the horizontal relationships, it is therefore necessary to inquire beyond the limits that relate to stability by examining the manner of forming the layer and

the relative chances of formation of the square layer and of the simple rhombic layer.

It may first be assumed that the accumulation of spheres is always initiated on a horizontal plane starting surface. Spheres brought to that surface and accumulating upon it as a first layer may come to rest either in positions of mutual contact or (at least for a time) in separated positions without contact. For the present, attention will be directed to the first of these alternatives, since only through mutual contact of spheres can be attained the ideal row and the ideal layer with which we have hitherto been dealing. Later, the second alternative will be considered.

Any two uniform spheres lying in mutual contact but aloof from others upon the horizontal starting surface afford two similar cuspate bays or re-entrants that determine two special positions along an otherwise monotonous convex "shore line" of the pair. A third sphere, transported along the starting plane into contact with the pair, is more likely to come to rest in one of the bays where it contacts with both spheres of the pair than at some chance place along the convex shore line where it contacts with only one; for with but one point of contact, its retention of that position is unlikely—the slightest recurrent impulse of the transporting force which brought it will tend to push or roll it along from this single-point contact until either it deserts the pair or it gets into one of the bays and becomes tangent to both of the two earlier spheres, in which latter position it takes on a quality of relative fixity. The placing of the third sphere in one of the cuspoid bays formed by the first two spheres now produces a 60° equilateral arrangement, and thus at once is determined the essential pattern of the simple rhombic layer; such placing also increases from two to three the number of re-entrant bays available for the reception of additional spheres. Each such succeeding sphere brought to this group upon the starting plane has the same greater tendency to take up its place in one of the bays than to make contact with only one of its predecessors. As the group thus increases, some of the new-coming spheres may come into contact not only with two but with three or even four of the spheres already assembled in 60° arrangement; also, as the group grows, the number of bays increases so that the chance of acquiring

additional spheres in this orderly arrangement becomes steadily greater.

It thus results that in the accumulation of spheres on a horizontal starting plane, there is a strong tendency to produce an orderly layer of the simple rhombic pattern. Moreover, the fact that in such an arrangement each interior sphere is "locked," as it were, in the six embayments between six neighbors gives to this layer a quality of steadiness that tends to preserve and perpetuate this pattern. As contrasted with the production of the first layer in this simple rhombic pattern, the likelihood that a square layer or any general rhombic placing other than the 60° or "simple" rhombic pattern would either get started or continue to grow by systematic repetition is a relatively small chance. These generalizations can be readily confirmed by pouring out uniform spheres upon a table.

For the time being it may be assumed that the simple rhombic pattern grows to any desired dimensions as the first layer—that the first layer, in fact, contains only this pattern in uniform orientation throughout.

It might be natural to assume that, since the simple rhombic pattern is thus strongly favored for the first layer, it will be at least equally favored for the second and succeeding layers. Such an assumption, however, cannot be granted offhand. Some circumstances tend to increase the probability that whatever type of arrangement is incorporated in the first layer will be repeated in the second and succeeding layers; but other factors tend, on the contrary, to lessen this probability. The resultant of these opposing tendencies cannot always be forecast.

It is evident that the second (and each succeeding) layer cannot accumulate on the smooth horizontal plane postulated as the starting surface for the first layer; instead, it accumulates on the upper surface of the first layer—a surface of alternating hummocks and hoppers. The hummocks and the hoppers are each spaced at systematic intervals determined by the pattern of the layer just completed. If the minimum horizontal intervals between hopper-centers are in all cases exactly $2R$ (as they are in the square layer), then a sphere can settle into *every* hopper and thus form an orderly second layer (this would produce the packing of Case 3). But if the

intervals between hopper-centers are different from $2R$, such an outcome cannot be attained.

As already noted, the top of a layer in simple rhombic arrangement exposes just twice as many hoppers as can be occupied by spheres of the second layer even when this second layer is itself in most compact arrangement, i.e., in simple rhombic arrangement. (It is this 2:1 ratio that permits the twinning of Case 6.) All these hoppers are identical in shape; so any one hopper is as likely to be selected for the repose of a sphere as is any of its neighbors.[27] Yet, if the second-layer spheres do not in every instance faithfully select *alternate* hoppers (leaving the intervening alternates empty [cf. Fig. 7]), the orderliness of the second layer will be broken, and this will happen even though the first layer may be ideally arranged throughout according to the simple rhombic pattern. If the orderliness of the second layer is broken, it will not be repaired in the third layer (or in succeeding layers except by faintest chance.) Instead, the disorder will be perpetuated up into the higher layers and has more than an even chance of becoming further aggravated; for a layer of any given systematic packing ordinarily cannot be larger than the layer of the same packing on which it rests (i.e., without some support from the exterior there can be no overhang), and, if it cannot be larger and there can be no overhang, it follows that each succeeding layer will be smaller than the one below. Therefore, upward propagation from any given area of the simple rhombic layer, unless aided by just the right shape of support from the sides, tends to produce an upward-converging pyramidal "colony" of Case 6 packing; and between such colonies of decreasing size upward, the proportion of haphazard packing increases.

Granted, then, that the simple rhombic arrangement is the one most probable to be adopted in the first layer, the chance that this same arrangement will be continued in higher layers is dependent on whether each sphere settles in the "right" or in the "wrong" hopper. Since the right and wrong hoppers are equal in shape and number, the chance that the right hopper will be chosen would appear to be one out of two—that is, an even chance. But the follow-

[27] See p. 832 for partial qualification of this statement.

ing circumstances somewhat increase the chance that the right hopper will be selected in preference to the wrong one.

In the first place, if one sphere of the second layer has, quite by chance, already chosen a given hopper for its location, any other second-layer sphere brought along into contact with the first sphere is somewhat more likely (through lowered velocity due to impact) to come to rest in contact with this first sphere than to move away and come to final rest elsewhere; and if the second sphere does thus come to rest in contact with the first, any one of six hoppers located around the periphery of the first is available that will still keep the first and second spheres in contact, and any one of these six hoppers available for the second sphere is a "right" hopper to produce the simple rhombic layer. The same tendency will apply to all later spheres of the same layer.

In the second place, if the spheres are brought to the place of accumulation by being carried along the surface of the underlying layer from some source direction—as is most probable—then, although the *shape* of the hoppers on the top of a simple rhombic layer is identical, the hoppers are not equally favored in orientation to receive and hold spheres for a new layer. If the diagram in Figure 7 be regarded as a view of the upper surface of a simple rhombic layer, it will be seen that, with respect to spheres brought over this layer in a direction from the top toward the bottom of the illustration (let us call this from north to south), all those hoppers bearing even numbers are oriented apex-on toward the on-coming spheres, while the odd-numbered hoppers are oriented base-on. For the even-numbered hoppers, a southward-moving sphere is invited to roll down the south-sloping trough of the north apex of the hopper, but then finds itself brought squarely against the steeper south face of the hopper; from this position it will be dislodged only by being lifted up and to the south over this steep south face of the hopper, or else by being moved, by a reduced component of the transporting force, diagonally to one side or the other into an odd-numbered hopper. Each odd-numbered hopper, however, is shielded on the immediate north by the high summit of a sphere of its own layer; if the transporting force is strong enough to lift the on-coming sphere over this summit, it will probably have sufficient strength

(i.e., velocity) to shoot the sphere past the odd-numbered hopper that lies just south of the summit and on into the less shielded and more inviting even-numbered hopper lying next to the south. And any sphere which, despite this, does land in an odd-numbered hopper would need only to be rolled up the gently inclined trough on the south apex of that hopper and be thus carried over the low "pass" or "divide" into the even-numbered hopper lying next south; this removal of a sphere from an odd-numbered hopper would, however, be to some degree retarded by the protected position of the lower part of the sphere in the "lee" of the summit on the north, but such protection is probably relatively slight. It therefore follows that southward-moving spheres are more likely to come to rest in the even-numbered than in the odd-numbered hoppers. On the other hand, spheres moving from the south will preferably come to rest in odd-numbered hoppers. It is evident, then, that spheres coming from a given direction perpendicular to the east-west row will tend consistently to take up hoppers that are all "right" relative to one another, instead of part right and part wrong. This proves true for a direction of transportation that is *perpendicular* to *any* of the three sets of rows of the simple rhombic pattern. But for a given direction of transportation *parallel* to any of the three sets of rows, all the hoppers offer an equivalent orientation to the on-coming spheres; therefore there is no tendency toward consistent selection, and some spheres may choose "right" hoppers while others choose "wrong" ones. The final outcome is that for any *random* direction of transportation there is a better than even chance that the "right" hoppers will be occupied. But this will not prevent the occupation of some "wrong" hoppers, which will therefore tend to interrupt the upward propagation of Case 6 packing.

The two tendencies just discussed—the one due to arresting of spheres by impact, the other dependent on geometry of the layer surface—are probably fortified by one another rather than being each reduced by the operation of the other. Therefore there would seem to be considerably better than an even chance that the second and succeeding layers will propagate upward the simple rhombic arrangement assumed by the first layer. The exact deviation from the 50:50 chance cannot be computed, since it depends upon variable

factors, such as the manner, direction, and velocity of transportation of the spheres, the specific gravity and viscosity of the transporting medium, the specific gravity and elasticity (tendency to rebound upon impact) of the composing material, and other influences. But it is certain that only by the rarest combination of favorable circumstances will the simple rhombic layer be repeated absolutely without break throughout the course of upward stacking. If it were so continued faithfully in layer after layer, the normal packing of Case 6 would result throughout. Interruptions of its continuation will therefore produce more or less restricted colonies of Case 6 packing between which will lie zones of a different, and probably haphazard, packing.

The foregoing discussion is postulated, as will be recalled, on the assumption that the accumulating spheres of the first layer all initially come to rest in contact with one another. There remains to be considered the case in which the early spheres deposited on the starting plane take up positions separated from one another. Each such isolated sphere may become the nucleus for growth of additional spheres; and, in accordance with what has already been seen, each of these regions of growth is most likely to adopt the 60° arrangement or simple rhombic pattern. Thus there develops a number of growing layer-colonies most of which will be of 60° pattern. As these grow, they will eventually reach the boundaries of neighboring colonies that are likewise in process of growing. But unless the spacing of the initial isolated spheres or nuclei happened to be exactly correct, and unless also the orientation of the 60° axes of all the colonies chanced from the start to be identical, there will be an interruption of the orderly arrangement where one colony comes into contact and interferes with another. The most probable result, therefore, is that layer-colonies of 60° pattern will be separated by zones of different arrangement, generally haphazard but occasionally, possibly by luck, assuming an arrangement that permits development of one of the other systematic packings.

It is to be noted that the development of regions of haphazard arrangement interrupts continuity of the plane layer from there upward, for the reason that the hoppers on the upper side of the haphazard portions are of unequal diameters; therefore uniform

spheres will settle into them to different depths and thus get "out of layer." This will naturally encourage upward propagation of haphazard packing.

The building-up of an assemblage of spheres is, in the main, therefore, a conflict between the tendency to form colonies of Case 6 packing and the tendency to produce haphazard packing. Attention may well be directed for a moment to the latter tendency and its consequences. Haphazard packing is obviously different from place to place: Here it may be as far from any systematic and repetitive arrangement as can be conceived, while not far away it may come to be almost systematic—and a little farther on may indeed become an orderly, repetitive arrangement of one kind or another. If inspection were started in the opposite direction, one would say that a systematic colony had become at its margin slightly disarranged, and that this disorder, increasing, finally became the most haphazard conceivable. But the change from entirely systematic to extremely disordered may also take place, not gradually, but abruptly; and this may happen whether the actual order of accumulation is first systematic and then haphazard or the reverse. It follows, therefore, that, in the accumulation of a (constantly changing) haphazard arrangement, the distance- and angle-relations may at any place become such that systematic arrangement of one kind or another will then and there click into being and, once thus started, may continue to propagate itself as if only it had prevailed from the start of the assemblage.

It is probably in this way, in considerable measure, that the packings of all the other cases than Case 6 come into actual existence.[28] Most of these other packings, being highly unstable, are destined to form but small colonies. But Case 3, once started at a given layer, vigorously tends to maintain its growth; indeed, given two equal expanses of square layer and simple rhombic layer, Case 3 will grow more faithfully than Case 6, not only because its hoppers are wider and deeper (and thus more inviting and retentive, as well as making for greater stability), but also because, since every hopper on the square layer is a "right" hopper, there is no chance, as in Case 6,

[28] The other manner of producing these cases is through the influence of the shape of the container walls, as later discussed.

of a single sphere starting a bad zone of disorder by innocently getting into a "wrong" hopper.

If simple systematic accumulation could be imagined under conditions in which gravity had no other competition or opposition than that afforded by the support of a horizontal starting plane, the outcome in every instance would be tightest rhombohedral packing of Case 3 orientation, since that gives to the assemblage the lowest possible center of gravity. But this would demand formation of the square layer throughout.

Among the inherent geometrical qualities of the sphere are those which greatly favor the 60° or simple rhombic pattern over the square pattern. Moreover, Case 3 has low chance of developing unless the starting surface is a plane; whereas Case 6 is much less subject to this limitation. Consequently, in the contest between Case 3 packing preferred by gravity and Case 6 packing preferred by geometry, geometry generally wins, gravity contenting itself with securing that packing which gives the *next lower* center of gravity to that of Case 3.

By integrating all these probabilities of the character of the first layer with the probabilities as to manner of stacking from that first layer upward, we come to the conclusion that, with a start upon an ideal horizontal plane, as postulated, the resulting assemblage is most likely to consist of three-dimensional colonies of "tightest" Case 6 packing and intervening zones or surrounding shells of haphazard arrangement. The relative sizes and importance of the colonies of Case 6 and of the intervening haphazard zones cannot be predicted and may vary widely in different examples. Simple systematic packings of other kinds than Case 6 may occur, but their colonies are likely to be small and their proportion to the total assemblage likely to be relatively unimportant; Case 3 will always dominate among these.

The accumulation of any given layer in a colony of Case 6 packing is equally as likely to take place in the twinned as the untwinned position with respect to the two (or more) layers next below. Therefore, multiple twinning will almost certainly be plentiful in colonies of a size embracing more than a very few layers. The stabilizing effect of this Case 6 twinning is zero. But the twinning will generally

be repeated at random layer-intervals rather than in orderly repetition; and the ideally systematic alternation in which the boundary between each pair of layers is a twinning plane, specially discussed earlier (see p. 819), will be of rare occurrence and will at best prevail over only a few consecutive layers.

Effects of Other Influences

Without attempt to set down in detail the underlying reasons, the following conclusions may be recorded with regard to the effects which various other conditions have upon the type of first layer and the type of completed packing that are produced.

A plane starting surface that is inclined (instead of horizontal, as hitherto assumed) will minimize the chance of development of initially isolated nuclei or groupings and will favor, instead, the growth of a single group, beginning at the downhill limit of the starting surface. This, in turn, will tend to diminish the number and importance of intervening regions of haphazard arrangement. Moreover, with a good development of simple rhombic pattern attained on the first layer, the chance of duplicating this on the second and succeeding layers would be greater on a somewhat tilted than on a perfectly horizontal layer. Moderate inclination of the starting surface, in fact, favors the production of important colonies of Case 6 packing. If the inclination of the starting surface is too steep, however, the tendency will be decreased to complete one layer before beginning the next, and this will tend to encourage disorderly, rather than systematic, arrangements; if the starting surface is steeper than 45° to the horizontal, it can no longer be regarded as the starting surface but will instead become a lateral limit or container wall.

Rapidity of assembly undoubtedly affects the orderliness of the arrangement. On the whole, order is promoted if the rate of accession of spheres is such that each sphere has assumed its final position before a new sphere comes in contact with it—in other words, if its position is influenced by its predecessors but not by its successors. Rapid dumping is therefore unfavorable. To some extent, however, the impact of successors may help to urge laggard spheres into contact with the orderly assemblage already formed; but too high

velocities, with resulting excessive rebound on impact, will probably tend toward disorder.

The method of laying down the spheres will also affect the orderliness of the assemblage. If the spheres are dropped, like hailstones, from above on the starting surface, each sphere will be more likely to take a chance position with respect to other spheres and thus contribute toward disorder of the first layer than if the spheres are brought to their place of rest by being rolled along the starting surface. And if dropped from above on any layer already accumulated, each sphere will be more likely to settle into the first hopper it reaches, regardless of whether this be a "right" or a "wrong"[29] hopper for the propagation of orderly assemblage; whereas, if the spheres are brought by transportation along the top of the preceding layer, this lateral force will be more likely to bring each new sphere into contact with those already accumulated in its own layer, and this will favor further propagation of whatever orderliness has already been attained. But something will depend on whether the spheres are gently drifted along the surface of the underlying layer or whether they come violently bouncing along over that hummocky surface: Too gentle a transporting force is liable to leave many spheres in the "wrong" hoppers; whereas too vigorous rebounding may likewise promote disorder.

In all that has preceded, it has been assumed that the foundation or starting surface on which the accumulation has been built is a true plane; and all the successive "layers" have been assumed to be plane also, in that the spheres of which the layer consists have their centers in the same plane. But just as haphazard arrangement may cause abandonment of the rectilinear row, so may it cause abandonment of the planar layer. Moreover, unless practical considerations are to be entirely subordinated to the ideal, it is essential to examine briefly into the effects of accumulation on a starting surface that is other than plane, and correspondingly on an already accumulated lot of spheres not lying in a plane layer. Except that the foundation or starting surface is, in general, more controlling than are the lateral limits or side walls of the aggregation, these deviations from ideal plane surfaces have the same kinds of effects on the manner of

[29] Of the simple rhombic layer; in square layer, all hoppers are "right."

packing regardless of whether they are at the bottom or at the sides of the assemblage. Therefore, they will be discussed collectively in the section that immediately follows.

Effect of Container Wall[30]

So far as the foundation or starting surface is concerned, this may not be a smooth plane, as heretofore assumed, but may be of any type and degree of irregularity: intersecting planes presenting upward either concave or convex intersections; curved or warped surfaces, likewise concave or convex upward and of either continuous or interrupted curvature; the surface of a prior accumulation of solid units (spheres or near-spheres) of smaller size, and thus marked everywhere by alternation of small hummocks and hoppers; or the surface of a prior accumulation of larger units (or a mixture of large and small units) with deep depressions (more or less filled) and high projections.

Wherever a portion of such an irregular starting surface consists of a true plane of not too steep declination from the horizontal systematic packing may develop upon this portion in the manner already considered. But whenever this particular plane portion of the starting surface gives way to a plane at a different inclination, this colony of systematic packing will likewise come to an end (except by rarest lucky combination of just the right areas and angles of the planes), and haphazard packing will ensue or will separate the one colony from another colony. And wherever a curved surface terminates the plane surface, either by intersection or tangency, a systematic colony that may have been built on the plane portion inevitably ceases and gives way to haphazard packing.

Irregularities of the starting surface, occasioning alternate depressions and elevations, cause the initial growth of isolated nuclei in the depressions; and even if some of these should have systematic arrangement, it has already been seen that the isolate-nuclei condition is provocative of disorder in the subsequent assembling.

Haphazard areas in the first "layer" (no longer a plane layer) of spheres upon the irregular starting surface promote continued disorder upward except where, here and there, by happy chance com-

[30] This section may be read more easily by frequent reference to the illustrations in Figs. 29–36.

bination of distance and angle, the disorder becomes "absorbed" or "ironed out," and merges into a more or less local and restricted colony of some type of systematic packing.

In general, curvature of the starting surface, whether gentle or sharp, and whether concave or convex upward, is highly inimical to the development and growth of simple systematic packing. And still more generally it may be said that any and all departures of the starting surface from a smooth and continuous plane favor increased proportion of haphazard packing. Nevertheless, the strong inherent tendency toward assembly of spheres in the 60° relationship gives to the Case 6 packing a vigor and aggressiveness that is constantly being revealed here and there. The result of an irregular, as contrasted with a plane, starting surface may therefore be said to be, on the average, the production of smaller (and probably less numerous) colonies of Case 6 packing and the increased development of haphazard packing.

So long as the spheres are supposed to consist of actual matter and to be of greater density than the medium surrounding them, and therefore subject to the downward pull of gravity, the nature of the starting surface, which may be regarded as the bottom wall of the container, thus exerts a profound and inescapable influence on the packing built upon it. The same is not true in just the same degree for the side walls. Nevertheless, as a practical matter, any assemblage of spheres of greater vertical thickness than the depth of the depressions of the starting surface must have some independent lateral limits or walls, unless the assemblage is rapidly to terminate upward as a pyramidal body having the starting surface for a base. Obviously, practical interest resides chiefly in instances where lateral walls or limits must exist.[31]

Plane side walls are more conducive to orderly packing than are curved walls. This is true whatever the inclination of the side walls to the starting surface. But obviously, side planes which meet a plane starting surface at 90° extend a degree of invitation to packings

[31] These limits may be man-made, such as the sides of a tube or box or other receptacle; or, if we are thinking geologically of spherical pebbles or grains of sand, the side limits may be natural, such as cliffs or large boulders, or a sharp steepening of the floor of the sedimentary basin.

of Cases 1, 2, 4, and 5; side planes which meet the starting plane at angles of 60° or 120° are conducive to Case 2 and Case 3 packings; while planes making with the starting plane the angle of slope of the octahedron, 70°32′ or 109°28′, favor development of Case 6.

The angle at which the side walls meet one another is also influential: Intersection at 90° favors the development of the square pattern for the first layer, and thus Cases 1, 2, and 3 would be favored. Intersection of the side walls at 60° with themselves and at 90° with the starting plane would be friendly toward Case 4 and Case 5, while 70°32′ with the starting plane would very strongly favor Case 6 packing.

But under the best of circumstances, unless the extent of these planes (as measured along the intersection with the starting plane) happens to be just right,[32] there will be a cessation of systematic packing at the lateral limits of such planes and perhaps even a propagation of disorder from the sides into their fields. And in any of these instances, if the bottom or starting surface is curved rather than plane, the advantageous influence of plane side walls is lessened, if not entirely lost.

A simple outline of plane side walls is more likely to permit orderly arrangement than is a complex polyhedral form.

If the container sides are curved surfaces instead of planes, imperfect packing will be favored and the colonies of Case 6 packing will tend to be of reduced size. As a rule, the sharper the curvature and the more complex the surface, the greater the disorder will be; conversely, curvature deviating very slightly from a plane may induce a packing that approximates very closely to some one of the simple systematic cases, most commonly to Case 6.

Not infrequently, the "zone" of haphazard packing may be less than $2R$ wide (or thick) and may consist, in fact, merely of a narrow zone along which the margins of neighboring colonies just fail to fit and merge into a single larger colony. In certain other instances, the departure from perfect order will consist only in voids of abnormally large size where a colony comes up to a boundary wall.

Of the multitudinous possible phases of container effect, the illus-

[32] The required values are: $2nR$ for the 90° intersection with the basal plane; $2.31nR$ for the 60° intersection; and $2.12nR$ for the 70°32′ intersection, where n is an integer.

trations in Figures 29–36 (pp. 881–87), give only a general sampling. These figures are direct copies of photographs of a single plane layer of spheres, bounded by container walls of the shapes shown. They thus depict some of the varied influences of the container wall on the arrangement in the first plane layer. But by thinking of any side of the limiting wall as being itself a section of the starting surface, the observer can gain a general and approximate[33] idea of the effect of irregularities of the starting surface on the sphere arrangement as it is built progressively upward therefrom. In the preparation of these figures, the spheres were poured out freely and fortuitously on the plane-glass starting surface, which was tapped gently in order to encourage reasonably compact arrangement, and as many spheres were put into the container as it would accommodate under these circumstances in a single layer. More vigorous jarring might make it possible to add one or two more spheres in each figure—especially those in which there is poorest packing; but in the main, and with allowances for the higher degree of orderliness occasioned by the immediate presence of an ideal starting plane, these relations as they stand are fairly typical of pretty well compacted natural assemblages.

It is highly important to note that haphazard packing, once established in the midst of an assemblage within a rigid container, cannot, as a rule, be completely eliminated and translated wholly into one or more systematic types of packing by any amount of mere jarring. Once the spheres attain certain relations to their neighbors, they become "locked" or "keyed" in these relations, like stones in an arch, by the very presence of these neighbors, which themselves are likewise keyed in place. Openings more than large enough in every direction to accommodate a whole sphere may, by the chance manner of sphere accumulation,[34] be left entirely unfilled, and are protected thus indefinitely by the "arching" of the surrounding spheres.

Only by drastic rearrangement, through violent jigging or some other action which temporarily *lifts* many of the spheres by a con-

[33] Approximate only, because these figures show only spheres whose centers lie in the plane of the paper; they thus show perched positions and the settling into plane cusps, but not settling into three-dimensional hoppers.

[34] Especially when there is impetuous dumping.

siderable fraction of their radius, may the voids in the haphazard portions be reduced to minimum dimensions. And even when such maximum compacting has been achieved, systematic packing throughout will not have been attained *unless the container has identically the same shape, and a very special exact relation to the size, of the unit cell of the kind of systematic packing involved.* This is an extremely unlikely condition to be encountered in a container; in consequence, systematic packing is practically never found *throughout* a container.

It follows, therefore, that, despite the inherently unstable nature of haphazard packing, as measured by any one or all of the criteria listed on page 824, and despite the fact that its spheres, if free to be rearranged, could invariably be more closely packed in rhombohedral arrangement, nevertheless, many haphazard groups, as they lie in the midst of a larger assembly, have a striking degree of stability and permanence. But haphazard packing obviously has no fixed and characteristic degree or coefficient of stability, as has each of the cases of systematic packing. Instead, some haphazard arrangements are, under any and all circumstances, highly unstable and destined to be short-lived; other haphazard arrangements may possess a considerable inherent potentiality of stability; and, when surrounded and supported in just the right manner by stable systematic colonies or by fixed container walls, such arrangements may survive despite disturbing tendencies of very considerable magnitude. The translation of an open-textured haphazard grouping to a closer textured assemblage, whether still haphazard or partly or wholly systematic, participates as truly in the phenomenon of dilatation as does the translation of one type of systematic packing into another, more stable systematic type.[35]

One final important consideration requires attention in connection with the effect of the container. Up to this point, emphasis having been placed on the packing produced, the resulting voids *within the assemblage* have been chiefly in mind. There remains for examination the nature and arrangement of those voids that lie between the

[35] All such dilatation would, of course, fall in the class of *negative* dilatation; translation from a more compact to a less compact and less stable arrangement would constitute positive dilatation.

marginal spheres of the assemblage and the container walls. In the general example, the spheres of an assemblage will not fit as closely against the container walls, whether bottom walls or side walls, as they will fit against other spheres.[36] Stated otherwise, there will be a larger proportion of voids along the container walls than within the general assemblage; and these marginal voids will be unsymmetrical or unique, with respect to all other voids in the assemblage, in the sense that on one side they will bear the impress of sphere surfaces while on the other side they will be shaped by the container wall.

These marginal voids therefore introduce modifying influences on both the porosity and the permeability of the container-held assemblage as a whole. Some idea of the magnitude of these influences may be gained from the following considerations:

The unit cell (cube) of Case 1 packing (having edge $2R$ and containing the sphero-pyramids of eight different spheres which collectively equal exactly one sphere) will just circumscribe its unit sphere. But the unit cell ("simple" rhombohedron) of Case 3 or Case 6 (likewise having edge $2R$ and likewise containing eight sphero-pyramids which total exactly one sphere) is not large enough to circumscribe its unit sphere. A rhombohedron of identical angles to those of the unit cell of rhombohedral packing would need to have a volume 126.64 per cent of that of the unit cell in order exactly to circumscribe the unit sphere. Such a circumscribing rhombohedron contains voids of a volume totaling $2.97R^3$ (a "porosity" of 41.43 per cent); whereas the voids in the rhombohedral unit cell aggregate less than half this volume, or only $1.47R^3$, giving a porosity of 25.95 per cent. Thus, a single sphere lying within the plane walls of its exactly circumscribing "simple" rhombohedron is surrounded by 102.04 per cent more void than would belong to the same sphere if lying completely surrounded by neighboring spheres in perfect rhombohedral packing. Stated otherwise, if the circumscribing rhombohedron be the container, the container-wall effect in this instance adds to the normal voids ascribable to the tightest rhombohedral

[36] This generalization is not true for certain orientations of Case 1 and Case 4 packings when the adjacent wall is either a plane or a curved surface convex toward the spheres; nor is it true at all for those very restricted local places along the container walls where the convex curvature of the wall is sharper than the curvature of the sphere surfaces.

packing a further volume of marginal voids amounting to 102.04 per cent! This is the extreme case where the container holds an "assemblage" of only one sphere. When the assemblage comprises eight spheres in tightest rhombohedral packing, as in Figure 3, Case 3 and Case 6, the circumscribing rhombohedron contains 88.00 per cent more voids than the aggregate voids in eight unit cells; therefore this 88.00 per cent excess must be ascribed to the "disturbance" right against the container walls. As the assemblage grows to larger and larger size by the inclusion of more and more spheres, all in tightest packing, the influence of the container walls relatively declines; in an

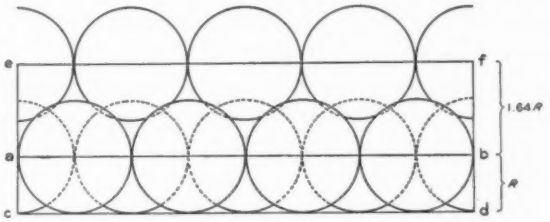

FIG. 13.—Relation of marginal voids to internal voids in tightest rhombohedral packing. Locus of marginal voids, *a–b–d–c;* locus of internal voids, *a–b–f–e*.

infinite assemblage, the increased porosity due to the larger voids at the container walls would be infinitesimal. A more general way of stating these relationships is: The larger the container with respect to the radius of the unit sphere, the smaller is the relative increase in porosity occasioned by the necessarily more open packing next to the container walls.

So much for the effect of the walls as a whole. In the instance just discussed, all six walls are involved, thus including a top limit as well as the bottom, or starting surface, and the four side walls. There remains for consideration in the same manner the increase in porosity along any single wall. If tightest rhombohedral packing be built on a plane starting surface (*c–d* in Fig. 13), there is 39.54 per cent porosity (the same as the porosity for Case 2 and Case 4) in the slice between the parallel planes *a–b* and *c–d*, while the slice between the parallel planes *a–b* and *e–f* has only the 25.95 per cent porosity of tightest packing. Stated otherwise, there is 93.14 per

cent as much void in the slice only R thick lying next the starting surface as in the slice 1.64 times as thick lying next above. Thus we have a measure of the greater porosity of the marginal region along the starting plane as compared with the porosity within the assemblage in tightest packing of Case 6. If the tightest packing is built on the square layer, therefore yielding Case 3 packing, the slice R thick next the starting plane has a porosity of 47.64 per cent (the same as that of "loosest" or Case 1 packing); and there is 29.59 per cent *more* void in the slice R thick next the starting plane than there is in the slice 1.41 times as thick lying next above. If the packing within the sphere assemblage is haphazard and thus of greater porosity than that of tightest packing, the porosity in the region between the assemblage proper and its walls is increased also, and probably on the average in at least corresponding, and possibly even greater, degree. This will be true even though the haphazard arrangement within the assemblage is itself a propagated consequence of the disturbance right against the walls.

Under all conditions, therefore, save those special and rare exceptions mentioned in the footnote on page 844, the porosity along the container walls is substantially higher than the porosity within the sphere assemblage proper. For *chance packings*, i.e., those natural fortuitous combinations of systematic colonies (chiefly of Case 6) with intervening or surrounding haphazard or actually orderless zones, no direct quantitative data are available as to the relative values of the porosities due to the marginal voids and those due to the voids within the assemblage proper.[37] It is, however, doubtless

[37] C. C. Furnas, in "Flow of Gases through Beds of Broken Solids" (*U.S. Bur. Mines Bull. 307* [1929]), studied the effect on porosity and permeability occasioned by introducing cylindrical pipes into beds (assemblages) of sized solid units. These units ranged, in various examples, from (approximately) spherical lead shot of diameter 0.138 cm. to broken lumps of ore of average diameter of 0.876 cm. The various cylinders used had internal diameters ranging from 5.6 to 113 times the diameter of these solid units. By making certain simplifying assumptions, combined with the results of experimental determinations of permeability (using gases as the fluid), he reached the following generalization (*ibid.*, p. 42): "When a container wall is introduced into a bed of material, 30 per cent of the solid material in the section next to the wall is removed, leaving the voids in that section correspondingly larger." If this conclusion be applied to a mass of spheres in tightest packing into which a (cylindrical pipe) container is forced, it would seem to imply that the normal porosity for that packing, i.e., 25.95

significant that chance assemblages of uniform spheres, in containers of simple shape and tens to hundreds of times the sphere diameter, yield measured porosities pretty close to 40 per cent[38]—perhaps about 39 per cent could be taken as a fair average figure for material compacted as much as is possible by tapping. Of such assemblages, Case 6 colonies with 25.95 per cent porosity make up a substantial fraction—certainly more than half in some instances, as estimated by inspection. In those instances where the Case 6 colonies account for just a half of the total volume, the remaining haphazard portions plus the marginal voids would have to possess an average porosity of 52 per cent to give the indicated total porosity of 39 per cent. Again on the basis of inspection, it seems improbable that the porosity of haphazard zones *within* the assemblages ordinarily averages as high as the 47.64 per cent of Case 1 packing. But if we should arbitrarily assume this 47.46 per cent porosity for these internal haphazard zones, the notably high porosity of the marginal regions is manifest. This is already evident in the qualitative way by inspection (see Figs. 29–36).

The contrast between the internal porosity and the porosity near the container walls is well disclosed in Figure 14, based on a photograph of an entirely random actual section cut through a chance assemblage of spheres whose true size[39] is indicated by the black circle

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per cent, would become 48.16 per cent in the "section next to the wall" of the container (i.e., $100-[74.05\times.70]=48.16$ per cent). And if an assemblage in chance packing had initially a porosity of, say, 38 per cent, the introduced cylindrical container would raise the porosity in the "section next to the wall" of the container to a value of 56.60 per cent. The present authors are not clear as to how far inward from the cylinder this "section next to the wall" is supposed to extend, beyond which, one would infer, Furnas assumed the original packing to be maintained. We are inclined to suppose that the introduction of the pipe caused disarrangement, with resulting net increase in porosity, through a considerable portion of the material within the pipe, and that the 30 per cent removal of solids referred to by Furnas should be averaged over a wider zone than the "section next to the wall."

In any case, the forceful introduction of a foreign "container" into the assemblage already packed is so different from the initial packing against the static walls of a container that close comparisons between the two cases are not likely to be very fruitful.

[38] Ranges outside the limits 35–43 per cent are rare.

[39] In another connection, we have computed, on the basis of reasonable assumptions, that the average diameter of the circles shown on a random section through a chance assemblage of uniform spheres is about two-thirds of the actual diameter of the spheres.

that touches the container wall at the upper right. (See analogous sections in Figs. 16, *b*, and 17, *a*, of Furnas, *op. cit.*) Figure 15 below serves to show the (theoretically) 100 per cent porosity just against a container wall, even though this be the basal starting surface and thus contain more spheres in contact with it per unit of area than touch any of the other walls. In this illustration, the dark dots represent the (enlarged) points of tangency of the spheres of the first

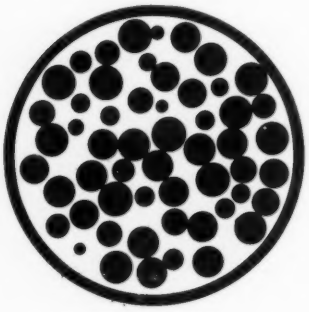


FIG. 14

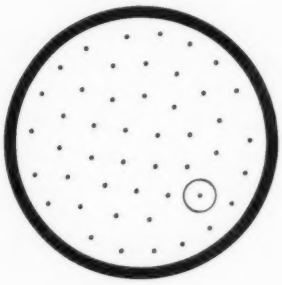


FIG. 15

FIG. 14.—Distribution of voids as shown on random section across chance packing in container.

FIG. 15.—Overwhelming proportion of void in section just above starting plane. Chance packing.

layer with the plane starting surface (which is the plane of the paper); the size of the spheres is indicated by the circle surrounding one of these dots; and the rather uniform, though chance, manner of arrangement of the spheres is indicated by the distribution of the dots. Successive sections taken at small fractions of R above the starting plane would, of course, show increasing areas of the sphere sections with corresponding decrease in proportion of empty space or void; Figure 15 as it stands can be taken to represent such a section just slightly removed from the starting plane.

Part II. Considerations Especially Affecting Permeability

The significance of permeability

Geological literature still contains such frequent confusions and abuses of the terms "porosity" and "permeability," with resulting adverse effects on conclusions, that it may not be amiss here to recall the distinctions between these two properties.

Porosity is the property of possessing pores or voids in the midst of "solid" or non-porous[40] material. Porosity relates to both isolate and conjugate voids; but only with respect to the latter is the commonly expressed statement true that porosity of a body is measured by the volume of water (or other fluid) which can be introduced into the pores of the body. As already stated, the voids in any assemblage of "solid" but uncemented spheres are all conjugate voids.

Permeability is the property of permitting a fluid to pass through the body which possesses this property. Since permeability[41] is possible only because of the presence of voids, there is a most intimate connection between porosity and permeability of a body. But isolate pores, by the nature of their definition, contribute nothing whatever toward permeability. Since the pores in any assemblage of "solid" free spheres are wholly of the conjugate, interconnecting type, each and every void in such an assemblage is a potential part of the complex channelway system through which movement of the fluid may be accomplished and permeability realized. Therefore, in such assemblages, the relationship between porosity and permeability is particularly close.

Notwithstanding the absolute dependence of permeability on porosity and the intimate nature of the relationship between these two properties, it is essential to have constantly in mind the fact that porosity and permeability do not vary proportionally.

[40] In strict application, the terms "porous" and "non-porous" are probably relative only, if consideration is given to the intra-atomic, inter-atomic, and intra-colloidal spaces. But for most practical purposes and in the ordinary sense, microscopically perfect simple crystals, tight-fitting crystal aggregates as in flawless cast metal, and glass free from bubbles and flaws may be taken to typify solid or non-porous material.

[41] In the practical sense, i.e., without regard to movement through those ultra-fine openings, inter-atomic, etc.

In an ideal assemblage of spheres of uniform size, freed from the complications of container effects, porosity is primarily and solely a matter of the pure geometry of angles. Magnify or demagnify the spheres to any desired degree but keep them in the same angular relationships and the porosity remains exactly constant, regardless of scale or absolute size. Indeed, the same is true even with the container effects included, if the same relative scale is maintained for both container and spheres.

But permeability depends on more than angular relationships; still more it is concerned with absolute dimensions. Change the scale of an assemblage, and permeability changes in the same direction, though porosity remains constant. It may be held that dimension, area, etc., are as truly geometrical concepts as are angular relations; and that permeability is as truly dependent on geometry as is porosity. But, while the geometrical aspects of permeability are assuredly important, they are not, as in porosity, the whole of the story. For in addition to these geometrical factors, there enter into the property of permeability the element of absolute direction and certain other properties of matter as well as attributes of force and energy; for porosity is a static quality while permeability is of dynamic and vectorial essence. Cohesion, adhesion, adsorption, surface tension, viscosity, density, temperature, pressure, inertia, momentum, velocity, acceleration, impact, friction—these, as well as direction and distance, enter into the determination of permeability. To do as some thoughtlessly do, in making permeability synonymous with porosity, is to deny the influences of all these important physical realities just listed.

Porosity is capable of specification by the simple ratio of percentage of void volume to total volume. But because so many of the influences upon permeability listed just above are capable of variation independently from the others, it follows that permeability is a highly complex property, which, in any given instance, can be completely specified only by a very considerable record; no simple ratio or coefficient is all-sufficient. And because the absolute distance is of such vital importance in connection with the manifestations or values of so many of these other influences, it follows that the factor of *actual dimensions* of the voids is of greater control on permeability

than is the *relative angular* relations that serve to determine porosity.

Despite their close interdependence, then, it is as yet impossible, with precision, to specify[42] permeability in terms of porosity or *vice versa*, except for that limiting extreme[43] where zero porosity gives zero permeability. While porosity and permeability commonly vary in the same direction, there is extremely elastic variation between the two properties, so that, under certain conditions, low porosity may be associated with high permeability; and very often, indeed, material of high porosity has very low permeability. These well-known facts are repeated here in order to emphasize particularly the importance of the absolute dimensions and the manner of interconnection of the voids in determining permeability.

THE INTERSPHERE VOIDS AS CHANNELWAYS

Permeability implies directioned motion—flow; and flow demands both an inlet and an outlet for the moving fluid. Thus the anastomosing composite void present in an assemblage of spheres must have access to the fluid at two separated places in order to meet this requirement of inlet and outlet. Granted this, every portion of the composite void throughout the assemblage is accessible to the fluid and may (potentially at least) assist in the flow.

While the void-complex as a whole is thus to be regarded as a unit, the contact of the spheres enmeshed in it perforates or subdivides it into what may be regarded as a maze of intercommunicating canals, each of which may likewise be regarded as a unit. But these, too, are relatively complex. In an assemblage possessing any given type of systematic packing, the device of analyzing both the void-complex as a whole and the component canals into the appropriate and characteristic unit voids for that type of packing, as discussed on pages 808 ff., greatly simplifies the mental visualization of flow through the assemblage, for on this relatively simple and systematic unit can be readily focused detailed study. It is then

[42] For what appears to be the closest approximation to a mathematical relationship of permeability to porosity see the formula by Fair and Hatch, discussed later, pp. 897–903.

[43] The other extreme, or 100 per cent porosity, merely implies free opening in space and has no useful significance with respect to permeability.

easier to understand the linking-up of these unit voids into canals, and of canals into the composite, thorough-going channelway system that leads from inlet to outlet for the whole assemblage, or for that part of it (i.e., that colony) which possesses this particular packing and orientation.

When the characteristics of the voids and their interconnections have thus been examined for the relatively simple and systematic conditions of orderly packing, the effect on permeability produced by the more complex and diverse but more natural varieties of packing may be more effectively investigated.

PERMEABILITY CHARACTERISTICS OF THE UNIT VOID[44]

Change in Cross-Sectional Area

The unit void, as already defined, is a form having fourteen sides or faces, eight of which are curved (spherical triangles) while the other six are the (imaginary) throat-planes arranged in pairs, each member of which is parallel to the other and lies on the opposite side of the unit void.

Each pair of opposite and parallel throat-planes of the unit void may be regarded as, respectively, the place of inlet and the place of outlet for the easiest flow through or along the unit void in that particular direction from the one throat-plane across to its mate. For flow in that direction, then, the unit void constitutes a given length of a specially shaped conduit terminated by the given pair of throat-planes; and it is its characteristics as a conduit that need to be examined. To distinguish this particular portion and type of conduit, it will be called a *duct element*. It constitutes a part of the entire channelway system within the sphere assemblage; and in any colony of systematic packing the duct element exactly connects, through the throat-plane at each of its ends, with identical duct elements of its neighbors adjacent on those two opposite sides. The duct element in systematic packing is thus a standard part of a continuous duct passing in the given direction through the colony. From the standpoint associated with principal inlet-outlet couples, or principal directions of flow, there are three duct elements in each

[44] This subject obviously deserves expert treatment. Unfortunately, the present authors are able to give little more than a layman's reasoning from obvious premises.

unit void, corresponding to the three sets of pairs of parallel throat-planes. Each of these three duct elements (although all three embrace the same unit void in common) may be, along its axis or central line, identical with or different from one or both of the others, depending upon the type of packing involved. For example, the unit void of cubic, or Case 1, packing contains three identical duct elements, each reaching across from one cube face to the opposite parallel face; but in Case 2, and similarly in Case 5, two of the duct elements are identical, though different from the third.

The curved surfaces of the duct element are so arranged in relations of mutual external tangency that the section of the duct element at the throat-plane is, in almost every instance, a minimum section; and successive sections inward increase in size toward the central portion and then again progressively decrease in size till a minimum section is again reached at the throat-plane opposite the first. (See successive sections of the unit voids for the several cases of packing in Figs. 17–20, inclusive.)

It follows, therefore, that the duct element is, as a rule, relatively constricted at the throat-plane, enlarges toward its middle portion, and then again retracts in cross section to the initial restricted size on reaching the opposite throat-plane.[45]

It is highly important to recognize this invariable alternation of constriction and enlargement of the duct element, for this condition has a definite influence on quality of flow. It will suffice merely to mention in support of this conclusion the variation thus occasioned in velocity, the alternating tendencies to streamline flow and to turbulent flow, and the change in law of resistance at the "critical velocity"—factors whose importance is well recognized in the problem of series flow in hydraulics. The changes in cross section are relatively abrupt, and therefore are by no means of merely theoretical significance, notwithstanding the fact that the probabilities of low absolute velocities in most actual instances reduce to some extent the influence of these and similar factors.

[45] In those few instances of which the generalization just stated is not true, e.g., a given direction through the unit void of Case 3 and Case 6 (Fig. 20, *B*), where the section at the throat-plane is larger than sections farther centerward, there is nevertheless present a wholly similar alternation of wide and narrow places within the duct element.

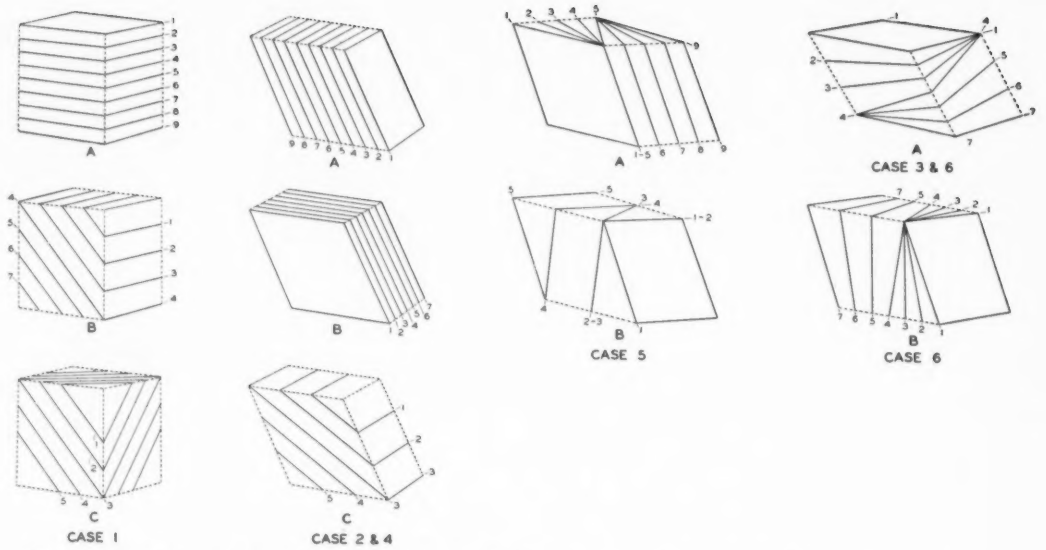


FIG. 16.—Key to sections across unit voids as shown in Figures 17, 18, 19, and 20. Outlined forms are those of the unit cells in orientation as shown in Figures 5 and 8; solid lines show positions of sections.

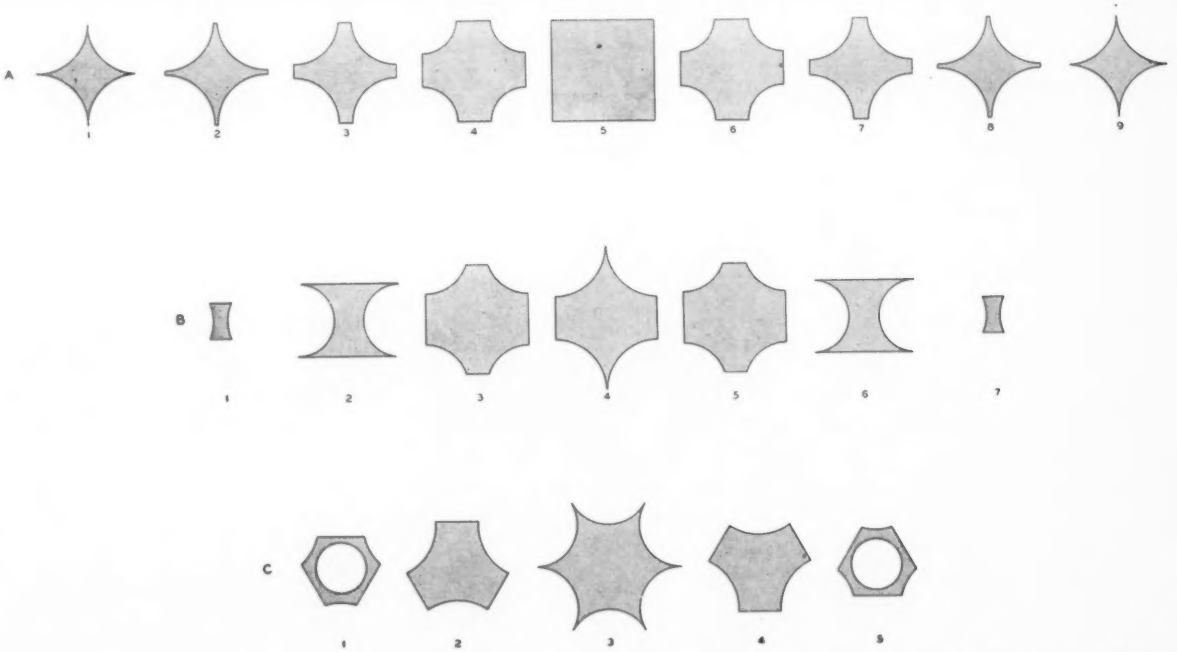


FIG. 17.—Successive sections across unit void of Case 1. For key to position of sections see Figure 16, Case 1, *A-B-C*. Refer to Figure 6 for unit void of Case 1.

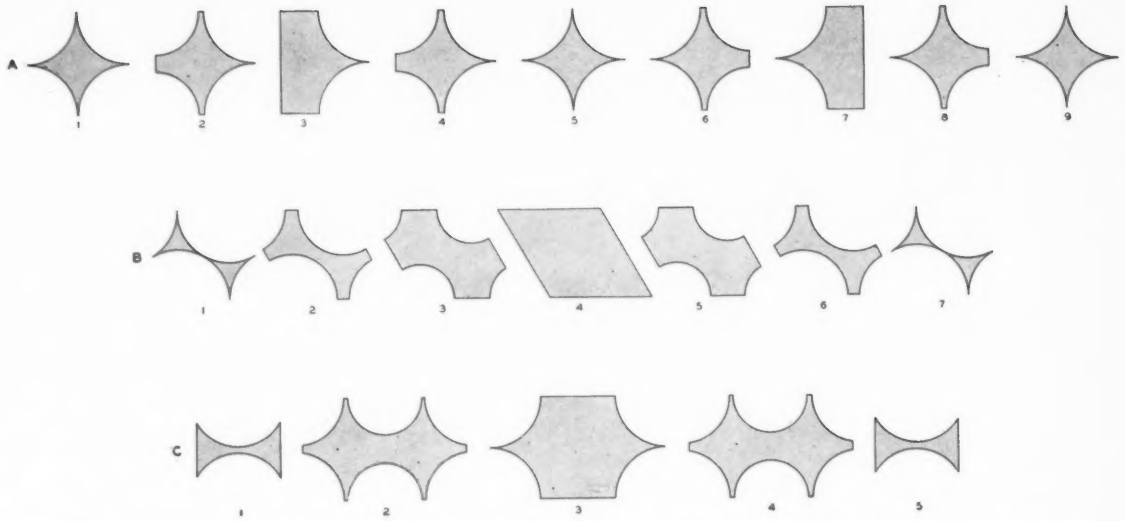


FIG. 18.—Successive sections across unit void of Cases 2 and 4. Refer to Figures 16 and 6 for positions of sections and shape of unit void

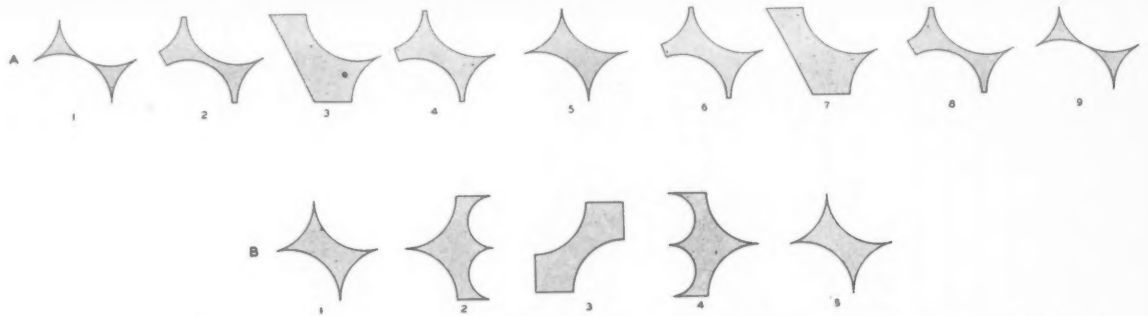


FIG. 19.—Successive sections across unit void of Case 5. Refer to Figures 16 and 6 for positions of sections and shape of unit void

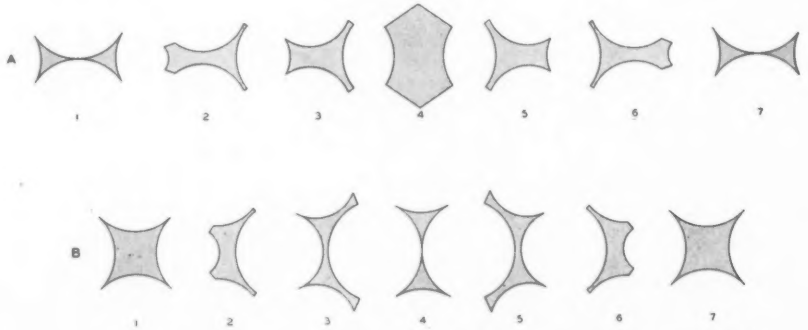


FIG. 20.—Successive sections across unit void of Cases 3 and 6. For positions of sections and shape of unit void refer to Figures 16, 6, and 8.

Change in Shape of Cross Section

The change in area of the cross section from point to point along the axis of the duct element does not take place by a simple swelling and shrinking of the conduit, as would happen in flow through successive sections of pipe of different diameters. Instead, the shape of the cross section of the duct element alters even more abruptly and drastically than does the area. This is strikingly disclosed in the series of successive sections portrayed in Figures 17–20, inclusive. Indeed, most of these shapes are so unusual and so special, from the standpoint of conduit cross sections, that it may be doubted whether adequate experimental investigation has ever been carried out on these or closely comparable sections. However, there can be no escape from the conclusion that the rapidly changing cross-sectional shape of the duct element must necessarily exert a retarding influence on flow. The details of shape, as well as the general form, vary markedly. In some sections the walls come close together and finally converge at sharp, tangential cusps; in other sections a much smoother outline prevails. Marked differences in wall friction, in internal friction, and therefore in velocity and in eddying, must consequently exist from place to place in the same cross section.

Those special and powerful manifestations of surface energy which cause a liquid to "wet" and adhere to the surface of a solid take on accelerating importance as the distance between *two* solid surfaces decreases. Such small distances are present in the narrowing cusps between tangent spheres, so that a larger proportion of the fluid in these narrow cuspate portions is virtually "frozen" to the sphere walls than is true in the more roomy central portions of the unit void. At the throat-planes, these narrow cusps are closer together than in any other sections through the unit void; therefore a larger proportion of the fluid is affected by these adhesionary or surface forces as it flows through the throat than in the more spacious regions on either side. The actual percentage of the throat-plane area or of the total void volume effectively influenced by these special forces varies in some inverse relation to the absolute area of the throat-plane, and thus to the sphere-size. It thus becomes merely a question of reducing the size of the spheres until the adhesionary surface influence of the (three or four) spheres extends completely

across the minimum section of the throat-plane. When this state of affairs is attained in all the throat-planes, *flow*, in anything like the usual sense, is no longer possible. Even the fluid in the central roomier part of the unit void, that might on its own account still be capable of flowing, becomes trapped, like water in a canal lock, between these tight and static parts; and thus the whole of the fluid mass is rendered immobile.[46] With still smaller sphere-size, the arresting influence of the surface forces would reach completely across even the largest cross sections of the voids and thus make fixity of the liquid doubly sure. Such conditions as those postulated are approximated in clays and serve to explain why, despite complete saturation by filling voids that represent a porosity reaching up to 50 and 60 per cent, flow through such material is virtually nil even though the external head may be high; also, why the water present is retained in the weak mass in spite of heavy compressive stress exerted by the column of overlying material. The results of these forces, as just described, have obvious analogies to capillary behavior, discussed on pages 904–8, but differ in being independent of an interface and in the smaller dimensional ranges involved.

Some of the duct elements possess successive sections that reflect the general shape characteristics of their closely neighboring sections; and some accomplish their changes without any marked rotational tendency. (See especially Fig. 17, *A*, and Fig. 18, *A*, *B*, and *C*.) In others the change in shape is most abrupt. And in several, the element of rotation enters, to introduce tendency toward vorticity of flow.

When all these complicating influences of shape are taken in conjunction with the changing area of cross section, the effect must be decidedly different from what would happen were the duct element of constant shape and area from end to end.

Divided and Re-united Flow

In every duct element of each of the cases, the cross section of the duct element at the throat-plane is distinct and separate from the corresponding section of neighboring unit voids in the assem-

[46] Mr. David T. Griggs suggests to us that it is because of this same adhesionary phenomenon that Stokes's law for the fall of spheres in a viscous fluid fails to hold when the size of the spheres becomes small enough.

blage. To look only at these throat-plane sections, one would gain the impression that each duct element is independent of the neighboring duct elements on the four sides around it and is connected only with the duct elements that join it on its ends. But of course each duct element is connected with *all* its immediate neighbors; so each duct element carries merely a particular fraction of the total flow passing through all the various duct elements aligned in the same direction; it is not, therefore, a question of a series of parallel but independent pipes or ducts, but rather of a series of parallel ducts each of which is connected at frequent intervals (through the common lateral throat-planes) with each of the four ducts that lie next to it, and thus with all the ducts in the colony. This permits the flow to veer and wobble from one duct into nearby ducts and back again, giving thus a sort of braided flow through the assemblage as a whole.

Moreover, for certain directions in some of the packings, the duct element has a "double" section at the throat-plane (see Fig. 20, *A*, 1) or somewhere centerward, suggestive of *two* duct elements within the unit void; but other sections show that these "double" sections represent only a local split or bifurcation of the duct element, which elsewhere shows its single, unit character. Thus, for these instances, alternate division and reunion of the flow occurs even within the same individual duct element.

Here, then, is another degree of the factor of varying area of cross section, with special opportunity for retardation of flow by complex eddying.

Curvilinear Flow

In Case 1, the direction of easiest flow through each of the three duct elements is represented by a straight line perpendicular to the throat-planes that terminate the duct element. The same is true for vertical flow in Case 4 as oriented and for the corresponding direction of Case 2. But even in these instances a part of the fluid is always being forced to adopt non-rectilinear motion because of the alternately diverging and converging walls of the duct element. In duct elements of the other cases, the principal axis or central line is

curved, and flow must adapt itself to this curvature. The throat-plane, being perpendicular to the spheres it intersects, is always perpendicular to the axis of the duct element at the latter's point of emergence; but because of the rake or cant of the unit cell of most of the packings, the axis or central line of flow shows a reverse curve. The curvature is gentle, introducing neither great increase in length of path of flow nor excessive angular change in direction. But beyond question, "tortuous flow" must to some extent exist and thus reduce the volume of flow from what would take place in an otherwise similar conduit of straight axis.

Diagonal Flow

Flow along the general direction of the axis of the duct element, i.e., from the "inlet" at one throat-plane to the "outlet" at the opposite throat-plane, will most easily accommodate flow-tendency aimed in that general direction. Flow thus accomplished may be termed "direct flow," with respect to the given duct element. In each unit void there are three such directions of easiest flow, and they stand at fairly high angles to one another—in no case less than 60°. But the positions of the general inlet and the general outlet for the assemblage as a whole, or for the particular colony in which a given unit void is contained, may be such with respect to the orientation of this unit void as to make favorable the exclusive adoption of no one of these three individual directions of easiest flow through this given unit void.

Under these conditions, the flow will tend to take a diagonal short-cut through the unit void. There will thus be established a situation approximating flow through an elbow. The outcome will be a resultant of two tendencies: (1) the tendency to hold to the axis of maximum cross section, which would imply flow part way along the axis of one duct element and then a turn to flow, for the remaining distance through that particular unit void, along the axis of that one of the other two duct elements which better accommodates the net direction of general flow and (2) the tendency to hold to the shortest path that will permit flow in that net direction, or a tendency to hug the inner bend of the elbow even at the cost of utilizing

thereby narrower parts of the available cross section.[47] Emphasis on alternative (1) would result in a definitely zigzag path through the succession of unit voids—zigzag in perhaps two, or possibly all three, of the directions of the duct elements of the colony. Emphasis on alternative (2) would carry flow by a straighter and shorter but more constricted diagonal route, which might be the sidewise-diagonal or the still more oblique and constrained cornerwise-diagonal of the unit void.[48] The resultant compromise between these two tendencies will occasion a more marked "tortuous flow" than that already discussed where flow follows the curving axis of the duct element; it will lessen the average cross section of the path adopted, as compared with that along any axis of a duct element; and it will exaggerate the changes in size and shape of cross section already discussed. The net result of these influences can only be to make diagonal flow more difficult than direct flow along a single duct axis.

With respect to the curved paths of flow discussed in this and the immediately preceding section, some interesting geometrical relationships may be considered. The qualifying condition in Newton's First Law of Motion regarding the deviation from motion in a straight line is, fortunately for the human race, very easily converted into reality in the case of a fluid. Nevertheless, the tendency and desire for truly rectilinear motion is not to be denied. It is therefore pertinent to examine, at least in the qualitative way, the extent to which—granted ideal absence of eddying—the ducts in a systematic assemblage of spheres would permit absolutely straight-line flow.

If one were to look directly into the end of a straight cylindrical pipe aimed toward a light, he would see the illuminated opening as a circle of the same size as the cross section of the pipe. This situation might be specified by saying that the axially projected opening or duct equals the cross section. But if the pipe were slightly bent, the illuminated opening would be reduced in size and (ordinarily)

[47] The absolute velocities will determine the extent to which these tendencies will be reduced by that to press against the outer bend of the elbow, where likewise the successive cross sections have narrow parts.

[48] Successive cross sections available for sidewise-diagonal flow through the especially spacious unit void of Case 1 are shown in Fig. 17, *B;* and analogous sections for cornerwise-diagonal flow through the same void are represented in Fig. 17, *C*. See key to orientation of sections in Fig. 16.

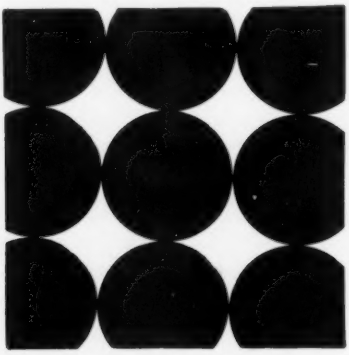
changed in shape; the projected duct would be smaller than the cross section. The greater the departure of the pipe from straight, the smaller the projected duct; until finally the pipe becomes so crooked that no light can travel directly through it—i.e., there is no longer a projected duct. If, instead of light, it were a fluid passing through such a pipe, the projected duct would represent the maximum cross section through which absolutely rectilinear flow could take place. And if, instead of a simple cylindrical pipe, we were dealing with a chain of voids within an assemblage of spheres, the projected or net rectilinear duct would likewise measure the maximum capacity for truly straight-line flow along the direction of the chain.

In Figures 21–24, inclusive, are shown the outlines of the maximum projected ducts that can exist in various directions through colonies of indefinite (or infinite) size for the several cases of packing. Each of these represents the net projection of all the duct elements lying along a given straight line through the colony when the projection plane is perpendicular to that line. Such projected openings may be called *net rectilinear ducts*. Spaced at regular intervals, as shown, they lie in parallel sets; and they penetrate uninterruptedly from one side of the colony to the other.

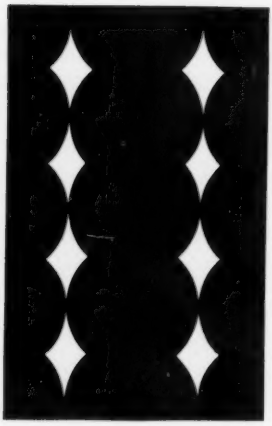
In general, the shape, size, and spacing of the net rectilinear ducts vary in different directions through a colony; but certain directions are identical in these respects with certain other directions, in much the same way that certain different directions through a crystal have the same index of refraction. In Case 1 and in Cases 2 and 4, there are, theoretically, an infinite[49] number of sets of net rectilinear ducts along varying oblique directions, each giving progressively smaller "diamond-shaped" outlines; the general nature of these outlines is illustrated for the larger shapes in Figure 21, *B* and *C*, for Case 1 and in Figure 22, *B*, *C*, and *D*, for Cases 2 and 4; the still smaller examples are not shown. All other net rectilinear ducts for all possible directions for all the untwinned cases of packing are shown, together with

[49] In Case 1, the points of tangency of the spheres lie in three sets of planes which elsewhere touch none of the spheres; the same is true for one set of planes in Cases 2 and 4. Therefore, in *any* rectilinear direction in such a plane, there would be uninterrupted lines of flow, just as there would be uninterrupted lines of sight in every possible horizontal direction through an orchard if the trees had the infinitely small thickness of geometrical lines.

the number of different directions in which the given pattern appears in that type of packing. For example, Figure 21, *A*, is obviously the



A



B

C

FIG. 21.—Net rectilinear ducts, Case 1. *A*, three directions; *B*, six directions; *C*, twelve directions.

view that one would have when looking (without perspective, i.e., in non-divergent light) vertically downward upon a colony of Case 1 packing; but the same pattern would appear also if looking along

either of the other two major 90° axes of this case; therefore, this shape and arrangement of the net rectilinear ducts occurs in three directions in Case 1. Likewise, the pattern of Figure 21, *B*, occurs

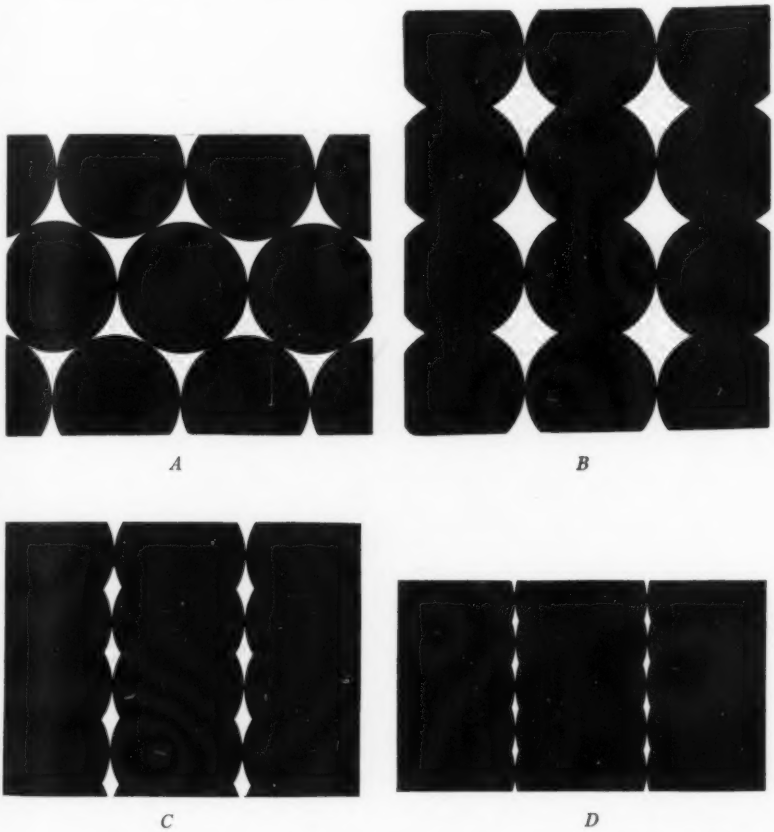


FIG. 22.—Net rectilinear ducts, Cases 2 and 4. *A*, one direction; *B*, three directions; *C*, three directions; *D*, six directions.

in six directions, etc. The number of directions in which each type of net rectilinear duct occurs is in each instance given under the corresponding diagram.

Nothing better serves to bring out the beauty of system and symmetry of these several fundamental cases of packing than these striking patterns of the net rectilinear ducts.

When attempt is made to translate these patterns into some quantitative expression of permeability, it becomes evident that there is not a direct and fixed relation between aggregate cross-sectional area of the net free ducts and the permeability in the given

FIG. 23.—Net rectilinear ducts, Cases 3 and 6. Six directions

direction.[50] But the symmetry and relative simplicity of the patterns promote the suspicion that a not too complicated relationship may actually exist and might, perhaps, be discovered by a sufficiently conscientious and talented mathematician. In a qualitative way, these patterns certainly do give a general impression of the relative permeabilities in given directions through the several packings.

Effect of Twinning and Tripling

Twinning and tripling, i.e., the stacking of the sphere layers in alternate orientations, obviously changes the angular relations of the duct systems in colonies of those packings where such alternative

[50] For example, many chance directions through colonies of the less porous packings would show no net free ducts on the perpendicular projection; yet none of these packings has zero permeability in any direction.

stackings are possible. This necessarily changes the permeability; it is like putting extra bends in a pipe. The effects upon permeability are invariably repressive. They are brought about in three ways: (1) increasing the importance of tortuous flow; (2) decreasing the

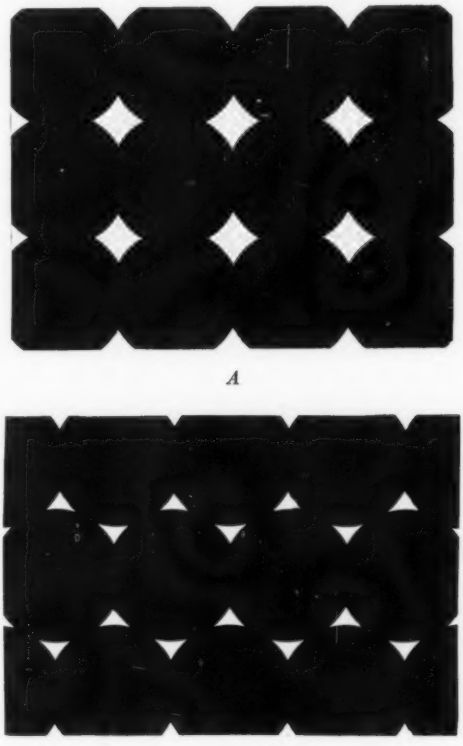


FIG. 24.—Net rectilinear ducts, Case 5. *A*, one direction; *B*, two directions untwinned, one direction twinned.

projectioned cross section, or the size of the net rectilinear duct; (3) increasing the rotational or screw nature of the composite duct, thus favoring vortex motion. The smaller the twinning interval, the greater is the reduction of permeability.

Since twinning has more than an even chance of occurring in colonies of appreciable size of those packings that permit it, the adverse effect of twinning on permeability has always to be taken into consideration for those cases.

The effect of twinning and tripling in generally reducing the cross section of the net rectilinear ducts is clearly shown in Figures 25–28, inclusive,[51] and by comparison of these with Figures 21–24. Some of the net rectilinear ducts of untwinned packing are indeed destroyed as such in the twinned and tripled arrangements. In general, as would be expected, tripling exaggerates the effects produced by twinning in the same case.

Here, just as in the corresponding patterns for untwinned packings, the symmetry and system of these net rectilinear ducts is noteworthy.

Summation of Effects

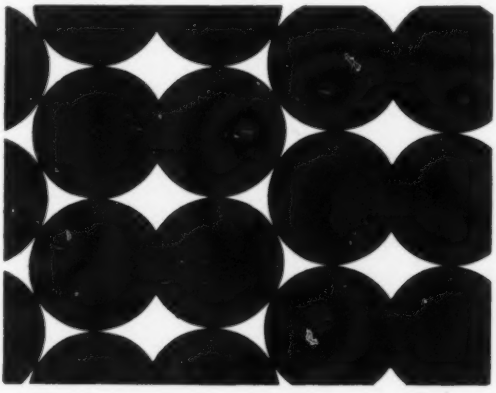
It is evident that the unit void of systematic packing, considered both independently and in relation to its interconnecting neighbors, is a pretty complex type of conduit and that its effect on flow is obviously difficult of mathematical analysis. But it is also clear that the various components of this complexity have the same sign and thus act cumulatively. Appraised qualitatively, the summation undoubtedly operates to diminish flow in very considerable degree, as compared with familiar and conventional conduit systems such as cylindrical pipes.

In a colony packed according to one of the systematic schemes, the adverse effect which any one unit void has on the flow through it is closely[52] indicative of the effect that will be produced by each of the other unit voids that lie (within the colony) along that particular path of general flow. This effect, of course, is additive; so the

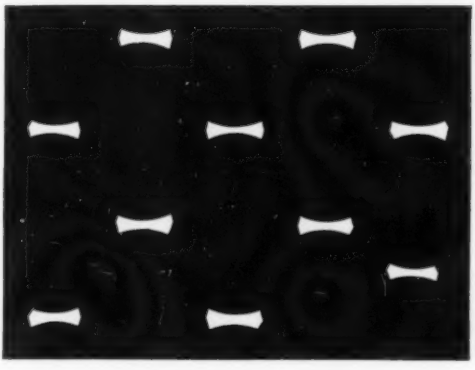
[51] These are the patterns produced when the twin planes are spaced at the closest possible interval of one layer. Some of the patterns produced by twinning are identical with those for different (simpler) directions in untwinned arrangements, and are therefore already illustrated in Figures 21–24; where necessary, cross-reference is made in Figures 25–28.

[52] The effect in all the unit voids is not exactly identical, for the effect in each is to occasion in the next a loss of head (pressure), which is a fundamental factor in each of the influences discussed.

longer the path (i.e., the greater number of unit voids traversed) the greater is the reduction in flow.



A

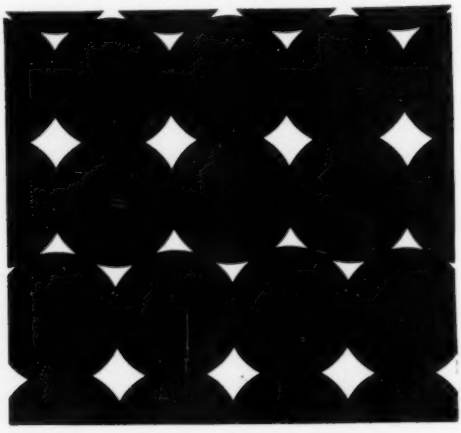


B

FIG. 25.—Net rectilinear ducts, Cases 2 and 4. *A*, two directions twinned; *B*, four directions twinned.

It would obviously be advantageous to have for each case of packing some simple expression of its permeability, so that the several cases could be compared as to permeability in the way that

they can be (by percentages) as to porosity. But in addition to the various reasons already given which make this highly desirable end difficult of attainment, there is the fact that, whereas each case of



A

B

FIG. 26.—Net rectilinear ducts, Case 5. *A*, two directions twinned, one direction tripled; *B*, two directions twinned.

packing has a single and characteristic porosity, it has no single and characteristic permeability—and this is true even if the size of sphere were invariable—for each type of systematic packing varies in

permeability in different directions, since the cross-sectional character of the unit void is not the same in all directions, as Figures 17–24 so strikingly show. Thus there can be at best no *single* permeability coefficient for any type of packing, but rather a number of coefficients, each applicable to a given net direction of general flow. The problem is therefore similar in certain aspects to the orientation of elastic properties in a non-isotropic medium.

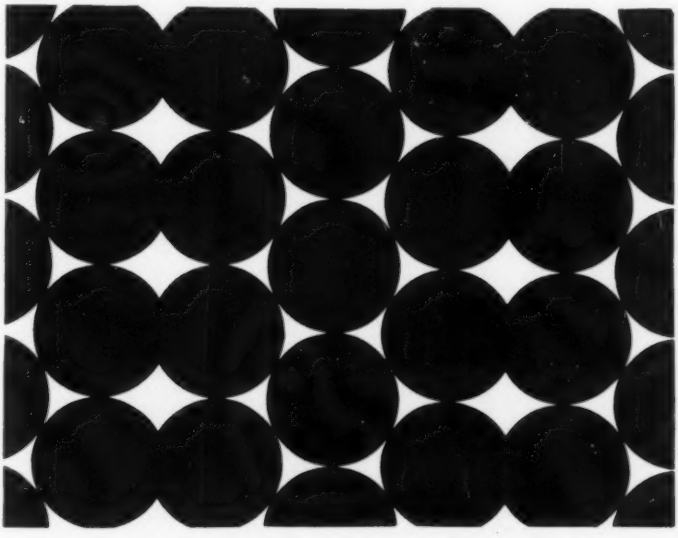
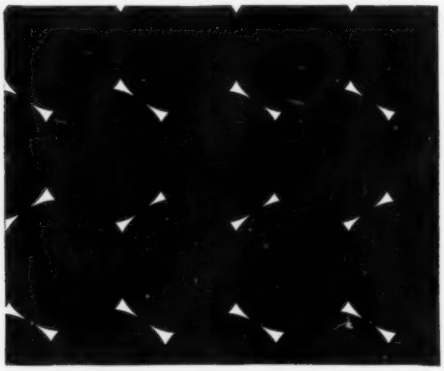


FIG. 27.—Net rectilinear ducts, Case 5. Two directions tripled

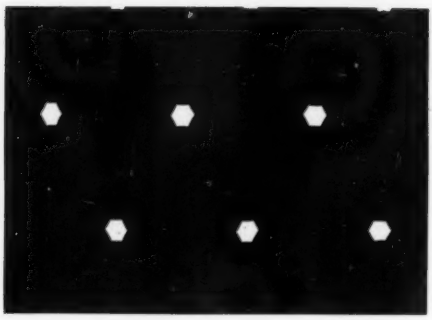
Nevertheless, it is possible to conceive of a permeability value which, for any given sphere-size and for constant head, temperature, etc., of the given fluid, would afford a generalized expression of the permeability of each type of packing as a whole, but without regard to its orientation to the direction of flow. Such a value would obviously have to comprehend and average the differing permeabilities in the various directions through such a packing;[53] thus, to use

[53] Twinned and/or tripled arrangements would also be taken into account to the extent that such are likely to exist in the average colony.

the analogy to elasticity, this could be regarded as the permeability value along the "axis of mean permeability" of the given type of packing. In the discussion that follows, it is this "mean permeabil-



A



B

FIG. 28.—Net rectilinear ducts, Case 6. *A*, three directions twinned; *B*, one direction twinned.

ity" that is meant, unless otherwise noted, when reference is made to the permeability of any given packing. By this means it is possible to compare in a somewhat approximate way the characteristic permeabilities of different packings.

Slichter,[54] in 1899, and Fair and Hatch,[55] in 1933, have devised formulas of flow, through sand, that are directly applicable, as well, to assemblages of uniform spheres. These formulas are discussed on later pages. From them, by assuming all other variables to be constant, values may be derived that correspond to the mean permeability as defined above.

TABLE III

INDICATED RELATIONSHIP BETWEEN POROSITIES AND MEAN PERMEABILITIES OF THE SEVERAL PACKINGS

Type of Packing	Percentage of Porosity	Difference in Porosity		Fair and Hatch		Slichter	
				Permeability Factor	Difference	Permeability Factor	Difference
		(Per Cent Units)	(Percentage)		(Percentage)		(Percentage)
(*I*)	(*II*)	(*III*)	(*IV*)	(*V*)	(*VI*)	(*VII*)	(*VIII*)
Rhombohedral (Cases 3 and 6)	25.95			3.19		1.19	
Tetragonal-sphenoidal (Case 5)	30.19	4.24	16.34	5.65	77.30	1.87	56.73
Orthrhombic (Cases 2 and 4)	39.54	9.35	30.97	16.91	199.17	4.32	131.49
Cubic (Case 1)	47.64	8.10	20.48	36.89	118.09	8.06	86.60
Rhombohedral (Cases 3 and 6)	25.95	21.69	83.58	3.19	1572.01	1.19	675.46

In Table III the several systematic packings are compared as to porosity and mean permeability. The porosity, given in column *II*, is, of course, ideally exact. Columns *III* and *IV* contrast successive pairs of cases, first in terms of per cent units, and second translated into terms of percentage. Column *V* gives the mean permeability

[54] *Op. cit.*

[55] G. M. Fair and L. P. Hatch, "Fundamental Factors Governing the Streamline Flow of Water through Sand," *Jour. Amer. Water Works Assoc.*, Vol. XXV, No. 11 (1933), pp. 1551–65.

factor for each of the packings, as derived from the formula of Fair and Hatch, and column *VI* shows the percentage difference in mean permeability from type to type of packing, as derived from column *V*. Column *VII* gives similarly the mean permeability factor according to Slichter (i.e., the reciprocal value of his constant K as given on his page 326, Table II). And column *VIII*, derived from column *VII*, expresses the percentage difference from type to type, according to Slichter's findings. It should be noted here that both formulas are established in terms of two variables—porosity and sphere-size. In Table III the sphere-size is assumed to remain constant for the data in each column. For different sphere-sizes, Fair and Hatch would multiply by $(3/R)^2$, and the corresponding factor for Slichter is $4R^2$. The packings are arranged in the table in order of their increasing porosity, except that rhombohedral packing is given again at the bottom of the table in order to simplify direct contrast between tightest and loosest systematic packing.

It will be seen that, as would be expected, for any given size of sphere the mean permeability varies with the porosity to the extent that the order of decreasing porosity for the six cases is also the order of decreasing permeability. This is another way of saying that the permeability varies in the same direction as the size of the unit void. The mean permeability of the untwinned packings therefore decreases in the following order: Case 1, Cases 2 and 4, Case 5, Cases 3 and 6.

Comparison of column *IV* with column *VI* (Fair and Hatch) and with column *VIII* (Slichter) gives us an idea of the rate at which permeability changes with change in porosity. For all these cases of ideally systematic packings and with the same sphere-size, it is invariably true that porosity and permeability change in the same direction and that the permeability changes faster than the porosity. But in haphazard and chance packings these rules do not necessarily hold, and may in extreme instances be reversed.

Columns *V* and *VII*, representing the relative permeability factors according to Fair and Hatch and to Slichter, respectively, are not in the same terms, and therefore are not comparable directly. But in columns *VI* and *VIII*, the two sets of data are brought to the comparable basis of percentage change from case to case. It is

seen that the two sets differ widely, the relative differences in permeability according to Fair and Hatch being invariably much greater than those according to Slichter. For reasons discussed later, it is believed that neither set of data represents better than an approximation to that ideal herein contemplated, namely, the mean permeability in a perfectly systematic arrangement of true spheres of uniform size. On theoretical grounds, the data of Fair and Hatch are deemed a closer approximation than those of Slichter to this ideal; and this would seem to be substantiated by experimental results.

The great difference in permeability between the cubic packing of Case 1 and the rhombohedral packing of Cases 3 and 6 is particularly to be emphasized, as showing the profound effect of packing and of dimension of void. Although Case 1 has only 1.84 times the porosity of Cases 3 and 6, its permeability is 6.76 times as great according to Slichter, and 11.57 times according to Fair and Hatch; in other words, from Cases 3 and 6 to Case 1, permeability changes 6.89 times as fast as the porosity changes according to Slichter, and 12.65 times as fast according to Fair and Hatch. It is noteworthy that the commonest type of systematic colony, Case 6, has both the lowest porosity and the lowest permeability that can be secured with any possible arrangement of free spheres of uniform size.

PERMEABILITY OF HAPHAZARD PACKING

From the standpoint of permeability, haphazard packing differs from simple systematic packing in three principal respects: (1) its lower degree or absence of ordered arrangement; (2) its generally looser texture or greater size of voids; and (3) its closer approach to isotropy because of the subordination of unique, systematic directions. Numbers (1) and (3) are closely interrelated, and (2) is a direct consequence of (1). These qualities affect permeability in two ways: the absolute magnitude of the permeability values, and the relation of these values to various directions of flow. Each of these influences will be briefly considered.

Magnitude of Permeability

If it were true that a given packing has twice the porosity of another packing because it has twice the number of identical voids

identically oriented, then the permeability of the first packing would be approximately, if not exactly, twice that of the second packing. But this premise does not and cannot hold in an assemblage of uniform spheres, whether arranged systematically or haphazard; instead, porosity differs between different modes of packing because of difference in the size rather than in the number of the unit voids. On the other hand, while it is true that a packing having voids the same in number but double the size of those of another packing will have double the porosity of the latter, it will have more than double the permeability, since the larger voids will carry more than twice as much flow as the smaller. And finally, it is true that a packing with voids of uniform size has the same porosity as a packing with the same number of voids and of the same *average* size, though actually some are small and some are large voids; but such packings will not have equal permeability, and the packings with the voids of different size will, as a general rule, have greater permeability than the packing with voids of uniform size, since the larger-than-average voids will gain a larger fraction of the flow than the smaller-than-average voids lose. All this is merely another way of illustrating and applying the fundamental fact that porosity depends on relative sizes but not on shapes of voids, while permeability depends on absolute sizes and is vitally concerned with shapes.

When the foregoing generalizations are applied to haphazard packing, the following conclusions emerge: (1) Haphazard packing, being of higher porosity than tightest rhombohedral packing, has a larger average size of void, since it can, at best, have no voids smaller than those of Cases 3 and 6 and must have many voids that are larger; therefore haphazard packing has a higher permeability, for the same sphere-size, than has tightest systematic packing. (2) Since most examples of haphazard packing will be in substantial degree systemless and disordered, thus always differing from place to place, it follows that the voids in such packing will be of non-uniform size; therefore such a disordered packing that may have the same porosity as one of the more open types of systematic packing will nevertheless have a higher permeability than that systematic type. (3) Although no imaginable haphazard combination of uniform spheres can possibly have a lower porosity (or therefore have a lower

permeability) than that of tightest[56] systematic packing, there can be haphazard combinations that will give a higher porosity than that of loosest systematic or Case 1 packing; but these will be local, relatively restricted, and relatively infrequent places of "arching," and therefore no large regions within[57] an assemblage will have greater porosity than that of cubic packing. Most haphazard arrangements doubtless have lower than the 47.64 per cent porosity of Case 1. But it is conceivable that in extreme instances voids larger than those of Case 1, produced by arching, could be so plentifully scattered through a haphazard assemblage, and by their abnormally large size be so effective in increasing permeability, that a considerable assemblage of this particular haphazard sort could have higher permeability than that of Case 1 packing. (4) In general, therefore, the permeability of haphazard arrangements will lie between the limits of the permeability of tightest and of loosest systematic packing, but on rare occasions may exceed the latter. Although the permeability will not be directly proportional to the porosity of the haphazard packings, the higher permeabilities will generally (though not always or necessarily) be associated with higher porosities, and vice versa. From a combination of experimental results and visual inspection, one may reach the guess that in the most common manifestations of haphazard packing, the porosity considerably exceeds the 39.54 per cent found in the packing of Cases 2 and 4, and the mean permeability for such haphazard packing is still greater than that for the Cases 2 and 4 packing (cf. Table III). But these generalizations hold only for haphazard groupings that are of such dimensions as to give to a path of flow through them a length of a considerable multiple of R, the sphere radius, so that something like an average sample of haphazard packing is presented along the pathway. In small haphazard groupings, the permeability may be anything between the minimum of Cases 3 and 6 and the maximum that exceeds the highest permeability of Case 1.

[56] Not infrequently in actual assemblages a region of packing develops that departs only slightly from true tightest packing—this being due to an angle or a spacing, imposed by the surroundings, that is not exactly right for the true simple rhombohedral arrangement. Such regions will, within their own limits, have a permeability not greatly higher than that of tightest packing.

[57] The conditions near container walls are not included in this generalization.

Orientation of Permeability

Reference has already been made to the definitely directional quality of the permeability in simple systematic packing. This is because of the constancy of relative angular relations between spheres and voids in systematic packing. In the usual and characteristic case of haphazard packing the low degree or complete absence of orderly repetitions reduces or completely prevents, as the case may be, the constancy of angular relations found in simple systematic packing, and therefore makes this haphazard assemblage more nearly "isotropic" with respect to permeability. Stated otherwise, any two chance directions through an assemblage of haphazard packing are much more likely to give approximately the same permeability values than are a similar pair of chance directions through an assemblage in any one of the simple systematic packings. But this needs one qualification; it is truest when the given haphazard occurrence is sufficiently extensive to afford, along any line across it, a fair sample of all the characters and orientations of the haphazard packing; and it is least true when the given haphazard occurrence is smallest or thinnest (always in terms of number of spheres rather than in absolute dimensions). For it is evident that if one were to take the void (or voids) between a relatively small number of spheres, this one void (or these few voids) might have very definitely directioned qualities even though the spheres were in a quite orderless arrangement.[58]

This relative isotropy of most haphazard groupings of any considerable size gives to such haphazard packing the quality of being able to utilize its (generally) high permeability effectively regardless of the direction in which the flow is aimed. Such groupings cannot be approached by the fluid from an inadvantageous side, as can colonies in simple systematic packing. This undoubtedly tends to give some permeability advantage to haphazard packing as contrasted with a simple packing, even one that may have the same

[58] It is, of course, to be recognized that those repetitive orderly arrangements which are of higher complexity than those here classed as simple systematic packings, and also those packings (cf. footnote on p. 877) which just fail to be one of the simple types, may have directioned qualities nearly as pronounced as those of the simple cases. But such phases of what is here called "haphazard packing" are relatively unimportant.

"mean permeability" value. The importance of this will be more evident in chance packing, where colonies of systematic packing and regions of haphazard packing are competing side by side.

PERMEABILITY OF CHANCE PACKING

There remains for consideration the permeability of the commonest or average arrangement, which we are calling "chance packing." Since chance packing is very definitely composite, comprising chiefly Case 6 colonies and intervening haphazard zones, together with such colonies of the other cases as may happen to develop, it is evident that the permeability of a chance assemblage will be some kind of a resultant of the permeabilities of its component portions. But this will almost certainly not be a simple weighted average of the individual permeabilities as determined separately for each of the component parts. There will probably always be a tendency to exaggerate the difference in individual permeabilities. In any event, since haphazard packing is invariably more permeable than tightest rhombohedral packing, the haphazard portions of a chance assemblage will always carry a larger porportion of the flow than the ratio of their combined volume to the total volume of the Case 6 colonies.

If a pipe were filled along alternate sections of equal length with Case 6 packing and haphazard packing, then, ignoring marginal effects due to the pipe walls, any section of haphazard packing could carry no more of the flow through the pipe than the preceding sections of Case 6 packing had allowed to be brought to it and than the succeeding sections of Case 6 packing would carry away. The permeability of the filling in such a pipe would therefore be increased only a little (relatively) by the effect of the haphazard sections as compared with the permeability that would result if the pipe were filled throughout with Case 6 packing. But if the same pipe were half filled with Case 6 packing distributed all along one side of the pipe and the other half were similarly filled with haphazard packing, so that the two packings lie side by side throughout the length of the pipe, then, again ignoring container effects of the pipe walls, the permeability of the filling as a whole would be decidedly greater than in the other instance, even though the inde-

pendent or individual permeabilities of the two kinds of packing involved and the proportions of the two kinds are the same as before.

In short, a given region will actually realize its inherent or potential permeability only if its surroundings permit. A region of lower permeability surrounded everywhere by material of higher permeability will carry all that its own voids can accommodate under the pressure at which the fluid reaches it. But a region of higher permeability completely surrounded by material of lower permeability will not realize its inherent permeability, but will have an effective permeability influenced, and in large measure determined, by the lower permeability of its surroundings.

Granted, then, the inherent difference in permeability between Case 6 packing and haphazard packing, it becomes important to examine more adequately than has been done thus far the manner of mutual distribution of these two kinds of packing in an assemblage of average or chance arrangement. Specifically, it is essential to know whether these packings lie side by side throughout an assemblage, thus giving what would be analogous in hydraulics to flow through parallel pipes; or whether flow has to pass alternately through one and then the other of these packings, analogously to series flow in hydraulics; or whether one packing consistently surrounds and isolates the other and, in that event, which type of packing is isolated and which does the isolating. Obviously, a wide variety of conditions can occur in chance packings, and only the general tendencies and commonest results need here be considered.

On earlier pages, reference has frequently been made to the presence in chance packing of colonies of tightest packing and intervening zones of haphazard packing. This characterization gives the essential clue to the architecture of the most common packing, but it needs a little elaboration.

From the analysis on page 831 it may be concluded that, without independent lateral support, a colony of Case 6 starting to grow on a reasonably horizontal basement will decrease in size upward until it comes to an apex and ceases. But such independent lateral support may often be fortuitously accorded, by container walls, by other colonies, or by haphazard groupings; for the geometric tendency toward tightest rhombohedral packing is so strong that, if

given a respectable chance, it will form. But such lucky support on the sides cannot be counted on to continue through a very great vertical range—sooner or later it will cease. From there upward, the best that can happen is a pyramidal converging of the Case 6

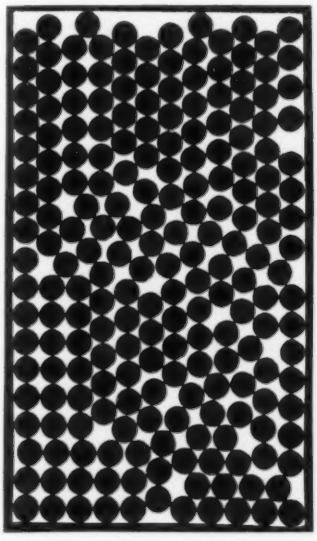


FIG. 29

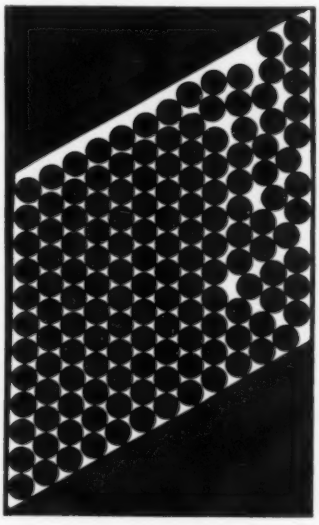


FIG. 30

FIG. 29.—Packing in a simple rectangular container. Note colony of Case 1 packing at lower left, favored by the 90° angle of the walls, but how this soon ceased forming, as the assemblage accumulated, and produced Case 3–6 colonies or haphazard packing.

FIG. 30.—Packing in simple 60°–120° rhombic container. Note how the 60° angle of walls at lower corner favored development of a Case 6 colony which, once thus started, grew uninterruptedly almost throughout the assemblage. But, despite the fact that the angles of the container walls are ideal for Case 6 packing, the fact that the *length* of the walls was not exactly right produced a region of haphazard packing on the right and a large void at the top.

colony until it ceases. From inspection of many examples, we conclude that the average colony of Case 6 packing not unduly influenced by special container walls tends to be chunky and roughly equidimensional regardless of whether it is a large or a small colony; there are exceptions of conspicuously different shape, but they are

not common and rarely are large. This tendency toward roughly equidimensional form of the simple rhombohedral colonies is fairly well illustrated in Figures 29–36, inclusive, where these colonies may be recognized by the invariable presence of the characteristic throat-plane section of absolutely equilateral concave triangular outline.[59]

Therefore, the average example of chance packing chiefly consists

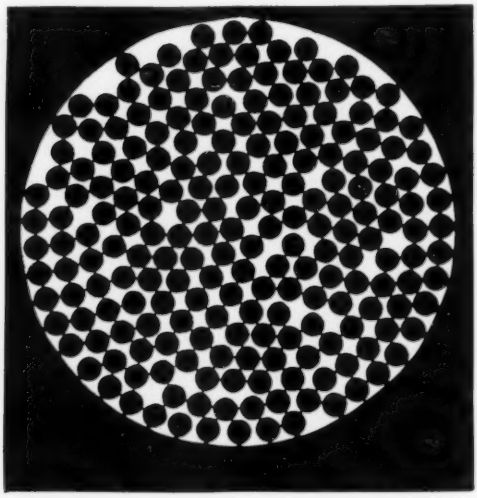


FIG. 31.—Packing in simple round container. Note the inability of the spheres to start and to retain any simple colony. Interference caused by the container wall is not limited to the immediately adjacent region but is propagated throughout the entire area. Several Case 6 colonies have formed, but are small because of interference.

of these compact chunky Case 6 colonies set in a surrounding meshwork of haphazard packing. It is important to note that it is the *colonies* which are nearly or quite isolated, while the haphazard mesh forms a conjugate interconnecting composite that winds here and there throughout the entire assemblage. The situation is somewhat analogous to a conglomerate, in which the chunky colonies

[59] Wherever, in the diagrams just cited, two sides of the triangle begin to separate, so that one void is connected through a narrow passage with another, systematic colony of Case 6 no longer prevails. In each diagram, the top of the assemblage as it was accumulated lies toward the top of the page.

of tightest packing correspond to the pebbles, while the enveloping three-dimensional mesh of haphazard packing corresponds to the conglomerate matrix.

With such an architecture, it is clear that the characteristic permeability of chance packing must be much more strongly determined by the relatively high permeability of the unbroken anastomosing mesh of haphazard packing than by the lowest possible permeability

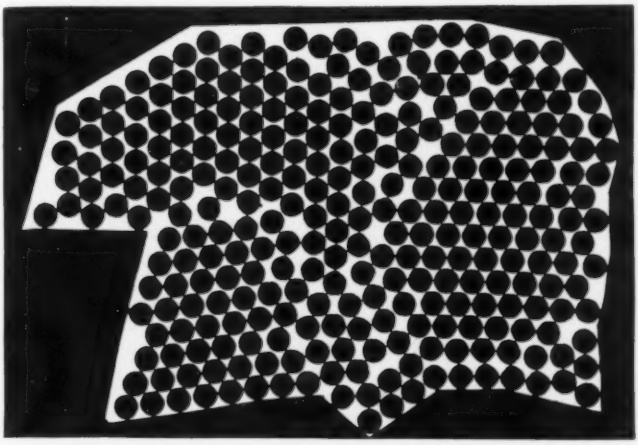


FIG. 32.—Packing in irregular container. The strong tendency to form Case 6 colonies is illustrated, but mutual interference of these as they grew produced relatively narrow intervening zones or "shells" of haphazard packing. Note the large voids (exaggerated "container effect") along the confining walls.

of the relatively isolated Case 6 colonies. (The subordinate colonies of other cases will ordinarily substitute for Case 6 colonies and will have the effect of rather mildly increasing the resultant permeability.) Obviously, the Case 6 colonies, by interrupting the continuity of the haphazard packing, will somewhat lower the permeability from what it would be if only identical haphazard packing were present throughout; but unless the Case 6 colonies are unusually numerous so that the intervening mesh of coarser texture is decidedly subordinate in amount, the lowering of permeability due to the colonies will not be extreme. All this means that much more of the

total flow will move through the somewhat tortuous route of the haphazard meshwork than will move by a more direct path that would carry it alternately through tight colonies and more open haphazard intervals.

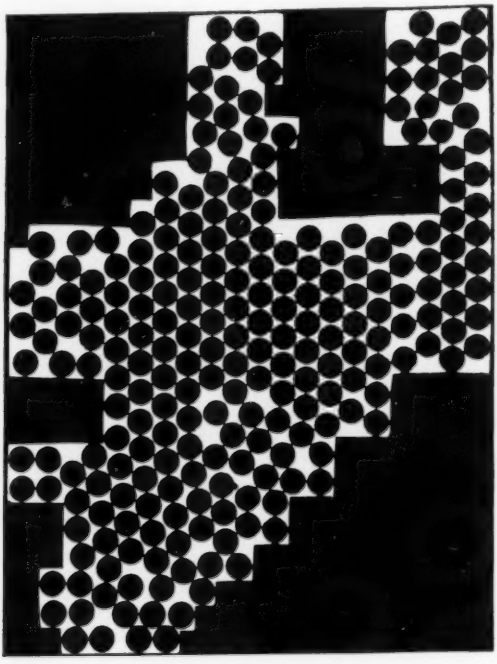


FIG. 33.—Packing in irregular container having 90° angles. Despite favoring angle, almost no Case 1 pattern develops. Unfavorable length of side walls occasions much disturbance and produces large voids. Relatively large Case 6 colonies form where far enough away from this marginal interference. Compare with Figure 34.

It is important to note that in considerable degree the haphazard quality of this meshwork is due to interference with orderly packing in the space between two or more orderly colonies which, for one reason or another, have ceased growing as such; these colonies, in short, induce the disturbing "container effect" upon the accumulation going on between them and cause in it an exaggeration of dis-

order. This serves further to increase difference in permeability of the colonies and of the mesh. It is also to be recalled that, because of the relative isotropy of haphazard packing (see p. 878) the meshwork of haphazard packing can operate with relative effectiveness throughout, and irrespective of the direction of the line between source and outlet for the assemblage as a whole.

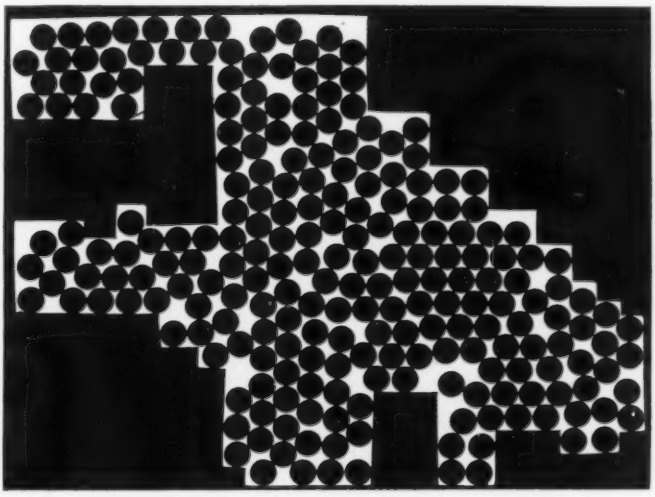


FIG. 34.—Packing in irregular container having 90° angles. This is the same container as in Figure 33, but having its top differently oriented. The pattern in detail is quite different from that in Figure 33, but the general nature of the pattern is similar, and due to the same influences of the container.

To sum up: the permeability of chance packing will ordinarily be very decidedly higher than the permeability of an equal volume arranged in tightest packing. Since the permeability of haphazard packing taken by itself probably averages higher than the mean permeability of Cases 2 and 4, and since the adverse effect of the interspersed colonies of Case 6 is, as a rule, probably relatively small, one may reach the guess that the mean permeability of chance packing lies well above that of Case 5, and perhaps is even higher than that of Cases 2 and 4.

Especially worthy of comment is the *localized* major flow through the assemblage, occasioned by the heterogeneous void distribution in colonies and mesh. This condition finds actual expression in the frequently mottled appearance of reasonably porous sandstones:

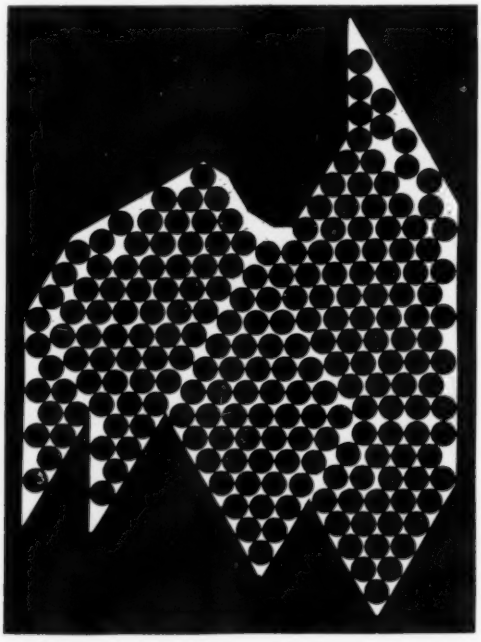


FIG. 35.—Packing in irregular container having several 60° angles. The favorable 60° angle encourages the formation of large Case 6 colonies; but unfavorable length of wall causes breaks in the continuity of these colonies even though the colonies have parallel orientation. On this account, the proportion of haphazard packing is exceptionally small. But note large voids at the container walls.

bleaching, discoloration, alteration, or introduction of new material may be seen, on a surface of what, to the eye, appears to be of uniform texture, to be distributed unevenly, in a sort of reticulated or mesh-like pattern, similarly to that of crazed pottery. Such localized places of special activity of solutions undoubtedly correspond to the more permeable portions of the mass, owing to the presence there

of haphazard packing, while the small included areas more free from the effects of the given activity represent the isolated colonies of tighter packing.

EFFECT OF CONTAINER WALL

Under this same heading in Part I, emphasis was placed on the high porosity due to the marginal voids next to the walls of the container. This increased porosity, as contrasted with that existing

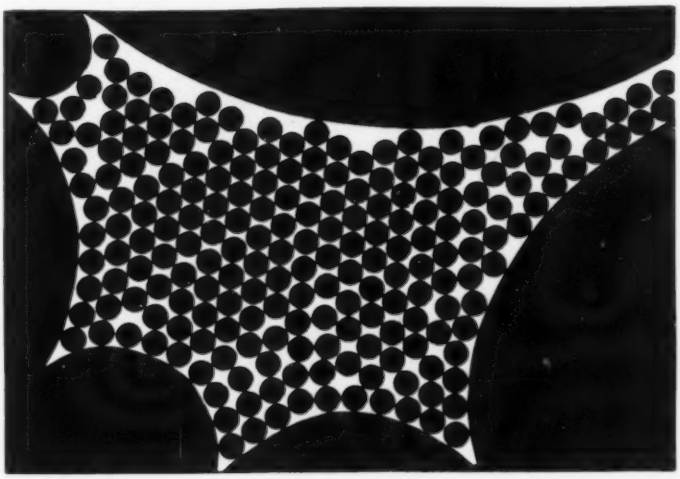


FIG. 36.—Packing in irregular container of curved walls. Case 6 colonies grow wherever possible (even if involving only three or four units); but the mutual interference and resulting open texture occasioned by their unlike orientation is a consequence of the limitations imposed by the walls. This figure may be taken to illustrate the packing of sand in a large irregular void between a group of pebbles. Note particularly the high porosity at the very top, i.e., immediately underneath the topmost pebble.

within the assemblage proper, is due to larger voids and greater cross sections at what would correspond to the throat-planes of the voids. Obviously, these conditions are strongly conducive to increased permeability along the container walls as compared with the permeability through the interior of the assemblage, and every known experience supports that conclusion. (See Figs. 14, 15, and 29–36 for illustration of larger and freer voids at the container walls.)

If the container wall is plane, or is a surface of gentle curvature, the possibility of more nearly straight-line flow along it than would generally be possible through the sphere assemblage would naturally favor the highest permeability along the wall; whereas a sharply irregular container wall would, in so far as mere directness of movement is concerned, be less conducive to marginal flow. However, the more irregular container wall would naturally cause greater interference in the packing and thus promote both high porosity and high permeability in its vicinity.

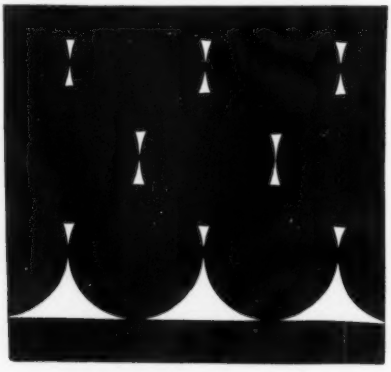


FIG. 37.—Effect of container wall. Contrast between net rectilinear ducts within the colony and those at container boundary, Cases 3 and 6. Compare with Figure 23.

The importance of especially high permeability at container walls can hardly be overemphasized. Of course, so far as concerns the permeability of the mass as a whole, this effect is of greatest relative importance in containers only a few sphere diameters across, and becomes progressively less important as the size of the container increases relative to the sphere diameter.

Unfortunately, there appear to be no quantitative data on the difference in permeability at container margins and within the assemblages. Figure 37 shows the relative sizes of net rectilinear ducts through a colony of tightest rhombohedral packing in Case 6 orientation built on a plane starting surface. The small double concave-triangles are characteristic of the assemblage proper; while the

larger triangles with straight base show, for the same angle of view, the net rectilinear ducts between the assemblage and the starting plane. The difference in cross-sectional areas is of the order of 5:1 in favor of the marginal ducts; therefore the relative advantage in permeability would be even greater still.

SLICHTER'S FORMULA FOR FLOW

Of the numerous attempts that have been made to derive a general formula for the quantity of water transmitted through soils, permeable rocks, and other open-textured materials, the analysis by C. S. Slichter[60] is especially pertinent in the present connection, since it was based directly on a theoretical consideration of the packing of uniform spheres. Slichter's general formula[61] is

$$k=[1.846]\frac{d^2}{K},$$

where k, known as the transmission constant, is the quantity of water at 10° C. that would be transmitted in unit time through a cylinder of the soil (spheres) of unit length and unit cross section under unit difference in head at the ends; d is the diameter of the soil grain (sphere); and K is a constant derived by Slichter from the porosity of the given packing.

This formula finds fairly frequent reference by engineers and others who have to do with the movement of fluids through permeable materials; and the formula, and especially the discussion of porosity and permeability which Slichter presented in connection with it, are particularly well known to geologists.

Slichter appreciated the theoretical nature of his analysis and derivation, and recognized that his formula would have to be tested against actual experiments in order to ascertain its degree of soundness and reliability. Many have tried to check the Slichter formula experimentally; the checks made in connection with the present study of permeability are discussed in the following paper. The common result has been a disappointing degree of disparity between

[60] *Op. cit.*

[61] The derivation of a formula for flow is only one of several useful and brilliant accomplishments by Professor Slichter in the paper cited.

what the Slichter formula predicts and what experiment actually finds. It is not the purpose here to discuss these disparities, but rather to examine critically the considerations of packing which Slichter took into account in deriving his formula.

The present authors are impressed by the fruitfulness of Slichter's approach to the quantitative measure of permeability through the consideration of sphere packing and by his ingenuity in translating the geometry of his problem into terms of the established principles of hydrodynamics with which he was so plainly intimate. It is our belief, however, that Slichter came to erroneous conclusions chiefly because of an inadequate and imperfect perception of the geometry of sphere packing, and that probably the failure of his formula to show good agreement with experimental data is largely a consequence of its development from a faulty analysis of the problem of packing. While recognizing the mathematical complexity of the task, the present writers lean to the opinion that, with an adequate understanding of the packing of spheres, a mathematical expression for flow could be derived that would yield a closer approximation to actual results than is given by Slichter's or any other formula thus far deduced. If the present largely qualitative discussion of the packing of spheres may help to incite some qualified expert to attack the question anew, the efforts of the authors will have been well rewarded.

In our opinion, Slichter made two fundamental errors: (1) he greatly oversimplified the geometrical characteristics of the unit void of tightest packing, which he used as the foundation for his mathematical analysis; and (2) he concluded that any natural or chance sphere assemblage of any given porosity has permeability characteristics equal to those of an orderly assemblage possessing the same porosity but attributable to unit voids all of *constant* size and shape. The first error affected the correctness of his formula; the second vitiated its applicability to chance packing. Moreover, independent of these two errors, we think that Slichter was inconsistent in developing his formula on the geometry of tightest rhombohedral packing and then proposing to apply it universally to a general rhombohedral packing which has obviously and vitally different geometry.

Slichter's primary difficulty with the unit void of tightest rhombohedral packing appears to have arisen from the fact that he failed fully to visualize its shape *as seen from within*, as a moving fluid would necessarily experience it. This is all the more surprising in view of his clearly presented illustrations of the elemental sphere grouping, the unit cell, and the shape of the unit void as seen from without (see his Figs. 54, 55, 58, and 59, respectively).

His basic mistake in this connection, which pervades his entire analysis, is that the duct element or "pore" through the unit cell of tightest rhombohedral packing (and by implication through the unit cell of "general rhombohedral" packing) is "approximately triangular in section." By "approximately triangular" he means that three-cornered shape between three equal circles each pair of which are tangent; two such shapes in characteristic relation are shown in the present paper in the simple rhombic layer of Figure 1, *B*, and also appear on all the simple rhombic faces of unit cells in Figure 5. This pair of concave triangles is the throat-plane section of the unit void of tightest rhombohedral packing when built as in Slichter's Figures 54, 58, and 59, and as in Figures 8, *B*, and 8, *D*, herein. Slichter, who continually spoke of the "triangular pore," plainly assumed that all other sections through the void are of this same shape, though of somewhat larger size—or, if not identically the same shape, then sufficiently identical so that the shape could be analyzed quantitatively by approximating it to an inscribed equilateral triangle.

Because he visualized the duct element as triangular from end to end, Slichter made the further mistake of concluding that each unit cell of the tightest rhombohedral packing (and, by implication, of the "general rhombohedral" packing) contains *two* such triangular duct elements or "pores" running approximately parallel to one another along curving lines through the unit void.

Both the conception of triangular shape of the duct element and the idea that there are two like duct elements in the unit void plainly came from viewing the void externally. Figure 20, *A*, shows a series of sections taken successively across the central line of a duct element of this tightest packing. Figure 20, *B*, shows another such series in a different direction. The pair of concave triangles constantly emphasized by Slichter appears in each series; but that particular

shape of section prevails over a length of the duct that is infinitely short—i.e., only at that section where two spheres are tangent at a point. The remaining sections through the duct element are of notably different shape and are of substantially greater area. Plainly, it is far from the fact to describe these duct elements as "approximately triangular in section." Moreover, the illustrations just cited show that there are not two "pores" but only one; at certain sections, the middle of this void pinches down to zero thickness; but, this being due to the external tangency of two circles, such pinching down prevails only at a point; the void is really a single void, of composite and complex shape (see Fig. 8, *E*).

Slichter recognized that the duct element alternately increases and decreases in cross section; that the concave-triangular section at the throat-plane (i.e., the half of Fig. 20, *A*, 1) lies where the cross section is smallest. This concave triangular shape, whose area he found to be $0.1613R^2$, being difficult to deal with, he sought to simplify by substituting for it a true equilateral triangle that would have the same transmission capacity. For the area of this triangle he had to depend on arbitrary judgment, and he adopted the figure $0.1475R^2$. We think that his reason[62] for deciding upon that particular value is faulty and the value too high.

[62] As between $0.1613R^2$, the area of the concave triangle, and $0.1247R^2$, the area of the true triangle inscribed in the concave shape, he thought that the equitriangular section having transmission capacity equivalent to that of the concave triangle should have an area greater than the mean between the two values given. We think that it should be less than the mean. For it seems to us probable that the narrow cuspate portions of the concave triangle which lie outside the inscribed triangle would have a transmission capacity *less* than half of that of an equal area having the average transmission capacity within the inscribed triangle. Slichter pretty plainly underestimated the restrictive effects on flow through the narrow portions of a given cross section. For example, on p. 319, having shown that a circular section will have no great advantage over an elliptical section of considerable eccentricity, he applied the same conclusion to various shapes of triangles. Yet he had already shown that a circular section will carry 38 per cent more flow than will an equitriangular section of equal area. Circle and ellipse of any but excessive eccentricity are both continuous curves, with no sharp breaks or narrow bays or cusps and with reasonably similar perimeters for equal areas; therefore they are relatively effective transmitters. But a triangle differs strikingly from a circle; its apices are far less efficient in transmission than is its central portion and its perimeter is substantially greater for the same area; therefore, in transmission capacity it compares very badly with a circle. If, instead of a true, straight-sided triangle, we have the "concave triangle," with its thin, sharply pointed cusps and still greater

Having thus arrived at an "equivalent" equitriangular area of the minimum cross section, Slichter next sought to ascertain the mean cross section, as the basis for determining the retarding effect on flow caused by alternately enlarging and decreasing cross section. By erroneously assuming the presence of two ducts through the unit cell, instead of one duct of more complicated shape, he was led to conclude that the change from minimum to maximum cross section takes place through an angular distance of "about 35°" (it would actually be 35°16′, or half of the octahedral angle, 70°32′), and happens twice within the limits of each unit cell.[63] But he then abandoned this 35° angle in favor of 30°, apparently for the sake of simplicity in ensuing constructions and computations. The effect of each of these departures from the true situation was, not to minimize the actual amount of enlargement from minimum to maximum cross section, but, on the other hand, to exaggerate the effect of the changing cross section through excessive number of assumed alternations from minimum to maximum cross section. Exaggeration also resulted from his assumption that the maximum and the mean cross sections lie in planes parallel to that of the minimum cross section; whereas such parallel planes actually lie oblique to the general line of flow through the cell and therefore could not be taken as true cross sections. On the basis of these several assumptions, by a process of integration he derived an area of $0.2118R^2$, which he took to represent the mean cross section of the duct. This area he compared with the $0.1475R^2$ adopted as the transmission equivalent of the minimum cross section, and found the larger figure exceeds the smaller by about 43.5 per cent. But, even if the other errors already mentioned did not enter, this $0.2118R^2$ is the area of an equilateral triangle inscribed within the (supposed) mean cross section (which he erroneously assumed to be a concave-triangle similar to, but larger than, the minimum cross section). The $0.2118R^2$ is thus neither the

perimeter, the transmission capacity in these narrow portions must fall off strikingly. This situation holds throughout the duct element, since nearly every section through it contains narrow cuspate portions.

[63] It is our view that, for the single duct element, the minimum cross section at the throat-plane changes to maximum cross section through an angle of 54-44′ and happens only once within each unit cell. This maximum cross section is the "concave-square" shown in Fig. 20, *B*, 1.

(supposed) mean cross section nor its equitriangular transmission equivalent. It cannot, therefore, be usefully compared with $0.1475R^2$; hence the indicated difference of 43.5 per cent between those two values is incorrect and too low.

On the basis of the findings just cited, Slichter concluded that the retardation due to changing area of cross section is negligible. The computation which led directly to this conclusion contained also another error in that the length of path of flow through which the change of cross section takes place is too short, as a consequence of taking too small an angular distance, as already mentioned in the footnote on page 893. But even apart from these difficulties, Slichter's mathematical treatment of this retardation is affected by the same underlying error of assuming that the duct element or "pore" is everywhere of the same *shape* and changes from place to place only in the matter of size.

From the principles of hydrodynamics, Slichter found that a cylindrical prism will transmit about 38 per cent more than will a prism of uniform equitriangular section of the same length and the same cross section. Therefore, since he had computed the flow through the duct as if the cross section were circular when it should be (as he assumed) a triangular cross section of the same area, he pointed out that an error of 38 per cent would be involved here, the computed value being by that much too high. But this, he said, was more than offset by an opposite error of 43.5 per cent, inasmuch as he had computed the flow for the (transmission equivalent of the) minimum cross section when, in fact, the mean cross section is (so he thought) of 43.5 per cent greater area. The resulting net error of about 5.5 per cent he stated would be just about balanced by omitting the use of a 6.5 per cent correction made necessary because a straight pathway of flow was assumed in his computation, when in reality the pathway, because of curvature, is 6.5 per cent longer. And so, he said, because these errors in the assumptions underlying the formula cancel out practically to zero, the formula may be taken as approximately correct; and the constants for use in it are therefore carried out to the fourth decimal place.

It is to be emphasized that most of the errors into which Slichter appears to have fallen arise from misconceptions or misconstruals of

the geometry or the intersphere void. But in any case, it seems hardly surprising that a formula derived as Slichter's seems to have been should fail to check with actual experimental results, even if the material were ideal spheres in tightest rhombohedral packing, as Slichter's actual derivations invariably assume.

Still less applicable can such a formula be to Slichter's "general rhombohedral" packing. For in an assemblage packed in that manner, the concave-triangular section which forms the starting-point and foundation of Slichter's formula for flow is entirely absent. Save only in those cases of the general rhombohedron where the face angle θ is very close to 60° do any of the void sections even approximate triangular outline; and in most of the possible values of θ, the voids are farther from triangular in cross section than is true for any of the six cases of simple systematic packing. Thus it is futile to hope that a formula developed on the invariable assumption of triangular cross section could be valid for "general rhombohedral" packing.

When application is attempted to less orderly and more natural packings, the situation is still worse. Yet it was for application to just such packings that Slichter devised his formula. Slichter said (*op. cit.*, p. 305):

> It would probably be admitted that no matter how complex a soil may be, there exists a certain ideal soil of uniform spherical grains that will transmit under given conditions the same amount of water that would be transmitted by the complex soil.

This is, indeed, admissible. But then, by the device of packing according to the "general rhombohedron" of variable face angle, he proceeded to set up an ideal systematic assemblage which has the same *porosity* as the complex soil. He then made the wholly unwarrantable and necessarily erroneous assumption that this ideal assemblage has the same *permeability*, because it has the same porosity, as the complex assemblage (or complex soil).

For in the complex assemblage—i.e., haphazard packing or chance packing—the voids are of variable size and shape—some small, some large, some relatively open in shape, others tightly cuspate. For purposes of *porosity*, it is entirely permissible to substitute for these various shapes and sizes the uniform voids (in a systematic packing)

each of which represents the *average* size of the voids of the complex assemblage. But for the purposes of *permeability*, such substitution of a standard shape and average size of void is entirely unjustifiable and wrong. For this could only mean that, for any given sphere-size, all assemblages with the same porosity have the same permeability; yet nothing could be farther from the truth.

Neither is Slichter justified in concluding (*op. cit.*, p. 310):

> If a given soil shows, on measuring, a porosity equal to m, we may assume, by the theory of probabilities, that the element of volume[64] which has a porosity m occurs in that soil more times than an element of volume having any other porosity.

For all experience indicates that, so far as uniform spheres are concerned, the most commonly occurring element of volume or unit cell is that of tightest rhombohedral packing, and that this is true regardless of whether the assemblage as a whole has high or low porosity. In short, the peculiar likelihood and stability of the 60 per cent rhombic sphere pattern suffices largely to nullify the abstract "theory of probabilities" in the sense used by Slichter.

It is perhaps essential as a final step to point out that Slichter's formula rests directly on the Poiseuilleian law of the flow of liquids through capillary tubes; therefore, even if the formula were absolutely error-free, there could be no assurance that the formula would be vindicated by experimental data on materials having voids well above capillary dimensions. Some of the conditions limiting the application of the Poiseuille formula are mentioned in the following section.

The primary purpose of this severely critical discussion of Slichter's formula for flow has not been to take issue with a fellow-investigator but rather, if possible, to stimulate further interest in a fundamentally sound and promising method of investigating mathematically the basic laws of permeability. Criticism of Slichter for errors of geometry is here deliberately coupled with acknowledgment of the difficulty of visualizing what would seem, at first thought, to be very simple space relationships; and the present authors feel in no sense certain that they have themselves escaped all the pitfalls.

[64] "Element of volume" is the same as the "unit cell" in the terminology of the present paper.

FORMULA OF FAIR AND HATCH

A highly ingenious mathematical formulation of permeability of various assemblages of sand is that presented recently by Fair and Hatch, to which reference has been made on pages 873 ff. By this means, as applied to its intended purposes, namely, the prediction of actual flow of water through beds of sand of the sizes ordinarily used for filtration, computed results are obtained which, we understand, check actual measurements of flow within less than 10 per cent. As such matters go, this is a gratifying degree of agreement.

The treatment by these authors may be regarded as divisible into two parts: a first part which deals with general relationships of flow through assemblages of separate solid units; and a second part which is concerned with details of shape and size of these units when represented as sand grains and with other matters of technical interest.

Professor Fair has kindly indicated to us that the first or general part of their treatment is entirely applicable to assemblages of uniform spheres. That obviously brings the matter directly within the scope of this paper. And inasmuch as, from that standpoint, a beautifully simple mathematical expression is developed to give permeability directly in terms of porosity for such ideal assemblages, the treatment merits careful consideration. That expression is herein referred to as the formula of Fair and Hatch. Let it be understood that the discussion which follows is not at all directed at the usefulness of the formula for the practical engineering purposes intended, but is strictly confined to the ability of the formula to express with mathematical exactness the quantitative relation of permeability to porosity in any given assemblage of ideally uniform spheres. We believe we shall be taking no liberties with the reasoning if we speak of "sphere" and "sphere assemblage," respectively, in place of "sand particle" and "sand bed," as used by Fair and Hatch.

It is our belief that the formula of Fair and Hatch represents in certain respects the best approach yet made toward quantitative expression of permeability in terms of geometry of the intersphere voids—but that it yields only an approximation, not a rigorous derivation. Inasmuch as continued search for an ideal formulation is to be desired, analysis of the present formula and its limitations seems advisable.

MANNER OF DERIVATION

The objective of Fair and Hatch in that part of their treatment here concerned is, in essence, identical with the objective held by Slichter. The latter proceeded toward his end by an investigation of the detailed geometry and properties of the individual intersphere pore or duct element. Fair and Hatch, on the other hand, seek the same outcome by treating the duct elements collectively in an aggregation and deducing generalized or statistical data for the average of the group. Both methods are equally dependent for their accuracy on exact adherence to the true geometry of the voids. In this respect, surprisingly enough, the generalized treatment by Fair and Hatch is more satisfactory than the individual treatment by Slichter.

With respect to flow, Fair and Hatch, like Slichter, start from the classic formula of Poiseuille for the flow of a fluid through a capillary tube of uniform circular cross section. This they utilize in the following form, convenient for their purposes:

$$\frac{h}{l}=\frac{k\mu v}{g\rho d^2},$$

where h = loss of head,
g = acceleration due to gravity,
μ = absolute viscosity of the fluid,
ρ = density of the fluid,
k = a constant of flow ($=32$),
l = length of capillary tube,
v = velocity of flow through the tube,
d = diameter of the tube.

They then proceed to translate this standard equation into terms directly dependent upon the relation of voids to solids in an assemblage of uniform spheres, in order to express permeability in terms of porosity, that is to say, in terms of the given manner of packing in the assemblage. The first four factors retain their identity and significance as given above; the last four factors, namely, k, l, v, and d, they treat in the manner indicated below.

1. *Velocity.*—Letting f = porosity (i.e., ratio of voids to total volume), and redefining v to be the velocity of approach, in the direc-

tion of eventual flow through the assemblage, of the fluid in the free space before reaching the assemblage, they show that v/f is proportional to the *mean* velocity of flow through the interstices of the assemblage; so they introduce v/f instead of v in the foregoing standard equation.

2. *Length.*—To avoid the difficult task of determining the actual length of path of flow, since most actual intersphere ducts through a sphere assemblage are sinuous and therefore longer than straight ducts would be, they assume that the distance of generalized travel may be taken as proportional to the dimension (thickness) of the sphere assemblage measured in the net direction of flow; so in the formula they use l as this dimension.

3. *Diameter.*—Since the intersphere ducts are not uniform circular cylinders of diameter d, as assumed in Poiseuille's formula, but are in fact of complex and varying cross section, Fair and Hatch effectively resort to the hydraulic radius; and they neatly show that the defining equation

$$\text{Hydraulic radius} = \frac{\text{cross-sectional area of duct}}{\text{perimeter of duct}}$$

may be translated, for the sphere assemblage as a whole, into the following highly significant and useful form:

$$\text{Hydraulic radius} = \frac{V}{A} \times \frac{f}{1-f},$$

where f = porosity, as before,
V = volume of the spheres,
A = surface area of the spheres.

Therefore, they assume that the diameter factor of the ducts is proportional to

$$\frac{V}{A} \times \frac{f}{1-f}.$$

4. *Constant.*—The constant k in the standard Poiseuille formula has a value of 32. The assumptions made by Fair and Hatch, which represent departures from the narrowly defined postulates underlying the Poiseuille formula, necessitate modification of that

value. It will be noted that the various factors proposed by them, to be introduced instead of v, l, and d of the standard formula, are not regarded as exact respective equivalents thereof but as respectively proportional thereto. It is therefore necessary to introduce what Professor Fair describes to us as "proportionality factors" in order to validate the direct substitution of their own v/f, l, and $V/A \times f/1-f$ for the conventional v, l, and d, respectively, of the Poiseuille formula. These proportionality factors they absorb collectively in a new factor K. The hydraulic radius of a duct of circular cross section is $d/4$. Therefore d^2 of the Poiseuille formula becomes $d^2/16$ if hydraulic radius is used instead of tube diameter; and hence $k = 32$ of the standard Poiseuille equation would become $K = 2$ for ducts of circular section when hydraulic radius is used instead of duct diameter—in other words, the proportionality factor appropriate for this particular instance would be $1/16$. But this value would hold only for ducts of circular cross section (for which the use of hydraulic radius would be no advantage); for different shapes of cross sections, the proper proportionality factor would not be $l/16$ but some different value, the magnitude of which apparently cannot be determined mathematically. Similarly, the proportionality factors involved in their substitutions for v and l are not subject to computation; it is to be presumed that these, likewise, would vary with different shapes and arrangement of ducts, that is to say, with different manners of packing of the spheres. On the basis of experimental work, Fair and Hatch conclude that the resultant effect of the several proportionality factors is to give to K a value which they fix at 5. Under the circumstances, it seems hardly proper to call this a *constant*—unless it may be permissible to assume that in their experimentation they had in every case an exact average of chance packing, in which case $K = 5$ might perhaps be regarded as the constant for such packing.

When the substitutions discussed above are made in the original equation, Fair and Hatch find that, apart from the value of K, the product $f^3/(1-f)^2 \times (V/A)^2$ contains the only elements of the equation that are influenced by sphere-size and manner of packing. Furthermore, since for spheres the value of V/A (which is the reciprocal of the specific surface) is constant at $R/3$ (where R = sphere

radius), and since porosity, f, is independent of sphere-size and dependent only upon manner of packing, it follows, according to Fair and Hatch, that for any given sphere-size, permeability is directly proportional to $f^3/(1-f)^2$. In short, this is their derivation of the quantitative relation of permeability to porosity. It is of striking simplicity, thus permitting its application by the non-mathematical layman to any assemblage of which the porosity and uniform sphere-size are known or can be ascertained. Its easy adaptability is shown in columns *V* and *VI* of Table III (p. 873).

LIMITATIONS OF THE FORMULA

For the purposes of the present paper, there remains to be examined how reliable this expression is, not as an approximation sufficiently satisfactory for certain practical ends, but as a rigidly precise measure of permeability in terms of porosity of any ideal assemblage of uniform spheres—precise in the same sense and degree that computation of the porosity for such an assemblage can be precise.

We are indebted to Professor Fair for the following analysis to be used in this connection:

> It should be obvious that the averaging influences relied upon by Fair and Hatch in setting up their equation on a Poiseuilleian basis are restricted by certain unevaluated influences. Poiseuille's equation itself is delimited by the requirement of streamline flow, which in circular tubes persists up to a Reynolds number of about 2,000. The Reynolds number is defined as the ratio of the inertia forces to the viscosity forces; or in mathematical terms as $vd\rho/\mu$. For a given fluid and velocity, therefore, streamline flow is attainable only up to a certain maximum diameter; for a given fluid and diameter conversely only up to a certain maximum velocity; and for a given diameter and velocity, finally, only to a certain minimum viscosity. The velocity and diameter terms introduced by Fair and Hatch include proportionality factors that are greater than unity in both cases. It follows, therefore, that the limiting Reynolds number for streamline flow becomes only a fraction of the value of 2,000 for circular pipes. It stands to reason, furthermore, that the controlling velocity in the Reynolds criterion, in the case of an assemblage of spheres, is that created in the smallest cross section of flow. This further reduces the magnitude of the critical Reynolds number. Streamline flow, therefore, is restricted within smaller limits. It does exist, however, and there is the experimental evidence of many workers that it is well maintained within the normal permeabilities of natural sand deposits and filters.

The Fair Hatch formula covers both the turbulent and streamline range of flow when expanded into the following form:

$$\frac{h}{l}=\frac{K}{g}\left(\frac{\mu}{\rho}\right)^{2-n} v^{n} \frac{(1-f)^{3-n}}{f^{3}}\left(\frac{A}{V}\right)^{3-n}.$$

Here n and K are coefficients depending upon the state of flow. For streamline flow $n=1$ and K approximates 5, as before; for turbulent flow n approaches 2 and K becomes smaller than 5. How far the proportionality factors that are introduced into the equation are substantially constant, as assumed by Fair and Hatch, can only be determined by experiments covering a wide range of packings. It should be obvious that for turbulent flow the rapidly changing shape and area of cross section of the ducts of an assemblage introduce new uncertainties of formulation both with respect to the concept of average velocity and *hydraulic radius*.

It seems clear that, without experimental determination of the value of K and demonstration of the existence of exclusively streamline[65] flow in each instance, this interesting formulation cannot reach definite, quantitative results. Therefore, it does not afford the much-desired mathematical derivation of permeability directly from the porosity or manner of packing.

For tubes of uniform cross section and capillary size, as used by Poiseuille, streamline flow of water persists up to high velocities. For the ducts of variable shape and size of cross section such as are present in sphere assemblages, Fair and Hatch tell us that they found turbulence appearing at moderate velocities when the spheres are

[65] The geometry of the intersphere ducts is powerfully conducive toward production of eddies and hence turbulent flow, through rapid expansions of conduit and rapid decreases in velocity in the direction of flow. Slichter, tacitly assuming streamline flow to prevail, concluded that the changes in cross section of the intersphere ducts have a negligible effect in retarding flow; but, as we have shown, his analysis leading to this conclusion involves serious underestimates of the changes in area and shape of the ducts, and therefore his conclusion is not dependable.

The nature of the typical intersphere duct may perhaps be best expressed in hydraulic terminology by analogy to a conduit consisting of a succession of special trumpet-shaped or bell-mouth sections facing alternately with and against the direction of flow; the smaller end of each such section joins with that of the next section in a smooth and continuous curve, but the facing larger ends abut at places around their peripheries as a sharp-angled cuspate joint, but elsewhere are separated by what may be termed "lateral branches" that lead into adjacent ducts parallel to the one being described; finally, these flaring sections are not truly trumpet- or bell-shaped, since their cross section is nowhere circular, but of ever changing and relatively complex shape (see Figs. 17–20). The handicap to true streamline flow at all points in such a conduit is obvious.

as large as 3 mm. in diameter. For openings of still larger size, as in conglomerates and boulder beds, turbulent flow would result from all but very low velocities; yet it is in just such coarse, open-textured assemblages that high velocities could be most easily attained in natural water movements.

On the other extreme, for very small sphere sizes, corresponding to silts, clays, etc., the specific surface becomes so great and the retarding influence of the surface forces so dominant that it would be expected that the formula must entirely lose applicability.

It remains finally to point out that a formula such as that by Fair and Hatch or the one by Slichter, even if entirely sound in all other respects, is necessarily incomplete with respect to the factor of orientation. If the attitude of the ducts is such that some one of the directions of easiest flow does not coincide with the direction in which flow is intended or with the direction leading from major source to major outlet, then only some fractional component of the abstract permeability of those ducts contributes to the actual flow. If the assemblage embraces a single systematic scheme of packing in a constant orientation, the particular inclination of the axes of this orientation to the desired or natural direction of flow will very effectively influence the amount of flow, regardless of what the formula might call for. Twinned and tripled packings, likewise, though not recognizable by the formula (since porosity is the same as in the corresponding untwinned packing), should give lowered permeability, independent of orientation of the mass as a whole. The scheme of Fair and Hatch, resting, as it does, on *averaged* results, would be expected to yield what would approximate *mean* permeability (see p. 872), provided the assemblage is large enough with respect to sphere-size to assure average chance packing and provided no layering or other tendency toward some special orientation has been introduced by the manner of accumulating the assemblage.

In the narrow strait between the surface forces (the rock Scylla) on the one hand and turbulence and orientation (the whirlpool Charybdis) on the other, it is evident that any attempt at rigorous formulation of permeability in terms of sphere-size and packing will have hazardous navigating. Nevertheless, it is earnestly to be hoped that new undertakings will continue to be made.

CAPILLARY PHENOMENA

Although the Poiseuille formula which underlies the work of Slichter and of Fair and Hatch assumes flow through ducts of only capillary cross section, the flow so contemplated is not "capillary" movement in the strict sense; it is, instead, hydraulic flow, being actuated by a pressure difference or "head" rather than by those more subtle forces of adhesion and surface tension.

For capillary rise and capillary migration, in the strict sense, therefore, the formulas just discussed and similar formulas have no application. Yet, in certain soil, water, petroleum, and gas problems, true capillary phenomena are of prime importance. A few observations may therefore be pertinent on the general way in which the peculiar geometry of the intersphere void affects and even controls capillarity.

Capillary manifestations are invariably associated with an "interface" or surface of separation either between a liquid and a gas or between two liquids of limited miscibility. The pertinent and critical dimension of a tube or duct with respect to capillarity is the dimension or cross section at the place where this interface stands and immediately ahead—i.e., in the direction toward which the interface meniscus is concave. If capillary movement actually takes place, the interface thereby takes up a new position, which then in turn determines the locality of critical dimension. The possibility of capillary movement of the liquid is inversely related to the magnitude of the cross section at that critical locality. That is to say, if the cross section there is sufficiently small (in terms of the work to be done if movement should take place), there will be capillary advance; conversely, if the cross section at that critical locality reaches or exceeds some limiting value (appropriate for each case), there will be no capillary advance from there forward—and this will be true even though the tube a little farther ahead and/or farther behind may be of sufficiently small cross section. Stated otherwise, if the (appropriately small) capillary character of the tube is anywhere interrupted by a portion of too large cross section, capillary movement cannot pass that interruption.

Capillary movement and hyrdaulic flow are thus reciprocal in

the respect that, whereas the former is favored in the narrowest and tightest voids and parts of voids, the latter is least favored at such places. Moreover, in large measure, the very conditions that favor the one tend to impede the other.

From these considerations, one might jump to the conclusion that in ducts of such configuration as those in an assemblage of spheres, (1) capillary movement and (2) hydraulic flow would co-operate, the former assuming the principal rôle in the narrow, cuspate portions of the voids and the latter taking dominance in the larger, freer mid-portions. But such simultaneous operation of capillary movement and hydraulic flow are physically incompatible; for to whatever extent hydraulic flow operates, it would serve to *fill* the voids with the liquid and thus destroy the interface essential for capillary movement. It is to be recalled, moreover, that save under very special circumstances there is no such thing as maintained capillary "flow." When the finite force that occasions capillary movement becomes balanced by the work that would have to be done (in the way of further lift of the liquid or overcoming still more friction) if movement were to continue, an equilibrium is attained and movement ceases, regardless of how inviting (in the capillary sense) may be the geometry of the ducts just ahead. Such equilibrium is ordinarily quickly reached.

In order to maintain a continued and far-traveling capillary movement that could appropriately be called *flow*, it would be necessary in some way continuously to remove liquid at the interface, either by its evaportaion into the adjacent gas or by some other means of abstraction, such as by chemical combination of the liquid with the material of the walls.

Now in the geological discussions of the capillary migration of fluids it seems too commonly to have been taken tacitly for granted that the voids in sands, clays, sandstones, and other permeable materials of reasonably fine grain constitute sets of long capillary tubes through any portion of which capillary movement may take place. One may be more specific by saying that it seems often to have been assumed that these ducts are of *constant* diameter so long as they

are contained within sands, etc., of the same grain-size. Yet such an assumption is, of course, entirely wrong.

To whatever extent the natural packing of sand grains approximates the packing of uniform spheres (and, in fact, the approximation in many instances is relatively close) the through-going ducts between the sand grains are certainly *not* of constant cross section, but show rapid alternation of swellings and pinchings throughout their length.

It follows, therefore, that while capillary movement might be important in a duct which everywhere had the same cross section as the *minimum* cross section of the ducts in a sand of given grain-size, no appreciable capillary movement would take place in the actual ducts of that sand, since the interface would stop short upon reaching the first (supercapillary) enlargement of the duct and would not budge from that position.[66]

A further handicap to capillary movement arises from the fact that the intersphere or intergrain ducts are not independent and individually isolated tubular units, but that they are, instead, effectively interconnected with one another. It is not, therefore, a question of maintaining capillary movement despite periodic enlargements of two or three times the area of the minimum cross section. It is, instead, a question of attaining any appreciable capillary movement whatever through openings which, at their places of periodic enlargement, quite effectively lose their bounding walls and become connected with similar openings lying near and far on all sides. In a somewhat exaggerated way, the chance for capillary migration through such an assemblage could be compared with the chance through a long line of very short sections of capillary tubing with each section separated from its next neighbor by an open gap greater than its own length. The short sections of capillary tubing in this

[66] It is to be noted that the situation with regard to capillary *advance* is quite different from that whereby, in a tube that has already been filled with liquid by some means or other, the presence of a constriction of some given capillary dimension (and not located higher than the range of capillary rise in a uniform capillary tube of that same dimension) can prevent *fall* of the liquid level regardless of how large the tube may be below that construction. Cf. O. E. Meinzer, *U.S. Geol. Surv. Water Supply Paper 489*, p. 23. In short, capillarity can *hold* what it cannot *acquire* for itself.

analogy would correspond to those minimum sections of intersphere ducts at the throats, and the intervening gaps would correspond to the enlargements in the middle of the duct elements where there is uninterrupted lateral connection with adjacent ducts.

It seems obvious, then, that the phenomenon of capillary movement in sands and similar assemblages is rendered decidedly complex and substantially more difficult of realization by the special geometry of the intergrain voids. Partial closing of these voids by cementation may, when accomplished in just the right way and to the right degree, serve to bring closer approximation to parallel sets of through-going and mutually-independent or isolated capillary tubes, in which cases capillary movement would, of course, be greatly facilitated.

There remains for final consideration the capillary movement that takes place in an assemblage of spheres or grains by other means than by utilizing the intergrain ducts as capillary *tubes*. It is, of course, well known that there will be a capillary rise, above the normal hydrostatic level, in the sharp angle between two plates (as of glass) held partly immersed and touching along a common upright edge. This rise will be greatest right at the line of intersection of the two plates and will lessen as the plates become farther apart, declining to zero when the distance between the plates becomes "supercapillary." Now this manifestation of capillary movement in no way involves a *tube;* that is to say, a duct completely surrounded on all sides is not an indispensable condition for capillary movement. The requirement that is indispensable is the presence of two solid surfaces lying within less than supercapillary distance of one another.

Let us suppose, then, that we have an assemblage of spheres of such large diameter that even the smallest cross sections of the intersphere voids are of supercapillary size. It might therefore appear, at first thought, that there could be no appreciable capillary movement in such an assemblage. It is true that the voids, when regarded as enclosed tubular ducts, are too large to cause capillary movement that would, as far as it went, completely fill the ducts. But these ducts are bounded at countless places by the converging and eventu-

ally tangent walls of spheres; and no matter how large the spheres, the narrow angle or cuspate region sufficiently close to the point of tangency must in every instance be of less than supercapillary width. Therefore, in this narrow wedge of space, capillary movement can take place. This state of affairs would correspond to what J. Versluys[67] calls "pendular saturation."

If the radius R of the spheres is relatively large, the distance away from the actual point of sphere tangency to which the capillary phenomenon can hold will be relatively small as measured in terms of R; on the other hand, if R is small enough, the cuspate wedge of liquid held by capillarity may extend far enough (in terms of R) away from the point of sphere tangency to just reach and merge with the identical cuspate wedges of liquid held around the nearest neighboring points of sphere tangency. In this way there might be a sort of perforated mesh of liquid extending through the entire assemblage, filling all the narrow cuspate (throat-plane) portions of the intersphere voids, yet leaving unfilled (except by air or other gas) the larger mid-portions of the voids. This Versluys would call "funicular saturation."[68] Depending upon the exact size of R (with respect to the characteristics of the particular liquid and the surface characteristics of the material of the spheres), the liquid might reduce but not completely close the opening at the throats, in which case there would still remain interconnection between all the partly gas-filled voids; or, on the other hand, the liquid might completely stretch across the voids at their minimum throats, thus completely isolating bubbles of gas (air) in the mid-portions of the unit voids. In the latter case there would be produced what is probably to be regarded as a manifestation of the apparently ill-understood "Jamin effect."[69]

It would seem that this type of capillary behavior merits further investigation than it has thus far received.

[67] *Amer. Assoc. Petrol. Geol. Bull. 15*, Part I (1931), pp. 189–200.

[68] *Ibid.*

[69] Cf. S. C. Herold, *Amer. Assoc. Petrol. Geol. Bull. 12*, Part I (1928), pp. 659–70; and J.Versluys, *op. cit.* See also K. Schulte, *Das kapillare Gelichgewicht, Gesundheits-ingenieur* (1926), pp. 318–19.

If this paper may be closed with a digression from the dominantly geometrical to the more geological viewpoint, it might be stated that certain considerations seem to suggest that capillary phenomena of the kind mentioned last above, in which the liquid partly, but not completely, fills all the voids (or most of them), may play an important rôle in certain geological operations outside those few for which it has been invoked in connection with petroleum. It may well be that this manner of movement of water and of distribution of water and air are to some degree involved in that region of rocks lying above the water level and known technically as the "vadose zone." If so, several of the puzzling questions concerning the mechanisms and the rate of oxidation in that zone might find a considerable degree of illumination.

EXPERIMENTAL STUDY OF THE POROSITY AND PERMEABILITY OF CLASTIC SEDIMENTS

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ABSTRACT

The purpose of this study is threefold: to experimentally evaluate the factors controlling the porosity and permeability of ideal clastic materials; to determine the modifications produced in natural deposits by variations from the ideal; to apply the principles of permeability to the problem of the distribution and localization of mineral deposits in clastic sediments.

The effect on porosity of absolute grain-size, variable grain-size, shape of grain, method of deposition, and compaction are experimentally determined for ideal materials and compared with those of river and beach sands and other clastic sediments. Various equations for permeability are discussed; and the controlling factors of temperature, hydraulic gradient, size and shape of grain, uniformity of grain-size, porosity, and stratification are experimentally evaluated for ideal materials, and the modifications caused by natural materials are delimited and discussed. Field observations on the effects of wave and river depositional processes on porosity and permeability are presented. Comparisons are made of the relative permeability of gravel, sand, clay, loess, till, and unsorted alluvium. Changes produced in the original porosity and permeability of clastic sediments by the metamorphic processes of compaction, cementation, and recrystallization are outlined. Finally, the principles governing the flow of fluids through unconsolidated clastic materials are applied to the flow of hydrothermal solutions through rocks of clastic origin in an attempt to use the principles of permeability to explain and predict the distribution and localization therein of hydrothermal mineral deposits.

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Introduction

Many geological processes are greatly influenced by the porosity and permeability of the rocks involved. This is particularly true in the case of sediments. Permeability plays an important rôle in the accumulation, migration, and distribution of oil, gas, and water. Porosity and permeability vitally influence the compaction and consolidation of sediments; and these processes are important in connection with oil reservoirs, foundations, dam-sites, and filter beds. The sometimes puzzling distribution of alteration effects and the location of epigenetic metalliferous deposits in sediments would be materially clarified by a complete knowledge of the permeability conditions existing at the time of introduction of the altering and mineral-depositing solutions.

However, both the porosity and permeability of clastic sediments are subject to marked changes as the sediments undergo geological evolution. Problems dealing with the porosity and permeability of sediments are therefore twofold: first, "What are the importance and limitations of the factors which govern the porosity and permeability of unconsolidated sediments?" second, "How have these properties been modified by later geological processes?" These problems become increasingly complex, and modifications of the theory based on simple and ideal conditions become more marked in proportion to the evolution undergone by the sediment. Nevertheless, no sound comprehension may be had of the more complicated actual problems unless the factors involved under ideal conditions are first understood.

As a first step, therefore, in the attempt to analyze the conditions

of porosity and permeability of natural sediments, study was given to the geometry of assemblages of ideal spheres of uniform size, and of the relations thereto of porosity and permeability. The results of that study are set forth in the preceding paper, entitled "Systematic Packing of Spheres—with Particular Relation to Porosity and Permeability," by L. C. Graton and H. J. Fraser. Frequent reference to those results will be made herein.

In the present paper, consideration is given to progressive degrees of departure from the ideal shapes dealt with in the article just cited—these range from spheres of different sizes, with the various sizes in different relative proportions—to the several kinds of natural clastic sediments, in which sizes, shapes, and proportions of sizes and shapes of the component grains vary between wide limits, with consequent important effects on packing and on porosity and permeability. Some attention has also been given to the manner in which waves and rivers transport, arrange, and pack their deposits; also to the effects of the various processes of consolidation of sediments upon their original porosity and permeability. Finally an analysis has been attempted of the influence of permeability on the movement of hydrothermal ore solutions. In the following pages these major problems are considered in a general way, while the individual factors affecting porosity and permeability are discussed in detail. So far as possible, the treatment is quantitative.

All experimental determinations were made in duplicate, and many were repeated several times in order to insure reliability. The data on the porosity of spheres are accurate to within $\pm .01$ per cent; those on the porosity of sands *in situ* to $\pm .05$ per cent; and the measurements of permeability are probably accurate to within 1 per cent for the particular packing involved in each case.

This paper includes many of the results obtained during parts of the academic years of 1928–29 and 1929–30 and the whole of 1930–31. The experimental work was performed in the Laboratory of Mining Geology, Harvard University. The field data were obtained in part from New England rivers and beaches, especially in the vicinity of Boston, Massachusetts, and in part from the Willamette and adjacent rivers in Oregon, and from the copper-bearing conglomerates of Michigan.

In these pages will be found much that was already known. Some of this had been developed in special connection with water supply, or with soils, or filtration, or foundations, and, above all, in recent years with particular reference to petroleum. But so far as the writer is aware, the range of information pertinent to the highly important problems of porosity and permeability has nowhere else been selectively assembled and arrayed with respect to useful application to these particular problems arising throughout the wide range of primarily *geological* interest.

Since the spring of 1932, the writer has been constantly engaged in the field, where neither time nor accessibility to the literature has permitted attention to be given to the growing current output regarding porosity and permeability. There is collateral evidence that some of these later publications are of high value and suggestiveness, and there can be no doubt that these subjects are now receiving both more attention and more skilful and reliable treatment, especially in the quantitative and experimental directions, than was the case not long ago. Individual mention should here be made of the brief synopsizing of a surprisingly comprehensive list of writings upon these subjects by Fancher, Lewis, and Barnes[1] and of a series of excellent papers by Wyckoff, Botset, Muskat, and Reed,[2] all of which will be found most welcome.

One need do no more, however, than to glance at representative titles of current articles in this field to recognize that the emphasis is being placed more and more on the details of technology, especially in the petroleum and natural-gas industries. Unquestionably, much of general geological applicability will come from inquiries of that kind, but it will almost inevitably be mingled with a large proportion of technological or other specialized minutiae. Generalized summaries from a broader viewpoint will be needed from time to time and will undoubtedly appear.

This general study of permeability was first made possible through a fund generously contributed to the Laboratory of Mining Geology,

[1] G. H. Fancher, J. A. Lewis, and K. B. Barnes, "Some Physical Characteristics of Oil Sands," *Pa. State Coll. Bull 12* (1933), pp. 66–167.

[2] R. D. Wyckoff, H. G. Botset, M. Muskat, and D. W. Reed, "Measurement of Permeability of Porous Media," *Amer. Assoc. Petrol. Geol. Bull 18* (1934), pp. 161–90.

Harvard University, by the Roan Antelope Copper Mines, Ltd., the Rhodesian Selection Trust, Ltd., both of London, and the American Metal Company, Ltd., of New York, and was arranged through the courtesy and the active interest in geological science of Messrs. L. Vogelstein and Otto Sussman, of New York. The investigation has further been aided by a fellowship from a fund established by Mrs. James J. Storrow, of Boston, and administered by the United States National Research Council, Division of Geology and Geography. The field investigation during the summer of 1930 was in part financed by a Carlton Thayer Brodrick Fellowship from Harvard University. Grateful appreciation is expressed to all who have thus enabled this inquiry to be carried out.

The writer acknowledges his deep obligation to Professor L. C. Graton, who suggested and outlined the problem, helped to obtain the financial support for it, and actively participated in the entire investigation. It is a pleasure also to acknowledge the assistance received from Professor Kirk Bryan, who kept in constant touch with the progress of the study and offered valuable suggestions and criticism. The writer is indebted to Professor D. H. McLaughlin for constructive criticism of the apparatus and the experimental procedure; to the late Professor O. D. Kellogg, for assistance in the mathematical study of packing; and to Mr. E. Graustein, for permission to reproduce a graph prepared by him on the effect of the shape of openings on the flow of liquids. Mr. E. B. Dane, Jr., kindly collaborated in preparation of a number of the illustrations.

Part I. Porosity

Preliminary statement

Porosity, as viewed from the geological standpoint, may be divided into two classes: original and secondary. Original porosity is an inherent characteristic of the rock and was determined at the time the rock was formed. Thus there is an original porosity in sand resulting from the physical impossibility of packing the sand grains in such a way as to exclude interstitial voids of conjugate nature; and in a pumiceous lava the evolved gases, unable to escape completely, formed vesicular voids of the isolate kind. Secondary porosity results from later changes undergone by the rock, and may either in-

crease or decrease the original porosity. Thus sandstone may decline in porosity by further cementation or may gain in porosity by fracturing or by dissolution.

The porosity of any rock, as well as the magnitude, form, and distribution of its individual voids, is therefore controlled by the origin of that rock and by the processes which have later modified it. In general, rocks whose constituent minerals crystallized in place—whether igneous, metamorphic, or sedimentary—have an original porosity of the same general order of magnitude, and relatively low. Clastic rocks, on the other hand, have an original porosity fundamentally different, not only in proportion of voids present but also in their shape, size, distribution, and continuity.

The eventual porosity of clastic sediments is determined by the combined effect of many factors, each of which may vary quite independently of the others. The problem is one which courts solution experimentally, and a considerable amount of work has been performed upon it. But unfortunately, the natural phenomena and the experimental observations and data have not been thoroughly correlated. The present investigation has had as one of its objects the improvement of this situation.

FACTORS GOVERNING POROSITY

Direct and reliable comparisons[3] between the porosity conditions existing in assemblages of uniform spheres and those of natural deposits cannot ordinarily be made, chiefly because perfectly spherical grains all of uniform size are probably never present in natural accumulations. The following factors modify the ideal conditions pre-

[3] Porosity may be determined in numerous ways. An excellent discussion of various methods and their relative accuracy is given by O. E. Meinzer (*U.S. Geol. Surv. Water Supply Paper 489* [1923], pp. 11–17). During the present investigation, the porosity of various mixtures of spheres was measured as follows: The specific gravity of the different sizes of lead spheres was carefully determined. The spheres were then placed in a container of accurately known volume, packed as desired, and weighed. It was found advisable to adopt a standard method of jarring. From the known weight of spheres in the container, their solid volume could be computed, and the porosity calculated. This method gave consistent results and was the most convenient to use under the conditions existing in these experiments. The porosity of various mixtures of spheres was determined by a method already described by the writer ("Sampling Incoherent Sands for Porosity Determinations"; *Amer. Jour of Sci.*, Vol. XXII [1931], pp. 9–17).

viously described, and control the porosity of unconsolidated natural deposits:

1. Absolute size of grain,
2. Non-uniformity in size of grain,
3. Proportions of various sizes of grain,
4. Shape of grain.

In addition, porosity is affected by the following more general factors:

5. Method of deposition,
6. Compaction during and following deposition,
7. Solidification.

Each of these factors will be considered separately and its relative influence and importance evaluated.

EFFECT OF ABSOLUTE GRAIN-SIZE

Theoretically, the actual size of unit has no influence on the porosity of uniform spheres. But this does not prove true for assemblies of natural materials. Actually, as the grain-size decreases, friction, adhesion, and bridging become of increasing importance, because of the higher ratio of surface area to volume and mass; and therefore the smaller the grain-size, the greater is the porosity. Thus, determinations by Lee[4] on thirty-six samples, ranging from coarse sand to silt, indicate total voids as follows: coarse sand, 39–41 per cent; medium sand 41–48 per cent; fine sand, 44–49 per cent; fine sandy loam, 50–54 per cent. The average porosity of the thirty-six samples was 45.1 per cent. Terzaghi[5] found for small particles a range in porosity from 50 per cent for particles larger than 0.02 mm. to about 95 per cent for material smaller than 0.002 mm. Trask[6] also found that porosity increases as grain-size decreases. Doty[7] states that increasing the grain-size increases the porosity; but his results are in part due to factors other than the grain-size, as shown by his experiments.

[4] A. J. Ellis and C. H. Lee, "Geology and Ground Waters of the Western Part of San Diego County, California," *U.S. Geol. Surv. Water Supply Paper 446* (1919), pp. 121–23.

[5] C. Terzaghi, *Engineering News Record*, December 3, 1925, p. 914.

[6] P. H. Trask, *Amer. Assoc. Petrol. Geol. Bull. 15* (1931), p. 273.

[7] R. J. Doty, "Establishing a Method for Testing Green Bond Strength," *Trans. Amer. Foundrymen's Assoc.*, Vol. XXX (1922), pp. 796–822.

As the grain-size decreases, the tendency toward bridging, with consequent looser packing, is undoubtedly augmented by increased variation in shape and differences in the method of deposition. The latter two factors are independently considered later.

NON-UNFORMITY OF GRAIN-SIZE

Whether the grain-size is uniform or non-uniform is of fundamental importance with respect to porosity. The highest porosity is commonly obtained when the grains are all of the same size. To such an assemblage, the addition of grains of either a larger or a smaller size tends to lower the porosity; and this lowering is, within certain limits, directly proportional to the amount added. These generalizations are made clear by consideration of assemblages containing spheres of two sizes.

If an assemblage of uniform spheres is compared with an otherwise similar assemblage which contains a considerably larger sphere, the large sphere will affect the porosity of the assemblage in two ways:

1. It will reduce the porosity by completely filling with solid material a region which could otherwise be occupied by a combination of small spheres and voids.

2. It will increase the porosity by causing looser packing in its vicinity than is "normal" for the small spheres alone.

Owing to the curvature of the surface of a large sphere, it is impossible for the small spheres around it to pack as closely as they would on a plane, or against other small, tightly packed spheres. Accordingly, these small spheres have two alternate ways to pack: if they pack closely around the large sphere, then farther away they must pack more loosely than they would pack if no large sphere were present at all, for the disturbing effect of the large sphere may be "propagated" outward for substantial distances from it; if, on the other hand, they retain approximately their "normal" packing in the regions at some distance from the large sphere, then they must pack more loosely in its immediate neighborhood.

The two influences just listed work in opposite directions; but they are not of equal value, and the net result is to decrease the porosity. As the large sphere and the small spheres become increas-

ingly different in diameter, the relative value of the first influence mounts and that of the second declines; therefore porosity progressively decreases.

The foregoing conclusions not only apply to the case of a single large sphere in an assemblage of smaller ones but also apply with increasing force as the number of large spheres in such an assemblage increases up to, but not beyond, a limiting proportion discussed later.

Critical Ratios of Grain-Size

If a number of (larger) spheres of equal diameter are arranged in any strictly systematic packing, there is a certain diameter-ratio for a (smaller) sphere which can just pass through the throats between the larger spheres into the interstitial unit voids. This ratio is termed the "critical ratio of entrance." For tightest packing (Cases 3 and 6 [see preceding paper, "Systematic Packing of Spheres"]) the critical diameter is $0.154D$, and for loosest systematic packing (Case 1) it is $0.414D$, where D is the diameter of the large spheres.

Similarly, there is a critical ratio for the diameter of a sphere which, although too large to pass into the void between spheres of a larger size, can exist in such void without disturbing the packing of the larger spheres; obviously, it would have had to reach that position at the time when the large spheres were assembling. This may be called the "critical ratio of occupation." This ratio for tightest or Case 3/6 packing is $0.414D$[8] for the large void and $0.225D$ for each of the small voids per unit; and for loosest, or Case 1, packing is $0.732D$. Assuming an assemblage of uniform spheres to be systematically arranged according to the general rhombohedral packing of Slichter[9] so that the porosity is 37 per cent (and face angle θ therefore 68°18′), the critical ratio of entrance would be $0.208D$ and the critical ratio of occupation would be $0.437D$. But since such a "general rhombohedron" would be highly unstable and thus improbable of occurrence, a sphere assemblage having a porosity of, say, 37 per cent would undoubtedly not represent a single systematic packing

[8] Note that the size of sphere that will just enter the throat of Case 1 packing will just occupy the unit void of tightest packing.

[9] C. S. Slichter, "Theoretical Investigation of the Motion of Ground Waters," *U.S. Geol. Surv., 19th Ann. Report*, Part II (1899), p. 312.

throughout, but rather would be chance packing, varying from place to place. This would involve different sizes of void from place to place. It has been shown ("Systematic Packing of Spheres") that chance packing normally involves colonies of tightest packing, which would therefore have the critical ratios appropriate thereto as given above, and intervening zones of haphazard packing, for which no significant or constant critical ratios exist—each individual void would have a critical ratio of its own that would depend on its own size and shape.

The number of uniform small spheres which can be packed in any one unit void of an assemblage of uniform larger spheres varies widely with the relative diameters of the two sizes.[10] In tightest packing, as already noted, one sphere, and only one sphere, of diameter $0.414D$ can exist in the void. In order that two spheres may occupy this void without disturbing the packing of the larger spheres, their diameters must be considerably smaller than $0.414D$; and for all spheres between $0.414D$ and this smaller diameter only one such sphere can exist in the unit void, thus leaving the rest of the space unoccupied. Similarly, as the unit void is filled with successively

[10] The late Professor O. D. Kellogg computed for the writer the approximate number of small spheres of the critical ratio of entrance ($0.154D$) that could be most closely packed in the unit void of tightest (Case 3/6) packing and found that the number is of the order of 45. The porosity of tightest packing being 25.95 per cent, this would mean that a maximum of about 45 spheres of diameter $0.154D$ could be placed in the peculiar shape of unit void in tightest packing which has a volume of $0.2595D^3$. In an experiment analogous to this computation, the writer used a natural assemblage of larger spheres which, after jarring to compaction, possessed a porosity of about 36.4 per cent (which is well within the usual range of porosities for chance packing). The critical ratio of entrance for this assemblage was taken at $0.208D$, which, as already stated on page 919 would be correct for a *systematic* packing of substantially the same porosity, viz., 37 per cent. It was found that, by jarring, an average of 55 of the smaller spheres could be made to occupy each unit void of the assemblage of larger spheres. The correspondence between the computed and the experimental results is reasonably good, in view of the difference in porosity of the packings in the two examples, and of the fact that the computed case assumed the most compact possible arrangement of the smaller spheres whereas in the experimental case the smaller spheres were packed "naturally" and surely less compactly.

In this connection the general observation may be made that in all the experiments recorded in this paper, the packings are those which result from chance assembly plus compaction to a stable volume by jarring. This inevitably gives a somewhat higher porosity than if each individual sphere could have been placed in exactly the right way to secure the most compact arrangement possible.

smaller and smaller spheres, there will be a given size of these latter which will permit a whole number of spheres just to occupy the void; for a size only a little smaller, the number will still be the same, but there will be more free space. Then there will be a still smaller size which will again permit a whole number of spheres just to occupy the void; and such exact "filling" of the void by a whole number of

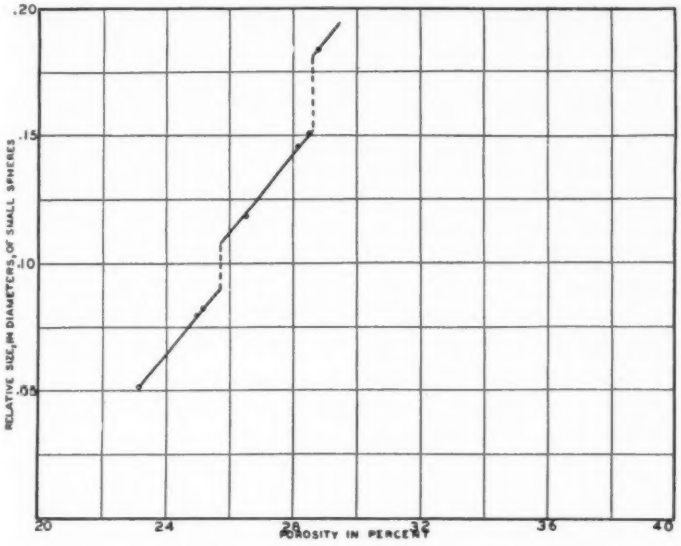

FIG. 1.—Influence of matrix grain-size on porosity of binary mixtures

spheres will continue periodically as the diameter of the smaller of the two sizes is decreased.

It follows that, although decrease in the size of the smaller of the two sphere sizes tends, *on the whole*, to decrease the porosity of the combination, the decrease should not be steady but jumpy. This was subjected to experimental verification. If the voids in an assemblage of spheres be filled successively with spheres of increasingly smaller diameter, the resulting porosity values for the combination are found not to fall on a smooth curve, but decrease by sudden steps, as shown in Figure 1. The procedure in this experiment was as

follows: The voids of a fixed assemblage of large spheres were filled with smaller spheres having diameter less than the critical ratio of entrance. The mass was jarred until no further compaction was obtained, more small spheres being added as the mass settled. The porosity of the combination was determined. Determinations were similarly made with successively smaller sizes of the small spheres. During all these trials the large spheres were fixed in position so that their voids did not change in shape or volume. The porosities were obtained by the specific gravity method. The small circles on Figure 1 show the several values secured.

The theoretical values for "critical" diameters cannot be rigidly applied to natural deposits. The figures already given for the critical ratios of entrance and of occupation assume systematic packing—a condition which does not prevail in natural deposits. It is obvious, for example, that a mass of chance packing with a porosity of 37 per cent could not have all of its voids filled by infiltration of smaller grains $0.208D$ in diameter. The voids of the Case 6 colonies could be entered only by grains with a diameter of $0.154D$, while the voids in the haphazard portions could be entered by varying larger sizes, depending on the particular size and shape of these individual voids.

However, it does follow from the preceding discussion that if the material filling the interstices of a conglomerate is mostly larger than 0.154 times the diameter of the large pebbles, or if there is any material larger than $0.414D$, then the "fines" have not been washed or sifted into the interstices of the large pebbles, but have been laid down contemporaneously. The same reasoning holds for clastic sediments of smaller grain than conglomerate.

PROPORTION OF GRAIN SIZES

Two-Component System: Spheres

When two sizes of spheres are mixed, the smaller size dominates the general structure of the assemblage so long as the proportion of its spheres is sufficiently great to keep most of the large spheres separated from one another; under such conditions, the small spheres and their own intervoids play the rôle of matrix while the large spheres constitute alien intruders which demand accommodation and impose disturbance on the matrix yet accept the general control

of the small spheres. In these circumstances there is a fairly uniform drop in porosity of the combination as the proportion of large spheres grows, for the reasons given on page 918.

When the proportion of large spheres increases beyond the limit just mentioned, two alternatives arise, depending on the relative sizes of the two lots of spheres. When the small spheres have a diameter less than the critical ratio of occupation, and the proportion of large spheres becomes just great enough to allow all the large spheres to touch one another and be self-supporting without any dependence on the small spheres, then the large spheres take over the control of the assemblage structure, and they retain it thenceforward with further increase in proportion of large spheres. At that particular proportion of the two sizes when this turning-point in control from small spheres to large spheres takes place, the small spheres just succeed in "filling" the voids between the large spheres without producing any distortion or disturbance of the packing of the latter. Thereafter, as the proportion of large spheres further increases, the small spheres no longer suffice to "fill" the voids of the large-sphere structure, and the extra space in these voids remains open. Therefore, from this turning-point onward, increased proportion of large spheres gives increasing porosity for the combination.

On the other hand, when the small spheres have a diameter exceeding the critical ratio of occupation, and the proportion of large spheres increases above the limit of domination by the small spheres, then and thereafter the two sizes mutually interfere with one another, and neither size completely controls the structure of the combination until the proportion of large spheres has attained 100 per cent. Thus, mutual interference tends to raise the porosity over what it would otherwise be; nevertheless, porosity still declines as the large spheres increase in proportion.

Certain of these relationships are shown in Figure 2, which rests on experimental determinations of porosity with changing proportions of two sizes. The curves marked (1) represent porosities when the smaller of the two sizes is $0.433D$; the curves marked (2) relate to spheres of relative diameter $0.158D$ for the smaller of the two sizes. The smaller of these two diameters is approximately equal to the critical ratio of entrance for tightest packing; the larger is only

slightly greater than the critical ratio of occupation for Case 6 packing and is not greater than the ratio for the zones of haphazard packing that surround the Case 6 colonies in the chance packing with which these experiments dealt.

Curve (1) for total porosity in Figure 2 shows a much slower rate of decline in porosity with increasing proportion of large spheres than does curve (2), for which the difference between the two sphere

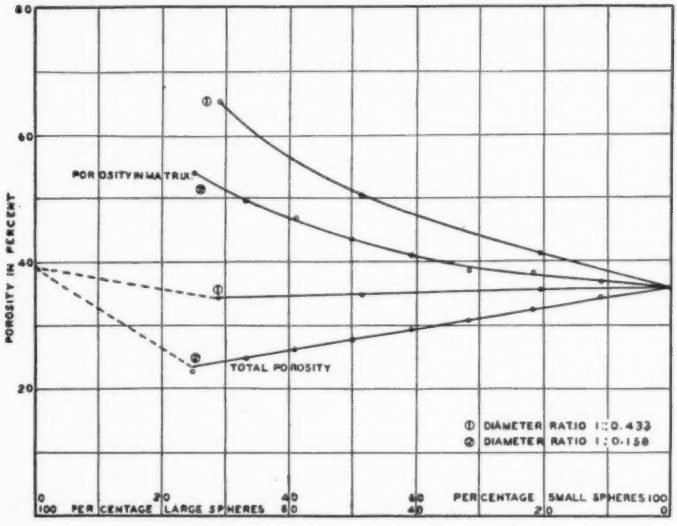

FIG. 2.—Relation between percentage of sizes and porosity

sizes is decidedly greater. This accords with the prediction on pages 918–19. When the proportion of large spheres has increased to about 3:1 (i.e., 72–75 per cent large spheres to 28–25 per cent small spheres, by volume), the large spheres take over control of the structure, as already explained; and the porosity curve thereupon abruptly reverses to an increasing slope, as shown by the broken lines. It will be seen that this increase in porosity is more marked for the combination of widely different sphere sizes than for the combination in which the two sizes are more nearly equal. This results from the fact that the respective curves must reach approximately the same

porosity when the mass consists of 100 per cent large spheres. That this is actually the case is shown in Figure 2, where both curves show about 39 per cent porosity[11] for 100 per cent large spheres.

In Figure 2 is also shown how the growing proportion of large spheres imposes increasing disturbance and looseness of packing among the small spheres that constitute the matrix. The two upper curves of the figure, designated "Porosity in Matrix," represent the proportion of free opening remaining in the regions between the large spheres after these regions have been occupied by the small spheres. It will be seen that as the proportion of large spheres increases, the porosity of the matrix mounts notably. This is because of the growing disturbance and loosening-up of the packing of the small spheres by the increasing number and closeness of the large spheres (see Figs. 3 and 4).[12] Indeed, as the proportion and the proximity of large spheres increases, the regions or voids between them may be regarded as "containers" for the small spheres; these "containers" are of decidedly irregular outline, quite ill-suited to afford a good "fit" with the small spheres, and this becomes accentuated as the "container" becomes smaller through still further increase in proportion of large spheres (Fig. 4). In short, we have here what amounts to a highly exaggerated container effect upon the packing of the small spheres—exaggerated both because the shape of the

[11] In these and all the other porosity experiments described in this paper there is present a small error due to container effect. (See "Systematic Packing of Spheres," pp. 839 ff.). This error has deliberately been reduced to small proportions by the use of a container having a diameter very many times that of the spheres involved. This container effect always exaggerates the true porosity of the assemblage proper. The error is greater for large than for small spheres, if the same container is used for both. This fact, together with probably slight variations in the chance packings, serves to explain why the porosities for 100 per cent large spheres are a little higher than those for 100 per cent small spheres.

[12] With respect to Figs. 3–6, which follow, it should be pointed out that these represent diagrammatically the situation in a section through the assemblage from top to bottom, the top of the assemblage always being toward the top of the page. The sections are idealized, however, in that all the spheres, large and small, have their centers in the same plane, the plane of the paper; if the plane of the paper were a truly random section, the spheres would show different diameters of section, many of them would not touch in the plane of the section, and a very much larger proportion of voids to solids would appear. (Cf. Fig. 14 of "Systematic Packing of Spheres"; also see Figs. 29–36 of same paper.) It is to be noted how, in Figs. 3–6 herewith, the voids are notably large directly under the large "pebbles."

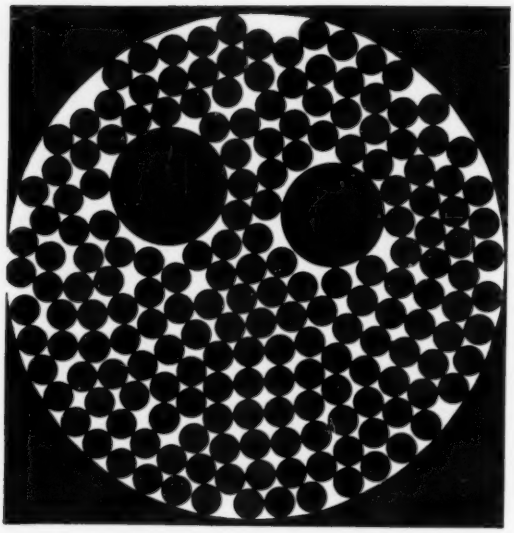

FIG. 3

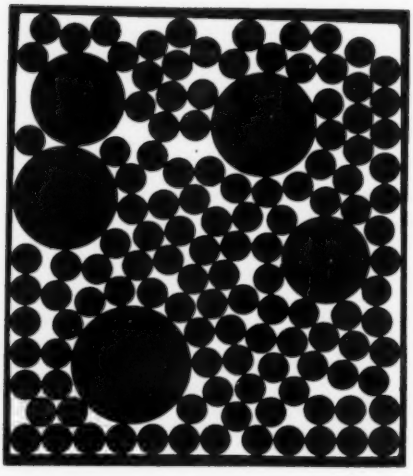

FIG. 4

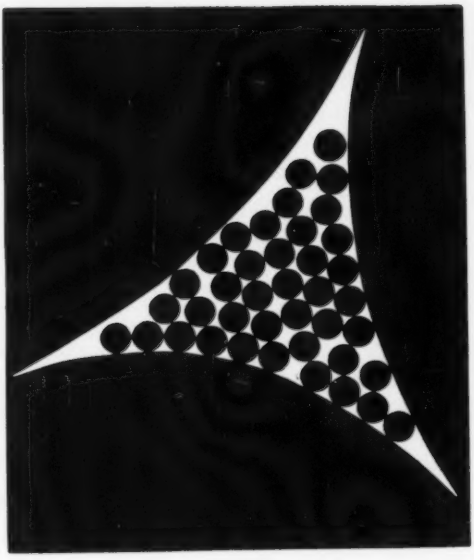

FIG. 5

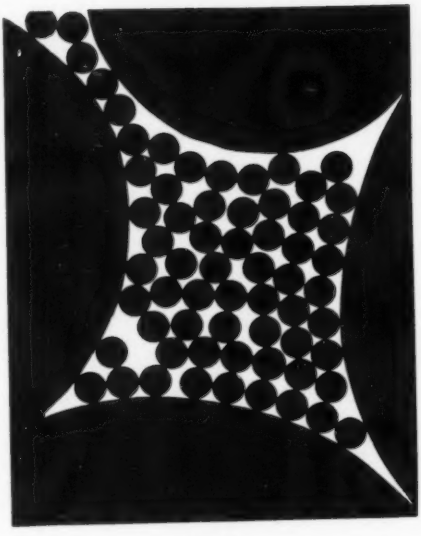

FIG. 6

"container" is so irregular, and especially because the size of the "container" is so small a multiple of the diameter of the contained small spheres and becomes progressively a still smaller multiple as the large spheres increase in proportion. The consequence is that the packing of the small spheres becomes very open-textured indeed, with measured porosities reaching up to 50 per cent, and even up to 65 per cent as the large spheres become numerous enough to support one another without support from the small spheres (see Figs. 5 and 6). For still higher proportions of large spheres, there would not be enough small spheres to "fill" completely the voids between the large spheres; therefore, any determinations of the porosity of the "matrix" in such assemblages would be without significance—for example, the "permeability" of such "matrix" alone would reach infinity when the proportion of large spheres becomes 100 per cent!

It is to be noted that the porosity of the matrix is higher (i.e., the disturbance of small-sphere packing caused by the large spheres is greater) where the two sizes of spheres are not widely different than where there is greater contrast between the two sizes. This, likewise, is what would be predicted from abstract considerations of the geometry.

If the relative proportions of two sizes of spheres is kept constant, the porosity of the combination will change with changing ratio of the two diameters. This situation is confirmed by the measurements plotted as Figure 7. In this experiment the volumes of the two sizes of spheres were always equal; the range in sizes started at 1:1 (i.e., only one size of sphere) and varied up to 19:1. More determinations would have been desirable here. But as far as the data go, they indicate that the decline in porosity is most marked within a range of size-ratios from about 2:1 to about 6:1, and that beyond about 8:1 growing difference in size reduces the porosity only slowly.

Two-Component System: Sand and Pebbles

The conclusions reached in the preceding discussion with regard to spheres should be applicable in a general way to those natural assemblages which consist of well-rounded units, such as the usual sands and pebbles. Precise measurements on ideal spheres could not be expected to show exact agreement with porosity determinations

on the less perfectly spherical natural grains; nevertheless, a reasonable concordance between the two should be expected, and a variation in a given direction when ratios of sizes or proportions are changed among spheres should forecast change in a similar direction when the ratios among the natural units are changed correspondingly.

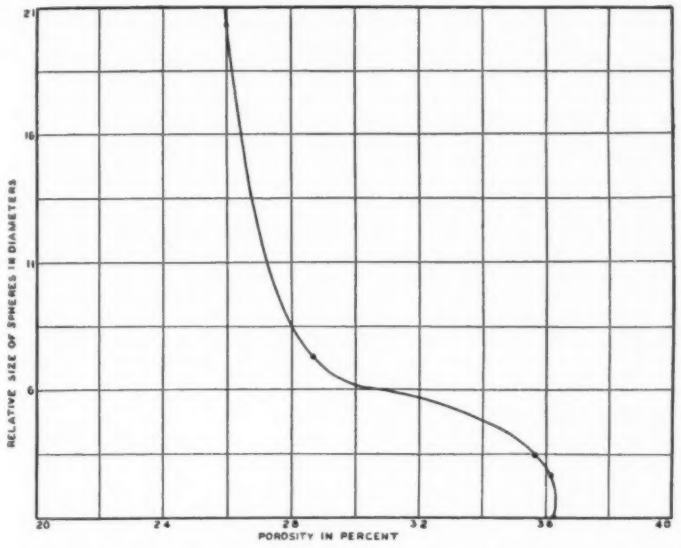

FIG. 7.—Relation between size of spheres and porosity in mixtures containing 50 per cent of each size.

In order to test these expectations experimentally, the degree of disturbance of the packing of sand caused by a pebble contained within the sand was determined for four different sizes of pebbles. The material was collected from a selected area on Revere Beach, near Boston, Massachusetts, where the sand was fairly uniform and contained occasional pebbles. At places where a just-protruding top betrayed the presence of a pebble, a sample was carefully removed without disturbing the packing of the moist sand, and was trimmed until approximately a half-inch layer of sand surrounded (all but the

top of) the pebble. The porosity of the entire sample was then determined[13] and recorded as the total porosity. The volume of the pebble was next determined and subtracted from the total volume occupied by the sand plus voids. The sand with its voids is called the matrix; and the ratio of total voids to volume of the matrix gives the porosity of the matrix, as shown in Table I.

The data in the table are arranged in order of increasing size of pebble. In the last three examples the pebble constitutes an important fraction of the total volume; and comparison of the data for

TABLE I

POROSITY OF SAND AROUND A PEBBLE*

Sample Number	Total Volume of Sample (Cc.)	Volume of Pebble (Cc.)	Portion of Total Occupied by Pebble (Percentage)	Porosity	
				Total (Percentage)	Matrix (Percentage)
(a)	(b)	(c)	(d)	(e)	(f)
17	65.30	2.02	3.09	40.55	41.83
18	70.92	49.56	69.88	13.18	43.78
16	92.30	58.67	63.56	16.55	45.17
19	191.37	125.42	65.54	16.52	48.06

* Average porosity of pebble-free sand in this locality: 41 per cent.

these three samples with the data for the first sample clearly shows that the presence of a pebble in an otherwise uniform area of sand causes a greater porosity in that vicinity and that, on the whole, the highest porosity in the sand is close to the pebble. (Note how this fact is illustrated in Figures 3–6.) The lack of exact correspondence between the figures of Column (d) and those of column (f) for the last three samples is probably to be explained by fortuitous differences in the nature and degree of disturbance of packing close about the pebble. This entire behavior is, in reality, a type of manifestation of container effect.

Multi-Component System

The porosity of various three-component mixtures of spheres was measured in the manner previously described. The diameters of the spheres were approximately 1.3, 2.3, and 8.1 mm., so that each size

[13] Fraser, *op. cit.*

was greater than the critical ratio of occupation for the next larger size in chance packing. The results obtained are given in Table II.

The higher porosities were invariably obtained in mixtures in which one size strongly preponderated; and with increasing complexity of the mixture, the porosity tended to decrease. It was hoped

TABLE II

POROSITY OF THREE-COMPONENT MIXTURES OF SPHERES

DIAMETER			POROSITY (PERCENTAGE)
1.3 Mm.	2.3 Mm.	8.1 Mm.	
10.51	9.63	79.86	33.32
20.88	21.45	58.29	30.10
25.45	29.55	45.00	31.45
38.45	39.74	21.81	34.66
8.75	82.18	9.07	38.84
21.61	55.23	23.16	35.29
29.57	41.39	29.04	33.84
39.06	22.89	38.05	31.37
81.42	9.46	9.12	37.37
54.57	23.23	22.26	34.04
41.31	29.84	28.85	33.13
23.42	38.81	37.77	31.66
43.90	43.99	12.13	35.93
38.45	39.74	21.81	34.60
27.82	35.91	36.25	37.40
45.72	9.45	44.82	30.87
39.81	16.32	44.78	30.44
31.90	22.97	45.14	31.43
10.17	44.48	45.34	33.39
26.88	29.13	43.99	31.34
34.80	21.44	43.76	31.27

that a given porosity would be found to result only from a given proportion of sizes, and thus enable a simple and direct prediction of the porosity (at maximum compaction) from the "screen analysis" of any mixture. But no ground for this hope was disclosed. While, in certain very general ways, there is a connection between the proportions of the various sizes and the resulting porosity, the relationship is inconsistent and unreliable. For example, selection from

Table II shows that a porosity of about 33 per cent can be secured from such diverse mixtures as those below:

Diameter			Porosity (Percentage)
1.3 Mm.	2.3 Mm.	8.1 Mm.	
10.51	9.63	79.86	33.32
29.57	41.39	29.04	33.84
41.43	29.84	28.55	33.13
10.17	44.48	45.34	33.39

Since it is thus apparently impossible to predict the porosity of a three-component mixture of spheres, it is equally, if not more, unlikely that any useful prediction can be made for more complex mixtures. This is confirmed by results gained by Stearns[14] on samples of sand, shown in Figure 8. The wide range of mixtures in sands which may possess the same porosity is strikingly disclosed.

It is evident, therefore, that it will be difficult to derive a mathematical expression connecting screen analysis and porosity. The ordinary sample of sand or soil contains at least six or seven sizes of material, according to the prevailing methods of recording mechanical analyses. Since the proportion of each of these size-groups may vary independently of the other groups, it seems probable that the problem is inherently too complex to be solved by a mathematical treatment, and that only an empirical solution can be hoped for.

SIGNIFICANCE OF GRAIN-SIZE EFFECTS

Before the further controls of porosity are considered, it may be advisable to translate certain of the effects due to size of grain more directly into terms of geological significance, for the importance of some of these influences is much greater than has been universally recognized.

The decreased average porosity occasioned by the presence of a single pebble or of numerous pebbles in a mass of sand is never attained without the accompaniment of a *local increase* of porosity of the sand in the neighborhood of the pebble or pebbles; that is to say,

[14] N. D. Stearns, "Laboratory Tests on Physical Properties of Water-Bearing Materials," *U.S. Geol. Surv. Water Supply Paper 596-F* (1927), pp. 163–69.

the sand cannot be so closely packed near and against the pebble as it is packed in pebble-free sand.

This increased local porosity is due, not to more voids, but to

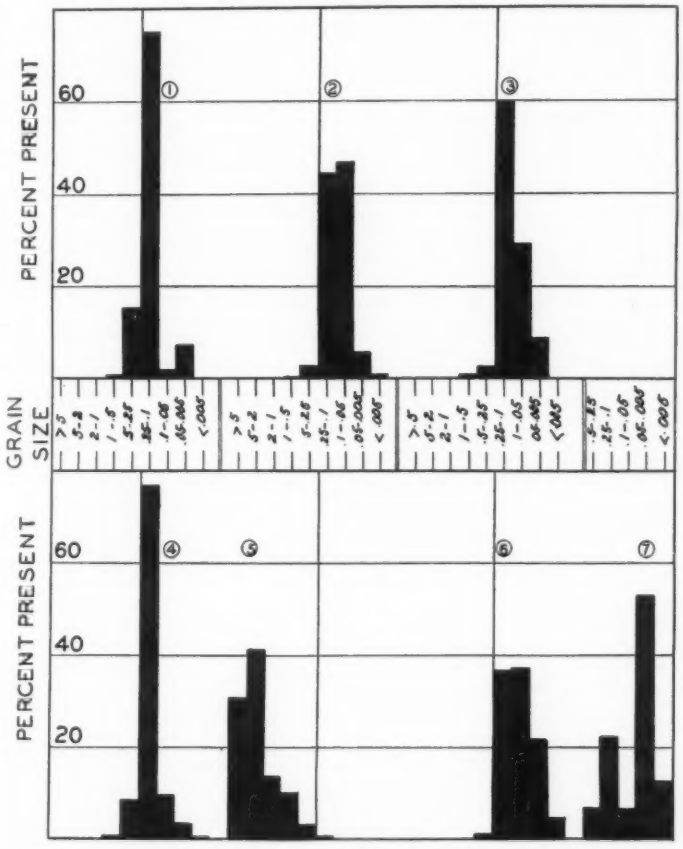

FIG. 8.—Variations in mechanical composition that yield porosities between 32 and 33 per cent. (After Stearns.)

larger voids than those normal to pebble-free sand of the same grain-size. Therefore, inevitably the relative permeability of these local regions of looser packing is even greater than is their relative poros-

ity, when the permeability and porosity of the pebble-free sand are taken as the basis of comparison.

As compared with pebble-free layers of sand of a given grain-size, conglomerate layers in which sand of the same size exists between the pebbles are of lower *average* porosity. But with relatively abundant pebbles, the porosity of the matrix is always greater than that of pebble-free sand and may mount to values very much greater (see Fig. 2). Therefore, although of lower average porosity than sand, a conglomerate invariably has decidedly higher permeability than the equivalent sand packed without pebbles.

The highest porosity and highest permeability are found close to the pebbles (see Figs. 3–6). Thus, experimental findings fully confirm and explain the countless manifestations of the superior permeability of conglomerates over sandstones; and in particular they give definite quantitative support to the general reasoning in explanation of that truth by Graton, Butler, Broderick, and others for the conglomerate copper lodes of Michigan[15] and the still more complete analysis by Graton regarding the gold-bearing conglomerates of the Rand.[16]

EFFECT OF SHAPE OF GRAIN

The shape of the grains in sediments is probably never truly spherical, but varies within wide limits, according to the character of the original mineral or material, the shape of the original fragments, the degree of wear that the grains have undergone, and the method of transportation and deposition. Irregularities in shape should result in a larger possible range in porosity, as irregular forms may theoretically be packed either more tightly or more loosely than spheres.

The degree of rounding generally varies for different grain sizes in any natural deposit, because of differences in the mineralogical composition of different grades. Because of their fracture, or cleavage, certain minerals break up more readily than others, and tend to form smaller-sized particles which are unable to withstand the rigorous wear necessary to round them, and consequently are continuously

[15] B. S. Butler and W. S. Burbank, "The Copper Deposits of Michigan," *U.S. Geol. Surv., Prof. Paper 144* (1929), p. 103.

[16] L. C. Graton, "Hydrothermal Origin of the Rand Gold Deposits," *Econ. Geol.*, Vol. XXV, Supplement to No. 3 (1930), pp. 50–65.

being broken up into smaller fragments without becoming rounded. Furthermore, variations in the perfection of rounding are due, in part, to the relative stability of particles of different size during mechanical corrasion. Marshall[17] found that the variation in rounding of the various sizes on a beach, where there were no pebbles with a diameter larger than 6.7 mm., was as follows:

1. All material coarser than 3.4 mm., well rounded.
2. Between 3.4 and 0.84 mm., mostly angular.
3. Between 0.84 and 0.42 mm., quite angular.
4. From 0.42 to 0.25 mm., fairly well rounded again.
5. All grades finer than 0.25 mm., composed of well-rounded grains.

These findings are in accordance with much of our previous experience and with Marshall's experimental results, in which he found that the rounded form of the coarser material was due to simple abrasion, while the angular form of the intermediate sizes was the result of impact, which, as he showed experimentally, acted for particles of such sizes far more rapidly than abrasion. The rounded form of the smaller sizes was caused by grinding, which supersedes impact when the grains are small.

It is difficult to determine the effect of shape of grain on porosity, because of the difficulty of obtaining angular particles of the same size. Screening is an unsatisfactory method of separation, especially for particles with one long axis, as it then separates the particles according to two of their dimensions but gives little separation of long and short grains of the same cross section. Elutriation yields somewhat better results, but the product is determined by a composite resultant of size and of shape. Therefore, determination of effect of grain shape on porosity is always attended by a second independent variable, that of grain size. A study of the porosity of natural deposits, which contain sand grains of different sizes and degrees of angularity, supplies little information concerning the effect of angularity on porosity.

The effect of angularity on porosity was determined, under conditions in which other factors were reasonably constant, on a series

[17] P. Marshall, "Beach Gravels and Sands," *Trans. of the New Zealand Inst.*, Vol. LX (1929), p. 363.

of carefully sized materials which ranged in shape from spheres to flat plates. The materials selected are shown in Table III. Lead shot and sulphur shot were used because perfectly spherical sand grains were not obtainable. For these two materials, the porosities at loosest packing are not reliably comparable with those for the other materials because they differ markedly from the other materials in specific gravity, which affects the compactness of the unjarred assemblages. The sands were carefully screened and the portion used which passed an 18-mesh screen and was retained on a 35-mesh screen. This gave a sample with an average diameter of 1.5 mm. The lead shot and the sulphur shot had this same diameter.

TABLE III

INFLUENCE OF GRAIN SHAPE ON POROSITY

MATERIAL	SPECIFIC GRAVITY	TYPE OF PACKING			
		Dry		Wet	
		Loose	Compacted	Loose	Compacted
Lead shot	11.21	40.06	37.18	42.40	38.89
Sulphur shot	2.024	43.38	37.35	44.14	38.24
Standard sand (marine)	2.681	38.52	34.78	42.96	35.04
Beach sand	2.658	41.17	36.55	46.55	38.46
Dune sand	2.681	41.17	37.60	44.93	39.34
Crushed calcite	2.665	50.50	40.76	54.50	42.74
Crushed quartz	2.650	48.13	41.20	53.88	43.96
Crushed halite	2.180	52.05	43.51	.....	.....
Crushed mica	2.837	93.53	86.62	92.38	87.28

The porosity was measured before and after the material was compacted by jarring, and under two distinct conditions: when the material was dry, and when it was saturated with water. The experimental procedure was as follows:

The dry material was slowly poured into a weighed and calibrated measuring flask, and allowed to pack naturally until the flask was full. After weighing, the material was compacted by tapping the flask, more material added as needed, and the jarring continued until settling ceased. The flask was again weighed. The volume of the grains was determined in a pycnometer, using tetralin as the

immersion fluid. As the total volume of the flask was known, the porosity was readily computed.

For wet packing, the material was first immersed in water to wet it thoroughly and remove adhering air. The measuring flask was filled with water and the material slowly poured in and allowed to settle naturally. After the desired packing was obtained, the sample was dried at 110° C., allowed to cool and take up moisture in the room, then weighed, and its volume determined. The results appear in Table III.

If the averaged porosity of the lead shot and sulphur shot in compacted packing is taken as representative of the porosity of uniformly sized spherical grains under the conditions existing in this experiment, then, for dry packing a porosity of 37.26 per cent is indicated, and for wet packing a porosity of 38.56 per cent. Taking these figures as a basis, two of the samples have lower porosities, and five have higher porosities. The shape of the "standard" sand grains readily explains the low porosities found throughout the tests for that material; the grains are pretty uniformly disk-shaped, and consequently possess many of the advantages of spheres in packing, but, at the same time, may be packed more tightly in the third dimension (with the flat sides together). The porosity of beach sand does not differ widely from that of spherical grains. More angular materials show in marked degree the effect of their angularity on packing, which is to increase the porosity.

On the assumption that the variations of porosity observed for compacted dry packing are entirely due to the influence of the grain shape on packing, there exists a range of 8.73 per cent units (excluding mica) in porosity, i.e., from 34.78 to 43.51 per cent. Of this range, 6.25 units is increased porosity due to irregular angularity, while 2.48 units is decreased porosity due to somewhat flattened grains (Table III). It is apparent, therefore, that ordinary, moderately well-rounded sands show little variation in porosity, because the deviation in their perfection of rounding is limited. The major consequences of angularity are bridging and poor original packing.

Wet materials, independent of the grain shape, pack more loosely than dry materials (see Table III). However, the difference between the porosity of rounded and angular material (wet and dry) in

compacted packing proves that wet packing increases the effect of angularity.

Flat and needle-like forms have the greatest effect on porosity. Porosities exceeding 90 per cent are common with particles of crushed mica. This amazingly high porosity cannot be reduced below 86 per cent, even by prolonged jarring. Pressures sufficient to burst a strong glass container were required to reduce the porosity of wet mica to 67.4 per cent. With such grain shapes, the surface area is very great, in proportion to the volume. Adhesion and friction between particles, as they settle and pack, increases enormously with such large surfaces. The weight of the overlying material produces less compression in underlying areas than in materials of a spherical shape, which, for a given volume, are packed more tightly and are therefore heavier. Aggregates of flat particles offer least resistance to compaction by external pressures. This may explain, in part, the constant relationship observed in clays between porosity and external pressures, as represented by the weight of overlying rocks.

Angularity may thus either increase or decrease porosity; most often it increases porosity. The only type of "angularity" found to cause a decrease in porosity is that in which the grains are mildly and uniformly disk-shaped.

EFFECT OF METHOD OF DEPOSITION

The process by which a given sediment has accumulated and deposited affects the porosity in two ways: (*a*) in the nature and variety of the materials deposited, which affects the entire deposit by controlling the range and uniformity of sizes present, as well as their perfection of rounding; and (*b*) in the manner in which the material is arranged and packed.

The extent to which depositional processes can affect porosity is partially dependent on their ability in sizing. Glaciers are probably the least effective in this respect, and consequently glacial deposits are characterized by a great variety of shapes and sizes of particles which are frequently intermixed to give a mass with relatively low porosity. River and beach processes are more effective in sorting and sizing materials and consequently their deposits show a higher poros-

ity. Transportation by air current offers the most effective sizing process, and subaërial deposits frequently have very high porosities.

The process and manner of deposition may have great influence on the strictly local arrangement or packing of the material. Certain phases of this subject may more appropriately be discussed in Part II (see pp. 988–90); but the following considerations are pertinent at this place.

The degree of porosity variation in material deposited on a beach or in a river channel depends, in part, on the geometric uniformity of the material deposited, and in part on the degree of compaction attained. In order to obtain a quantitative idea of this possible variation, porosity determinations were made at numerous closely spaced points on several of the beaches in the vicinity of Boston, Massachusetts.

The method of sampling and determining porosity was checked as follows: Three samples were taken immediately adjacent to one another on Revere Beach. The sand here averaged about 1 mm. in diameter and appeared to be well rounded and well sized. This appearance of uniformity was confirmed by the results, which showed porosities of 39.25, 39.24, and 39.42 per cent.

Local variations on a beach of well-sorted sand were ascertained as follows: four samples were collected from the surface of Lynn Beach (near Boston, Massachusetts). Samples 1, 2, and 3, taken successively 15 feet apart, and parallel to the water line, midway between high and low tide levels and about 25 feet from the low tide mark, gave the closely similar porosities of 43.28, 43.62, and 43.44 per cent, respectively. Sample 4 was taken 15 feet nearer the low tide level than sample 2; its porosity was notably lower, 41.07 per cent.

Material deposited by waves appears to have minor local variations in porosity, but may be essentially uniform over moderate distances. Porosities on a beach are much more likely to be uniform in a direction parallel to the water line than at right angles to it. This variation is due, not nearly so much to changes in the packing as to variations in the uniformity of the grain-size. Sample 4 of the Lynn suite owes its lower porosity to a decidedly lesser uniformity of grain-size that obviously is due to its location near the low tide level.

Samples taken on Marblehead Beach, Massachusetts, gave the values shown in Table IV. These determinations indicate the effect of progressive changes in size as deposition went on, and also the difference (between samples 1 and 4) occasioned by drying, which is further discussed under the next heading. Examination of the several kinds of materials made it evident that the finer sizes were of more

TABLE IV

SAND POROSITIES ON MARBLEHEAD BEACH

	Percentage
1. Fine-grained, wet, surface	44.21
2. Coarse-grained, damp, 1 foot below No. 1	39.01
3. Coarser-grained, damp, 1½ feet below No. 1	34.50
4. Fine-grained, dry, surface, about 30 feet landward from No. 1	41.56

TABLE V

POROSITY OF REVERE BEACH SAND ADJACENT TO BOULDERS

	Percentage
SAMPLES TAKEN AROUND A 6-INCH BOULDER	
1. Landward boundary, near surface	42.10
2. 3 inches below No. 1	42.66
3. Immediately underneath boulder	43.43
SAMPLES TAKEN AROUND AN 8½-INCH BOULDER	
4. Landward boundary, near surface	44.32
5. 4 inches below No. 4	42.71
6. Side boundary, near surface	43.16
7. Waterward boundary, near surface	44.76
SAMPLES TAKEN AROUND AN 8-INCH BOULDER	
8. Landward boundary, near surface	44.20
9. Immediately underneath boulder (material notably variable in size)	38.67

uniform grain, and that increasing heterogeneity of size attended increasing average grain-size. This deviation from uniformity of size is more responsible than the absolute sizes for the differences in porosity shown.

Details of sizing and packing at places where the controls thereon are highly local and complex are illustrated in Table V, which shows the varying porosities in the sand immediately adjacent to good-sized boulders.

COMPACTION ATTENDING DEPOSITION

Shrinkage of Sands

The volume of clays and silts undergoes a very considerable diminution on drying. This shrinkage is sometimes assumed to result wholly or in part from mineralogical changes that accompany desiccation. But beyond doubt the initial greater bulk or looser texture is usually and mainly due to (*a*) the buoyancy of the water, which has the same effect as reducing the specific gravity of the material by 1, and also lessens the velocity of settling, and (*b*) the cushioning and separating effect of the adhering film of water around each grain.

It has been stated[18] that sands show no shrinkage on drying. This appears to be incorrect. Wet sand packs less tightly than dry sand by percentages varying from 1 per cent or more of the total porosity (Table III). The difference in the perfection of packing increases as the particles become more angular; this effect may be due in part to increasing cohesion between the particles, which prevents them from adopting as tight a packing as they would otherwise assume.

The following experiments were performed to determine whether the difference in porosity is due to a difference in the perfection of packing—because of friction between the grains—or whether an actual shrinkage takes place during or following the drying of the sand.

Into a burette filled with water was poured a thick suspension of screened sand. The sand was packed by tapping the burette. When no further compaction could be obtained, the stopcock at the bottom of the burette was partially opened. Alcohol, followed by ether, was passed, under a low head, through the sand. When all the water in the sand had been displaced and the ether drained off, the burette full of sand was suspended in an oven and thoroughly dried. After drying, the sand settled slightly, and a further gentle tapping produced considerably more compaction. The lowest porosity obtained in the wet sand was 37.92 per cent. The same sand when dried had a porosity of 36.95 per cent. Duplicate runs gave very close checks.

The cause of this shrinkage is not entirely evident. Around every

[18] Cf. C. Terzaghi, "Principles of Soil Mechanics," *Engineering News Record* (1929, reprint), p. 56.

wet sand grain there is a film of water held in place by mutual attraction between the water and sand molecules. This film has been estimated to be of the order of 10^{-6} cm. thick;[19] and its removal would cause some shrinkage, the total amount of which would depend on the diameter of the sand grains, being greater the smaller the diameter of the grains. It was estimated that in the sand tested above, the removal of film would produce a shrinkage of about 0.01 per cent, or only about 1 per cent of the shrinkage actually observed. Terzaghi[20] states that shrinkage in sediments is caused by "capillary pressure" and that the maximum intensity of the capillary pressure depends on the grain-size. He concludes that in a fine sand this pressure would not be sufficient to produce any shrinkage. However, the experiments here described show that shrinkage does take place. On the whole, it seems most logical to assume that the greater buoyancy and viscosity of water prevent as close a packing in wet sand as in dry sand. In moist sand, i.e., sand incompletely wet, the surface tension or "binding effect" of the water film also enters, which still further increases the porosity as contrasted with that of dry sand.[21]

Whatever may be the cause of shrinkage, a very definite correlation with grain-size is apparent. Clays shrink several per cent of their volume. Sands shrink to a much smaller degree, and coarse materials, such as clean gravels, show no measurable amount of shrinkage.

Degree of Compaction on a Beach

Any given aggregation of clastic material has a fairly definite range of porosity which represents varying approaches to perfection in packing. It was found that a given sample of ordinary, moderately well-sized and well-rounded beach sand exhibits a range in porosity, when dry, between 46.5 and 37.8 per cent, depending on whether in loosest or in tightest chance packing; when wet, approximately the same range in porosity is shown. This represents a change in the total volume of the sand of about 14 per cent. In dealing with prob-

[19] P. G. Nutting, "The Movement of Fluids in Porous Solids," *Jour. Franklin Inst.*, Vol. CCIII (1927), p. 317.

[20] C. Terzaghi, reprint from *Engineering News Record* (1925), p. 64.

[21] C. A. Hansen, "The Physical Properties of Foundry Sands," *Trans. Amer. Foundrymen's Assoc.*, Vol. XXXII, Part 2 (1925), pp. 65–69.

lems of consolidation and settling of sediments, volumetric change of any such magnitude would be of much significance. Consequently, it is desirable to know where, within this considerable range, the porosity of a sand as newly deposited by wave action would actually fall. An answer to that question, or, in other words, the degree of packing achieved by wave deposition, was determined as follows:

A strip 4 inches wide by 3 feet long was selected, parallel to the water line and midway between high and low tide levels, on Revere Beach, near Boston, Massachusetts. The sand at this point is fairly well sized and rather fine, averaging about 0.5 mm. in diameter and with largest grains not exceeding 2 mm. The porosity of the sand in place was determined from samples taken at each end and in the middle of the selected strip. These three samples gave porosities of 40.19, 40.15, and 40.99 per cent, respectively; the mean porosity was thus indicated to be 40.56 per cent. After the strip had thus been sampled for porosity in place, a layer of sand over the whole area of the strip and 1 inch deep was collected as representative of that particular sand, to be used for porosity determinations in the laboratory, which were carried out as below. A glass cylinder 125 cm. tall (the volume of which, when filled to various heights, was accurately known) was partially filled with water into which a quartered portion of the wet sand was slowly poured. The sand was allowed to settle naturally, and the height of the column of sand was read off. The sand was then packed by tapping the base of the cylinder with blows of constant intensity and frequency. At 30-second intervals, the height of the sand column was read. Several runs on the sand gave consistent results. It was found that when the jarring exceeded a certain intensity, no further settling occurred, but rather, the sand was loosened. The packing obtained by this method could not be made any tighter by any other method of jarring. The experiments were repeated on the same sand after it had been thoroughly washed in fresh water, dried, and weighed. Six 30 cc. samples were then selected and their specific gravity determined by the pycnometer method, using tetralin as an immersion fluid. From these data the porosities in the wet and the dry condition were computed. The results of typical runs for wet and for dry sand are shown in Table VI.

These data show that the sand, as it existed on the beach, with a

mean porosity of 40.56 per cent, had already undergone sufficient compaction to have removed most of the bridging effect and to have arrived at a state where further compacting while wet could occur but slowly, although considerable latitude still remained for further consolidation and shrinkage on drying. The data also indicate that dry sand may be compacted much more quickly, especially in the early stages.

Clays show a much greater spread between the porosity of freshly deposited material and the possible minimum porosity. It is not un-

TABLE VI

PACKING-TIME OF POROSITY VARIATIONS IN SAND*

Minutes	Wet	Dry
0	46.00	46.30
½	43.45	40.70
1	42.50	39.53
1½	41.50	39.00
2	40.77	38.00?
2½	39.90	38.65
3	39.38	38.55
3½	38.85	.....
4	38.46	38.20
5	38.33	38.15
6	38.26	38.07
8	38.26	37.82
10	38.26	37.82
12	38.26	37.82

* *In situ* on beach, 40.56 per cent.

common for freshly deposited clays and silts to exceed a porosity of 85 per cent. Dried and compacted, the same materials will have porosities of 40–50 per cent, or less. Here again, total volume versus grain-size is to be considered. The smaller the volume of the grains, the greater the opportunity for later readjustments in packing, with diminution in porosity. Furthermore, the greater the size of grain—up to a certain limit—the greater its downward component of velocity at the time of deposition, and therefore the greater chance that it will come to rest in a relatively stable position, which in turn means lower porosity. Thus the chances of "honeycomb" packing are decreased.[22]

[22] C. H. Terzaghi, "Modern Conceptions Concerning Foundation Engineering," *Jour. Boston Soc. Civil Engineers*, Vol. XII, No. 10 (1925), pp. 397–402.

COMPACTION AFTER DEPOSITION

After a sediment is buried, it is subjected to a pressure equal to the weight of the overlying strata, which reduces the volume of the mass by elimination or diminution of the open spaces. This pressure induces closer packing, crushing, and deformation of the grains, as well as more or less varying degrees of recrystallization—when pressure becomes extreme.

In this connection, ordinary clastic sediments fall into two classes: those with high rigidity—sandstones and conglomerates—which are relatively little affected by pressure; and those with low rigidity—shales—in which deformation and compaction by pressure may be especially important.

Compaction in Sand and Gravel

Pebbles and sand grains are laid down in a more nearly stable state than are smaller particles, because they are buoyed up much less by the water. Accordingly, their porosity as initially deposited cannot be greatly reduced by compaction until the pressure is sufficient to crush the grains and cause more or less complete collapse of the arching across the voids. Athy[23] found that the sand derived from the St. Peter sandstone, when artificially deposited under water, could be made to settle about 11 per cent by continued jarring under atmospheric pressure. When it was placed under 4,000 pounds pressure, the compaction was only 2 per cent more, or 13 per cent in all.

It is difficult to make comparisons between the porosities of sands at the surface and at greater depths because sands vary widely in their mechanical composition and in the amount of cementing material which they contain. Sands of different porosities occur at different depths. Porosity measurements of slightly cemented sands from drill cores[24] have shown that many sands at 4,000 feet are more porous and less dense than sands which appear to be similar but which are taken from shallower depths.

It seems clear, then, that in the deposits of coarser material, such as sands and gravel, compaction after deposition is relatively unim-

[23] L. F. Athy, "Compaction and Oil Migration," *Amer. Assoc. Petrol. Geol. Bull. 14* (1930), pp. 1–24.

[24] Athy, *ibid.*, p. 8.

portant except in cases where the pressure applied has been really great. As a matter of fact, in the average sandstone or conglomerate there is only rarely to be seen evidence of pressure that exceeded the crushing strength of the component units. In the great majority of cases, therefore, postdepositional reductions of porosity are due to cementation by introduced mineral material.

Compaction in Clay

Observations indicate[25] that the porosity of shales at the same depth in any specific locality is approximately equal, and that porosity may be taken as a measure of the compaction which the shale has undergone. Inasmuch as relatively little extraneous material is available for the cementation of shales, because of their relative impermeability, Athy[26] has computed the porosity of a shale from its depth of burial by the following formula:

$$P = p(e^{-bx}),$$

in which P is porosity, p is the average porosity of surface clays, b is a constant, and x is the depth of burial.

Thus Athy[27] finds that, while the average porosity of surface clays is 45–50 per cent, the porosity of shale at 6,000 feet is approximately 5 per cent. Compaction equivalent to more than 20 per cent of the original volume has occurred by the time the clay has been buried to 1,000 feet, 35 per cent at 2,000 feet, and 40 per cent at 3,000 feet. Thus, the reduction of porosity in shales by compaction is a factor of prime importance. It is difficult to determine to what extent this reduction is due to physical rearrangement of the grains, and in how far it is the result of recrystallization; but the weight of present evidence favors the second alternative. Hence, it appears that in contrast to sandstones and similar rocks, compaction in clays is considerable and increases with pressure up to a limit as yet undefined. The conversion of shale into slate and phyllite is obviously a recrystallization, and original porosity virtually disappears. At that stage the processes pass out of the field of the present inquiry.

[25] H. D. Hedberg, "The Effect of Gravitational Compaction in the Structure of Sedimentary Rocks," *Amer. Assoc. Petrol. Geol. Bull.* 10 (1926), pp. 1035–72.

[26] Athy, *op. cit.*, pp. 1–24. [27] *Ibid.*, p. 31.

Part II. Permeability

DEFINITION

"Permeability" may be defined as "allowing passage, especially of fluids, through or into, without impairment of structure or displacement of parts."[28] In ordinary hydraulic usage, a substance is termed "permeable" when it permits the passage of a measurable quantity of fluid in a finite period of time; "impermeable" when the rate at which it transmits the fluid is slow enough to be negligible under the existing temperature and pressure conditions.

Much controversy and confusion regarding the permeability of rocks might be avoided by a recognition of the difference between absolute and hydraulic impermeability. The term "hydraulic impermeability" can be applied only with reference to a specified differential pressure and rate of flow. It probably is impossible to prove that any material is absolutely impermeable. For example, steel might be regarded as impermeable to air; yet Bridgman[29] found that at 60,000 atmospheres of pressure, air can be forced through steel walls at a measurable rate. Clay is commonly supposed to be impermeable to water, but careful measurements show that under an adequate hydraulic head clay will transmit water at a definite rate. Thus, in defining the permeability of a substance, it is necessary to state the conditions under which the permeability exists.

MEASUREMENT OF PERMEABILITY

APPARATUS

The different types of apparatus for measuring permeability may be classified into two groups, depending on whether they operate under a constant head or under a variable head. The apparatus used during the present investigation was operated under a constant head. In principle it is similar to that designed by Meinzer,[30] but it differs rather widely in some details—especially in construction. The general arrangement is shown in Figure 9.

[28] *Twentieth Century Dictionary.*

[29] P. W. Bridgman, "Experiments on the Effects of Extremely High Pressure," *Compressed Air Mag.*, Vol. XXVI (1921), pp. 10223–25.

[30] Stearns, *op. cit.*, pp. 144–48.

In this method of measuring the coefficient of permeability, inflow of water takes place at the bottom of a measured column of the ma-

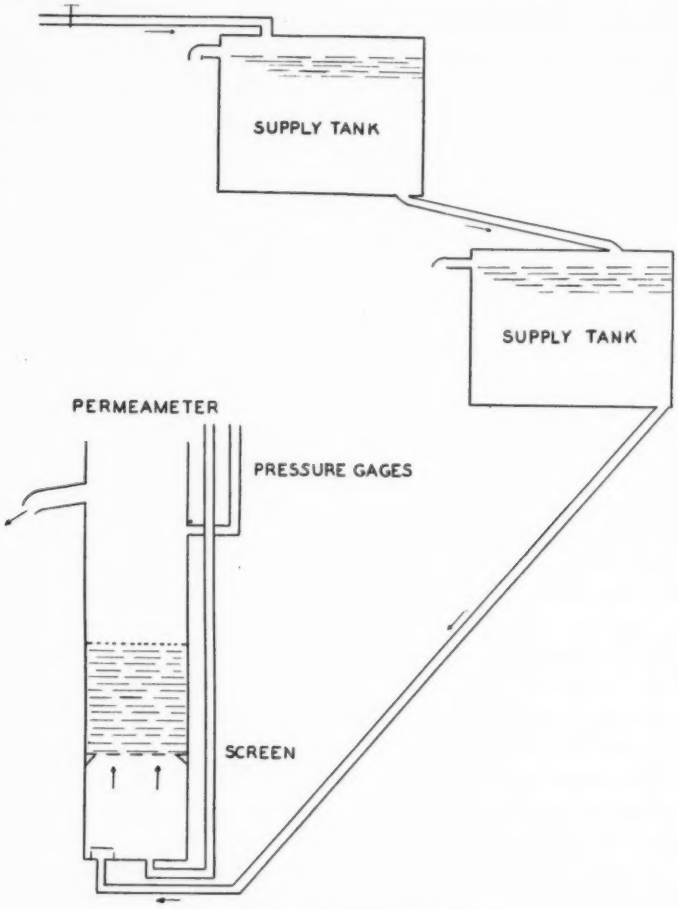

FIG. 9.—Diagram of permeability apparatus

terial and a free overflow is permitted at the top. The difference in head at the bottom and top is indicated by two pressure gauges. The head is regulated by an adjustable supply tank equipped with an

overflow. Observations are made on the loss in head, rate of flow, temperature of the water, and porosity of the material.

The percolation chamber consists of a Pyrex glass cylinder, 51 cm. long and 7.3 cm. in diameter. At a short distance from its base, this cylinder is slightly constricted, so as to support an aluminum ring on which rests a fine wire screen which serves to hold the sample in place. The two pressure gauges are also made of glass and have millimeter scales attached. Above the mouth of the intake water pipe, a rigid shield is fixed, so as to break the force of the upward-flowing water and thus give more uniform pressure at the base of the sample. Before entering the percolation chamber, the water passes through two successive overflow tanks to insure the utmost constancy in velocity of liquid delivered to the sample. The water used has previously been heated to reduce its air content; and it is used at a temperature which never permits its warming up and the liberating of air while the experiment is in progress. As a further precaution, fine wire screens are placed across the path of flow through the overflow tanks to catch any possible freed air before entrance to the percolation chamber. Finally, the screen underlying the sample and the outer voids of the sample itself may be viewed through a lens during the experiment to make sure that no air bubbles are forming.

PROCEDURE

The material to be tested for permeability is first weighed and its specific gravity determined. It is next thoroughly wetted with water at the required temperature, since it proves almost impossible, especially with fine materials, to remove air from the apparatus if the material in the dry condition be put into the percolation chamber. The sample is then slowly poured into the water-filled percolation cylinder, compacted by jarring if desired, the top of the column leveled, and the height of the column measured. The actual volume of the material is computed from its known weight, and the porosity of the assemblage then calculated. The water is turned on from the supply tank; and, after flow becomes constant, the rate is measured as the time required by the overflow to fill a 1,000-cc. flask. The height of water in the pressure gauges is read, and the temperature of the water obtained from a thermometer suspended in the percolation chamber.

A difficulty with this apparatus, and all others of its type, is that the walls of the container produce a zone of loose packing in their vicinity. Moreover, the voids along the walls differ from those between the grains, both in form and continuity, so that the permeability along the walls is certain to be greater than in the main mass. The magnitude of this error depends on the relation between the diameter of the container and the diameter of the individual grains. Although it is difficult to evaluate this error, the recognition of its existence is important.

A glass percolation chamber has the advantage of permitting observations of the sample and supporting screen for chance air bubbles, and of the packing of the sample for stratification or irregular distribution of sizes. These observations are of special importance when measuring the permeability of samples composed of mixed sizes which tend to stratify.

This type of apparatus proved quite satisfactory in the present work and yielded closely concordant results for successive runs on the same samples. But its use is not advocated for the determination of permeability of materials which require a considerable period for the flow of the water, under which circumstances serious difficulties may be expected from dissolved air. Where measurements of such nature are required, it will be necessary to free the water absolutely of all dissolved air and to control the temperature very carefully throughout the duration of the determination.

All experimental data used in the following discussion represent an average of two or more runs.

INTERPRETATION OF DATA

Darcy,[31] one of the earlier workers in the field of permeability, found that when water, at any given temperature, percolates in parallel flow through a permeable body of prismatic form, its rate of flow may be expressed by the equation $Q=KFs$, where Q is the volume of flow per second, F is the cross section of the body, and s is the hydraulic gradient, equal to loss of head divided by length of the body. The factor K has been called the *coefficient of permeability* and represents the velocity of percolation under a hydraulic gradient of unity.

[31] H. Darcy, *Les fontanes publiques de la ville de Dijon* (Paris, 1856).

Numerous attempts have been made to express K in terms of measurable units. In 1899, Slichter[32] computed the coefficient of permeability of sands by assuming that all of the grains were perfect spheres and that they were distributed in a regular fashion. On these assumptions he derived the following formula:

$$q=(1.0094)\frac{pd^2s}{\mu hC_s},$$

in which q is the quantity of water in cubic centimeters per second,
p is the difference in pressure at the ends of the column in centimeters of water at 4° C.,
d is the mean diameter of the soil grains in centimeters,
s is the area of the cross section of the column in square centimeters,
h is the height of the column of material in centimeters,
μ is the coefficient of viscosity,
(1.0094) is the logarithm of the factor 10.22,
C_s is a "constant" for various porosities of an ideal soil and whose computed value for porosities of 26, 36, and 47 per cent is respectively 84.3, 28.8, and 11.8.

In this formula Slichter has substituted for Darcy's K, the expression

$$(1.0094)\frac{d^2}{\mu C_s}=781.3\frac{d^2}{C_s},$$

when μ at 10° C. equals .01308. When the porosity is 47 per cent, K equals $66.0d^2$; and with a porosity of 26 per cent, K equals $9.26d^2$.

Hazen,[33] by experimental measurements on clean sands, in which the porosity was around 36 per cent, found that K equaled $116d_w^2$ in which d_w is the effective size of grain, defined by Hazen as that size of grain having a mean surface. He further found that when the grains were of approximately the same size, K equaled $150d_w^2$. To prevent confusion, it should be noted that Slichter used the symbol d_w in reference to natural materials. He defined it as the size of grain

[32] Slichter, *op. cit.*, pp. 295–384.

[33] A. Hazen, "Some Physical Properties of Sands and Gravels with Special Reference to Their Use in Filtration," *Mass. State Board of Health, 24th Annual Rept.* (1892–93), p. 554.

that, in a mass composed of various sizes, would give the same coefficient of permeability as would be obtained if the mass were composed of a single size of grain having a diameter d_w.

Terzaghi[34] derived a semi-empirical formula for the permeability of sands in which K had the following value:

$$K = \left(\frac{C_T}{V_o}\right)\left(\frac{V_o}{V_t}\right)\left(\frac{N-0.13}{\sqrt[3]{1-N}}\right)^2 d_w^2 = (800 \text{ to } 600)\,\frac{V_o}{V_t}\left(\frac{N-0.13}{\sqrt[3]{1-N}}\right)^2 d_w^2,$$

in which d_w is the effective size of grain in centimeters,

N is the void volume and equals voids divided by total volume,

V_o is the coefficient of viscosity at 10° C.,

V_t is the coefficient of viscosity at t° C.,

C_T is a constant whose value was found experimentally to vary between 10.48 and 6.026.

This equation is based on the assumption that in the void channels, the widest part of the channel is at least five times as wide as the narrowest part, and consequently the loss in head per unit of length of the narrowest parts would be at least twenty-five times as great as in the widest parts. If we compare the foregoing equation with that given by Slichter, it will be found that Terzaghi has written

$$\frac{V_o}{V_t}\left(\frac{N-0.13}{\sqrt[3]{1-N}}\right)^2$$

for $\frac{1}{\mu C_s}$ and uses a different value for his constant; otherwise the two equations are similar.

The permeability coefficient curve obtained by plotting K against void ratio as derived from Terzaghi's equation agrees closely with that derived from Slichter's equation, provided they are plotted with one point in common. Thus, it seems that Terzaghi's simplifying assumptions are warranted to at least the same extent as those of Slichter.

Terzaghi derived from the preceding equation what he termed the

[34] C. Terzaghi, "Principles of Soil Mechanics"; reprint from *Engineering News Record* (1925), p. 31.

"reduced coefficient of permeability" K_r by calculating the coefficient of permeability for a porosity of 50 per cent at a temperature of 10° C., regardless of the actual porosity of the sample. Thus, by introducing into the equation above the special values of N equals 0.50 and V_0 equals V_t, the value for K_r, as experimentally determined by Terzaghi, became 174 to $100d_w^2$. The value of K_r, being independent of porosity and temperature, should serve as a basis for investigation of the influence of other factors, such as the perfection of rounding. Terzaghi concluded that the values 174 to 100 represented the limits of these factors in clean sand, but that the presence of even a small amount of clay would tend to invalidate the equation.

Fancher, Lewis, and Barnes[35] have recently defined the coefficient of permeability for streamline flow as

$$K = \frac{QZL}{\theta A \Delta P},$$

in which Q is the volume in milliliters of fluid (at the mean pressure of a gas),

L is the length in centimeters of the sample parallel to the direction of flow,

θ is the time in seconds,

ΔP is the difference $(p_2 - p_1)$ in absolute pressure in atmospheres at the inlet and outlet of the sample,

Z is the absolute viscosity of the fluid in centipoises,

K is the general coefficient of permeability to any fluid for a specific sand.

When the quantities in the foregoing equation are unity, and therefore K equals 1, it has been suggested by Wyckoff, Botset, Muskat, and Reed[36] that the unit be termed a "darcy" in honor of Darcy, who first formulated the law for flow through clastic materials.

Fancher, Lewis, and Barnes[37] also find that $K = \frac{gd^2}{2C}$ in which g is the gravity; d is the average grain-size calculated so as to give consideration to numbers as well as absolute size of individual grains;

[35] *Op. cit.*, pp. 65–167. [36] *Op. cit.*, p. 167. [37] *Op. cit.*, p. 160.

and C is a constant, experimentally determined for each sample, which they state to be 585 for lead shot and 1,200 for all unconsolidated sands when P is measured in atmospheres and Z in poises at 20° C.

Substituting their values for C in the equation above and correcting for P measured in centimeters of water and Z in poises at 10° C. (so as to be consistent with the conditions adopted throughout this paper), we find that K equals $64.6d^2$ for lead shot and $31.5d^2$ for unconsolidated sands.

It is noteworthy that no expression for porosity appears in their equation. In fact, they state[38] that no direct or simple relation was found between porosity and permeability. From their data, however, it appears that their value of C both for lead shot and unconsolidated sands is based on an average porosity of about 34–35 per cent. It is also interesting to note that they find the constant C to have the same value for all unconsolidated sands irrespective of their shape or surface. However, it must be pointed out that the authors are dealing primarily with consolidated oil sands and not with unconsolidated clastic materials.

Another equation which has been suggested[39] for the streamline flow of water through sand follows:

$$H = C_H LTFV\left[\frac{S}{100}\ \text{sum of}\ \frac{P}{d}\right]^2,$$

or, expressed in terms of the coefficient of permeability,

$$K = \frac{1}{C_H TF\left[\frac{S}{100}\ \text{sum of}\ \frac{P}{d}\right]^2},$$

where K is the coefficient of permeability,
H is the loss of head in terms of water column,
L is the length of permeable column,

[38] *Ibid.*, p. 140.

[39] G. M. Fair and L. P. Hatch, "Fundamental Factors Governing the Streamline Flow of Water through Sand," *Jour. Amer. Water Works Assoc.*, Vol. XXV (1933), pp. 1551–65.

T is the correction for temperature here used as viscosity divided by density, but which is practically the same as the viscosity for any given temperature,

F is the porosity factor equal to $\frac{(1-f)^2}{f^3}$, where f is the void ratio,

V is the velocity of approach over the total area of the column,

S is the shape factor varying from 6 for spheres to 7.7 for angular sands,

P is the percentage by weight of a given diameter,

d is the diameter (for sands it is taken as the geometric mean of the sizes of adjacent screens),

C_H filtration constant experimentally determined as 5 divided by gravity.

This equation, which is based on an analogy with streamline flow through pipes, differs from preceding equations in that the size, proportion, and distribution of the voids is expressed in terms of hydraulic radius and porosity. Hydraulic radius is defined as cross-sectional area divided by wetted perimeter and, when applied to columns of spheres, equals

$$\frac{\text{Void volume}}{\text{Surface area of spheres}} = \frac{\text{volume of spheres}}{\text{surface area}} \times \frac{f}{1-f},$$

since volume of voids is f times total volume and equal to $\frac{f}{1-f}$ times volume of spheres. Determination of the volume and surface area of spheres requires that their diameter be known. Where the grains are not spherical, their surface area and volume, which depend on their shape, become more difficult to determine. The shape of non-spherical and irregular grains may be defined in terms of area divided by volume, which for spheres and cubes is $6/d$, tetrahedrons $14.7/d$, octahedrons $7.3/d$, dodecahedrons $2.7/d$, cylindrical or square rod whose length is $10d$, $4.2/d$, and a square plate $.1d$ thick, $24/d$, where d is the diameter or face length. For sands the authors have suggested a variation of from $6/d$ to $7.7/d$ for rounded to angular sands.

Where the sand is not uniform, it is screened into fractions and the geometric mean of adjacent screen sizes is taken as the average diameter.

PERMEABILITY EQUATIONS

The preceding equations are all founded on Darcy's fundamental equation and represent attempts to define his coefficient of permeability in units capable of direct measurement. Three different methods of expressing the conducting power of the pores are represented. Slichter assumes that the pore channelways[40] can be compared to continuous but curving channels, and that increases in porosity from 27 to 48 per cent result in a steadily increasing cross-sectional area of channelway. Terzaghi assumes in part that the narrowest parts of the pores determine the rate of flow and that consequently the pore channelways in a column of spheres may be likened to the channelways in an equal column of sieves, the sieves being spaced at intervals comparable to the diameter of the spheres, but the actual interval being determined by porosity. Fair and Hatch assume that for any given porosity the distribution of the pores is negligible and that the average cross-sectional area of the pores, as expressed by the hydraulic radius, controlls their water-conducting power.

Table VII gives a comparison between the coefficient of permeability as computed from some of these equations and that determined experimentally for three different columns of single-sized spheres and for columns composed of a mixture of two sizes in the proportion of 1:1 by volume. The effective size, d_W, of the two component columns is computed from Slichter's formula $K = 771 \frac{d^2}{C_S}$. The value of K_r as computed by Slichter, when void volume is 0.50 and temperature is 10° C., is about 80 d^2. The experimental coefficients of permeability were determined at temperatures varying from 8.5 to 10.5° C. for different determinations, but the computations for each system are at the measured experimental temperature.

To use Terzaghi's equation it is necessary to have a value for his constant C_T. The average experimental value of this constant was 5.16 for single-component systems containing spheres of measured

[40] An analysis of Slichter's geometrical derivation of his formula is given in the preceding article, "Systematic Packing of Spheres, pp. 889 ff.

diameter and 5.02 for one- and two-component systems when the diameter was computed by Slichter's formula. Since it was necessary to use computed diameters for two-component systems, the value of the constant was taken as 5.0 in calculations with Terzaghi's equation.

Table VII shows a reasonably close agreement between the experimental and calculated coefficients of permeability of columns of uniformly sized spheres when the coefficient is calculated by either

TABLE VII

PERMEABILITY OF COLUMNS OF SPHERES

Measured Diameter (Centimeters) (1)	Computed Diameter d_W (Centimeters) (2)	Void Ratio (3)	Experimental Coefficient of Permeability (K) (4)	K_S Slichter (5)	Variation from K (Percentage) (6)	K_T Terzaghi (7)	Variation from K (Percentage) (8)	K_F Fair and Hatch (9)	Ratio to K (10)	K_E Terzaghi (11)
.31.....	.324	.4329	5.00	4.43	−11.4	5.29	+5.7	10.36	2.06:1	$81.8d_W^2$
.21.....	.216	.4310	2.24	1.97	−11.6	2.17	−3.2	4.32	1.92:1	$83.8d_W^2$
.16.....	.162	.3898	.88	.83	− 8.0	.90	+2.4	1.58	1.84:1	$80.6d_W^2$
.31+.81..	.375	.3808	4.83	.....	.....	4.51	+0.5	11.65	2.60:1	$80.0d_W^2$
.21+.81..	.346	.3154	2.01	.....	.....	1.96	−2.3	3.00	1.40:1	$82.3d_W^2$
.16+.81..	.240	.3106	.92	.....	.....	.86	−5.4	1.73	1.88:1	$83.6d_W^2$

Slichter's or Terzaghi's equation. The equation of Fair and Hatch gives values for K which are nearly double the experimental values and which become increasingly higher as the size of the spheres, and consequently the voids, increases. Their equation, however, is based on streamline flow; whereas it seems reasonably certain that in pores whose average cross section is as large and variable as in these columns of spheres, turbulent flow is inevitable. Consequently, it would appear that in order to satisfy the experimental data the percolation constant should be increased from 5 to about 9.7.

Although the equations of both Slichter and Terzaghi give reasonably satisfactory values for the coefficient of permeability of small uniformly sized spheres, serious difficulties arise in determining what value is to be taken for d_W, the effective size, when the equations are

applied to masses of variable grain-size. Slichter defines the effective size as that diameter which, if composing the entire sample, would endow it with the same permeability as it maintains with a mixture of sizes. Evidently this conception of effective size includes, not only the effect of the various grain sizes present, but also the effect of imperfect rounding, irregular packing, irregular-shaped channels, and all other divergencies due to the variations of natural materials from the ideal case. But how can the effective size be determined? Experiment shows that it is not the mean size of grain. It can, of course, be obtained if the coefficient of permeability is known, since $d = \sqrt{\frac{C_S K}{771}}$; but there is then no need for it, since it is primarily used to compute the coefficient of permeability. The values for K in two-component systems when calculated by Terzaghi's equation and using a value of 5.0 for C_T agree fairly well with the experimental values; but this of course is to be expected, since the value of d_W is in part determined by that of K.

The third equation, that of Fair and Hatch, overcomes the difficulty of determining the effective size by representing the size and shape of the voids in terms of porosity and hydraulic radius and expressing their values in terms of porosity, an empirical shape factor, and the screen-size of the sand. The values of K, as calculated by this equation, for two-component systems are erratic. The three sizes of small spheres used in the foregoing mixtures are such that the smallest spheres fit easily into the tightly packed voids of .81 cm. spheres and leave a moderate amount of free space; the next larger size slightly more than fills the voids and causes a slight disarrangement of tightest packing, while the .31 cm. spheres will not fit into the voids without causing a much looser packing. Consequently, it is not surprising to find that the value of K for mixtures of .16 and .81 cm. spheres, in which void conditions are nearest normal, is closest to the average ratio established for columns of uniformly sized spheres; low for the .21 and .81 cm. mixture, where the voids are unusually small; and greatest in the .31 and .81 cm. mixture, where the packing is very loose and the voids are large.

It thus appears that all of the foregoing equations give reasonably satisfactory values for the coefficient of permeability of columns of

uniformly sized spheres but that the equations are unsatisfactory for mixed sizes. Some indications of the cause of these discrepancies may be had by a further examination of the factors which control the coefficient of permeability of spheres.

FACTORS AFFECTING PERMEABILITY

The flow through a unit cross section of clastic material is modified by temperature, hydraulic gradient, and coefficient of permeability. The latter is affected by the following: uniformity and range of grain-size, shape of grain, nature and uniformity of packing, surface condition of the grains, stratification, consolidation, and cementation of the material. Before it is possible to deal adequately with the permeability of clastic materials, it is necessary to know the extent to which each of these factors may affect the coefficient of permeability.

TEMPERATURE

Temperature changes affect the flow rate of a fluid by changing the viscosity. Viscosity is liquid friction; and its existence, depending on motion, becomes zero when motion ceases. Consequently, it has little effect where low velocities are concerned. It can have no influence on the ultimate effect of capillarity, although it may affect the rate of capillary penetration which takes place by streamline flow.

The loss of head for streamline flow is known to vary as the first power of the mean velocity; temperature through its effect on kinematic viscosity is important because the rate of flow is inversely proportional to viscosity. For turbulent flow in pipes, the loss of head varies as the nth power of the mean velocity, n having a value varying from 1.70 to 2.0.[41] Of this loss of head, the frictional resistance due to viscosity is proportional to $(P/\mu)^{n-2}$, where P is the density of the water and μ is the coefficient of viscosity. The value of the exponent $n-2$ varies between 0.3 and 0. It is thus evident that for turbulent flow frictional losses, due to viscosity, will be very small—no matter what the value of the term P/μ may be—as compared

[41] C. Y. Haiao, "The Flow of Water in Pipes and the Effect of Corrosion" (Ph.D. thesis, presented to Department of Sanitary Engineering, Harvard Engineering School, February, 1930), p. 165.

with losses due to eddying, since the absolute value of the exponent of v (velocity) varies between 1.70 and 2.0.

Accordingly, corrections for temperature depend on whether the flow is streamline or turbulent. This is frequently difficult to ascertain. Where the velocity is low and the material is of fine grain-size, streamline flow may be assumed. Then, if the flow at 10° C. is taken as equal to 1, the flow at 27° C. will be 1.5; at 40° C. it will be 2; at 153° C. it will equal 7.2. But, where the velocity is high, or the material coarse, turbulent flow becomes increasingly important, and extensive experimentation may be required to determine the proper correction for clastic columns.

HYDRAULIC GRADIENT

Rate of flow is commonly assumed to be directly proportional to hydraulic gradient. This implies streamline flow where the rate of flow varies as the first power of the velocity and is inversely proportional to viscosity. But a number of the earlier investigators[42] indicated that the viscosity of water decreases with pressure. However, Bridgman[43] shows that viscosity decreases with pressure only at low temperatures and pressures. At higher temperatures and pressures the viscosity increases with pressure. These changes in viscosity were obained under much greater heads than those encountered in ordinary ground water or laboratory tests, so that unless hydrothermal activity is concerned they may be ignored.

In the present investigation tests made on the rate of flow under different heads through a column of small shot (1.5 mm.) were in accord with Poiseuille's law ("Increases in the rate of flow are directly proportional to increases in the pressure"); but the agreement was not perfect, the flow at higher pressures being somewhat smaller than it should have been. This divergence from the theoretical value was undoubtedly due to the fact that the flow of water through the shot did not take place entirely by streamline flow, as postulated by Poiseuille. Measured through shot with a diameter of 8.1 mm., the flow was considerably smaller than that required by Poiseuille's law.

[42] R. Cohen, *Annalen der Physik und Chemie*, Vol. XLV (1892), p. 666; Rongten, *ibid.*, Vol. XXII (1884), p. 510; Warburg and Sachs, *ibid.*, p. 518.

[43] P. W. Bridgman, "The Effect of Pressure on the Viscosity of Forty-three Pure Liquids," *Proc. Amer. Acad. Arts and Sci.*, Vol. LXI (1926), p. 92.

Stearns[44] shows a close agreement between increases in the rate of flow and increases in the head when the sample is composed of sand of medium-sized grains.

Mills[45] found that in pipes of a diameter greater than 0.1 inches the flow does not always increase proportionally to the first power of the velocity; above a certain critical velocity the adhesion of the water to the pipe is broken, the slow-moving layer of water around the periphery is destroyed, and eddying begins. During the next range of increasing velocities there is a considerable loss of head, owing to eddying, proportional to a power of the velocity greater than 2. Above a certain maximum velocity this eddying effect reaches a maximum. For further increases in velocity the coefficient of V^2 is constant.

Hazen[46] states:

> As the size inceased beyond this point . . . . [effective size of about 3 mm.], ... the velocity with a given head does not increase as rapidly as the square of the effective size; and with coarse gravels the velocity varies as the square root of the head instead of directly with the head, as in sands. The influence of temperature also becomes less marked with the coarse gravels.

Terzaghi[47] found that if water is passed through a column of cohesionless sand the rate of flow increases steadily until a critical hydraulic gradient of about 1, depending on the density of the sample, is reached; at this point the rate of flow suddenly increases. He concluded that the structure of the sand becomes loose at this point, i.e., the grains are lifted and sustained in a loose packing by the force of the upward-flowing water, and that this condition persists as the hydraulic gradient is further increased. On gradually lowering the hydraulic gradient below the critical value, the ratio of percolation to hydraulic gradient remained practically unchanged, indicating that the sand structure remained loose.

Furnas[48] found a similar behavior when gas was passed through a

[44] Stearns, *op. cit.*, p. 154.

[45] H. F. Mills, "Flow of Water in Pipes," *Mem. Amer. Acad. Arts and Sciences*, Vol. XV (1924), p. 2.

[46] *Op. cit.*, p. 554.

[47] C. H. Terzaghi, *Jour. Boston Soc. Civil Engineers*, Vol. XII, No. 10 (1925), p. 399.

[48] C. C. Furnas, "Flow of Gas through Beds of Broken Solids," *U.S. Bur. of Mines Bull. 307* (1929), p. 21.

column of broken solids. When the gas pressure became sufficiently great, the particles in the column were lifted and the texture loosened so that the pressure suddenly dropped and became very unsteady. He further found that this lifting velocity for a uniform bed of loosely packed material was independent of the depth of the bed.

Kessler[49] found the rate of flow of water under pressure through limestone and sandstone to be directly proportional to the pressure.

From the preceding considerations it is evident that rate of flow is directly proportional to the hydraulic gradient in sands and other clastic materials of similar size when the velocity is low, but that with coarser material or greater velocities the rate of flow becomes proportional to the n^{th} power of the hydraulic gradient when n has a value less than 1.

GRAIN-SIZE

The general assumption that the rate of flow through a column of spheres is directly proportional to the square of the diameter of the spheres follows from the fact that when the diameter of the spheres is doubled the throat-plane area increases fourfold. The assumption is true in so far as rate of flow is dependent on the size of channelway.

SHAPE OF GRAIN

Any departure from the spherical shape affects permeability by varying the size and shape of the interstices and by causing looser or tighter packing. It has already been shown that at equal diameters of grains and equal porosity, the coefficient of permeability decreases with increasing uniformity of the pore spaces.[50] Hence, as the form of the grains departs from that of a true sphere, the permeability will increase.

The effect of grain shape, as represented by the shape of the cross section of the interstice, on the rate of flow is shown in Figure 10, where the computed percentage of laminar and turbulent flow through a circle with a diameter of unity is plotted against the perimeter of different cross sections of equal area.[51] Under turbulent con-

[49] D. W. Kessler, *U.S. Bur. of Standards Tech. Paper 305* (1926), p. 164.

[50] See *infra*, pp. 934 ff.; see also Furnas, *op. cit.*, p. 53.

[51] The data in this graph were computed and plotted by E. Graustein, of Boston, Massachusetts, to whom the writer is indebted for permission to reproduce the graph in this paper.

ditions, maximum flow is obtained through a rectangular opening; the least flow is obtained through an orifice which approaches an equilateral triangle in form. Other shapes permit a rate of flow intermediate in value between these two extremes. With laminar flow, the relationship is not so simple or so direct, but the greatest flow is

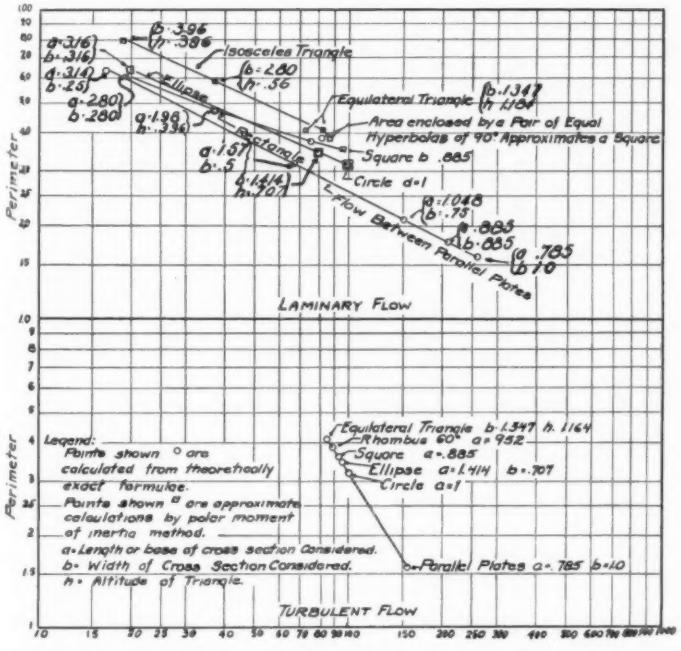
Perimeter
Isosceles Triangle
Ellipse
Rectangle
Equilateral Triangle
Area enclosed by a Pair of Equal Hyperbolas of 90° Approximates a Square
Square b .885
Circle d=1
Flow Between Parallel Plates
LAMINARY FLOW
TURBULENT FLOW
Legend:
Points shown ○ are calculated from theoretically exact formulae.
Points shown ⊡ are approximate calculations by polar moment of inertia method.
a=Length or base of cross section Considered.
b= Width of Cross Section Considered.
h= Altitude of Triangle.
Equilateral Triangle b 1.347 h. 1.164
Rhombus 60° a=.952
Square a=.885
Ellipse a=1.414 b=.707
Circle a=1
Parallel Plates a=.785 b=1.0

FIG. 10.—Percentage of flow in different cross sections of equal areas compared to the flow in a circle of a diameter of unity. (After E. A. Graustein.)

likewise through a rectangular opening. The relative capacities of other shaped openings for flow are shown in the figure.

Some of the discrepancies between the computed and experimental values of the coefficient of permeability have been assigned to the effect of variations in the rounding of grains. Slichter computed the coefficient of permeability of a column of spheres with a porosity of 46 per cent to equal $60.3d^2$. Hazen, working on a clean loose sand,

found the coefficient to vary between 116 and 150 d_w^2. The porosity of the sands on which he worked is not definitely known but would doubtless be of the order of 38–40 per cent. Terzaghi concluded that, at a porosity of 50 per cent, the values for the coefficient would range between 100 and 174 d_w^2 in sands. Using Terzaghi's equation, the coefficient for spheres has been shown to be about $80d^2$.

Thus it seems evident that the permeability of even the most angular sands, solely because of their angularity, would probably not be greater than two or three times that of a well-rounded sand.

POROSITY

The relative importance of the factors which influence porosity have already been evaluated. It is obvious that no prediction of the permeability of a material can be made solely from a measure of the percentage of voids irrespective of their size, shape, or continuity; permeability, although vitally dependent on the percentage of voids, is governed quantitatively by their size, shape, and continuity.

The resistance of a pipelike channel to the flow of a fluid is, in general, directly proportional to its length and perimeter and inversely proportional to its area. Per unit length, therefore, it is proportional to the perimeter divided by the area of the passage. The reciprocal of this figure is a geometrical constant known as the "hydraulic radius"; the term can be strictly applied, however, only to a passage with a fairly uniform diameter. When the passageways vary widely in cross section, as do the intergrain pores in a column of sand, the diameter of the narrowest portion on the channelway controls the flow.

The pores in a mass composed of individual particles are not uniform in diameter. In ideal conditions, the maximum circular cross sections of pores in tightly packed spheres vary from $0.15D$ to about $0.41D$ (D equals diameter of sphere). Furthermore, from each pore there are several horizontal branches to adjacent pores, but these branch pores do not assist in the transmission of fluids perpendicularly through the column. Therefore, much of the pore space present in a column does not aid flow in a particular direction; that portion which does, however, may be termed "useful porosity."

Any plane passed at random through a column of spheres reveals

alternating areas of solid material and voids. The area of the voids is not a true measure of the total area available for the transfer of fluids, because part of the voids are undoubtedly closed above or below by other grains which block the passageway. If, however, a plane is passed through the centers of spheres in one of the "rhombohedral" layers of tightest packing, i.e., a "throat-plane," the void area in this plane is a true measure of the minimum cross-sectional area of the channelways or of what may be called the "useful porosity." If it is assumed that this throat-plane has finite thickness, then its porosity may readily be computed. Theoretically, for tightest packing, the total porosity is 25.95 per cent; the effective porosity, 9.30 per cent. The total porosity in loosest packing is 47.64 per cent; the effective porosity is 21.46 per cent. Between these limits, changes in the effective porosity are directly proportional to changes in the total porosity, assuming uniform packing.

Slichter[52] computed that changing the total porosity of spheres from 26 per cent to 47.6 per cent would increase the permeability sevenfold. It is impossible to attain these porosity variations in a mass of spheres of uniform diameter packed under natural conditions. Experiments show that with spheres 1.3 mm. in diameter, a change in porosity from 36.6 to 38.2 per cent (brought about by different packing) increased the permeability 19.1 per cent. With spheres 3.1 mm. in diameter, an increase in porosity from 39.0 to 41.0 per cent increased the permeability 22.3 per cent. On this basis the increase in permeability for unit change in porosity is somewhat greater for the smaller spheres, but the difference is within the limits of experimental error. These figures do not agree with Slichter's calculations, which suggest that the increase in permeability should be over 50 per cent. The difference between the experimental and theoretical figures is probably due to the difference between the actual changes in packing and the change assumed by Slichter, who considered that the increased porosity was obtained by a uniform enlargement of the interstices. As pointed out in the preceding paper on "Packing of Spheres," changes in the porosity of a mass of uniformly sized spheres are not accomplished in this manner, but occur by variations in the number and size of the colonies of tightest pack-

[52] C. S. Slichter, *U.S. Geol. Surv. Water Supply Paper 67* (1902), p. 25.

ing and their surrounding shells of looser packing. It has been shown that the range of porosity in ordinary sands is not 21 per cent, as computed by Slichter, but rather rarely exceeds 9 per cent, and commonly is not over 3–4 per cent, after the sand has been deposited. These figures, of course, apply only to sands that have not been affected by cementation or recrystallization. Consequently, the total influence of porosity on permeability is much less than Slichter predicted. His figures show that increasing the porosity from 33 per cent to 43 per cent would increase the permeability 2.4 times; this value may be taken as the extreme limit of variation due to this factor. The ordinary porosity range of any pebble-free sand probably does not give variations of more than 100 per cent in the permeability.

Experience has shown that when changes in the permeability of a sand are plotted against changes in the porosity, a straight-line relationship results, but that the slope of the resulting line depends on the physical characteristics of the sand and consequently must be determined for each sample.[53]

The foregoing statements apply only to the porosity change in any one sample. Samples of widely divergent porosities owe this variation to differences in the shape and uniformity of their particles, and/or differences in consolidation and cementation. Consequently, no correlation can safely be made between two samples, on the basis of their porosity, unless it is certain that all their other physical properties are identical.

UNIFORMITY OF GRAIN-SIZE

Two-Component Systems

The influence of the uniformity of grain-size on the coefficient of permeability is very great in a column of clastic material. Just as porosity is less for a mass of mixed sizes, so may the permeability be lower, within certain limits, when material of a different grain-size is added to an existing mixture.

Around each large sphere, as around the walls of a container, there is a layer of voids considerably larger than those existing throughout the mass of the small spheres. Small spheres lying adjacent to the

[53] Hansen, *op. cit.*, p. 72.

large sphere can touch it at only one point. The spacing of these contact points is limited by the diameter of the small spheres. Consequently, a very permeable zone exists around a large sphere, because the large voids there connect freely in all directions along the surface of the sphere.

In addition there exists throughout the adjacent small spheres, for several of their diameters away from the large sphere, a zone of looser packing than is characteristic for the small spheres when packed by

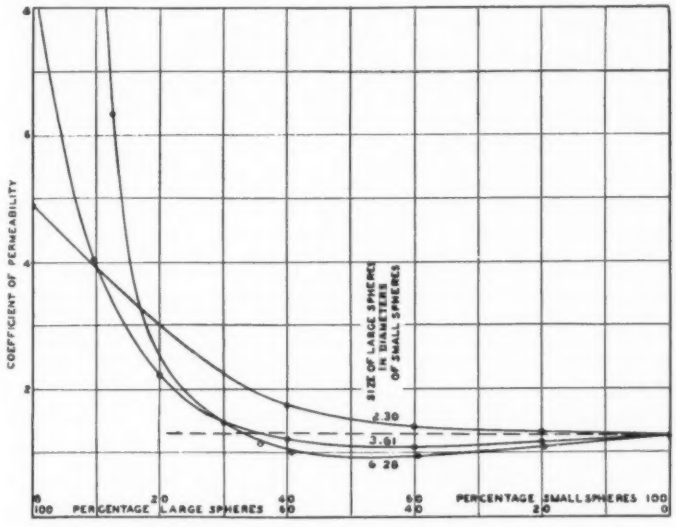

FIG. 11.—Relation between proportion of two constituents and permeability

themselves. Both of these factors tend to oppose the reduction in permeability caused by the impermeability of the larger sphere. The relative value of these opposing factors may be determined for a limited number of cases from Figure 11.

The curves in Figure 11 were obtained by measuring the permeability of a mixture of two sizes of lead shot in varying proportions by volume. The coefficients of permeability so obtained are plotted against the percentage composition of the mixture. The upper curve shows the effect of adding to a mass of small spheres larger spheres

whose diameter is 2.3 times that of the smaller ones. As the percentage of large spheres increases, the permeability increases—slowly at first, and then much more rapidly—because the large spheres which are only about twice the diameter of the small spheres are more effective in increasing the size of the channelways by disturbing the packing than in decreasing the permeability by blocking previously existing channelways. Hence, while there is less open space remaining throughout the mass after each addition of large spheres, the remaining voids form larger and more effective channelways, and we have the "anomalous" situation of the permeability increasing as the porosity decreases.

In the lower curve (Fig. 11) the addition of spheres with a relative diameter of 6.23 to a mass of smaller spheres (with actual diameter of 1.3 mm.) results at first in a lowering of the permeability. This lowering continues until the mixture is composed of about 50 volume per cent of each size. Then the permeability begins to increase until at about 31.5 volume per cent of small spheres, the permeability of the mixture becomes equal to that of the small spheres alone, and from that point on is greater. The porosity of the mixture, however, decreases until the proportions are about 25 per cent small spheres and 75 per cent large, at which point the large spheres are in contact and their voids filled as completely as possible with small spheres. Thus, between mixtures containing 50 per cent and 25 per cent of small spheres, the condition again exists in which the total void space decreases while permeability increases. Beyond the point where the mixture contains less than 25 per cent of small spheres, the value of the porosity loses significance because the number of small spheres present is insufficient to fill all of the interstices of the large. Also, on this curve, between 100 per cent and 31.5 per cent of small spheres, the "effective size" for the mixture would be a theoretical size smaller than the smallest spheres present, and its size would change for each change in the proportions of the mixture.

The middle curve (Fig. 11) was obtained for a mixture containing spheres whose diameters were in the ratio of 3.6:1. The maximum lowering of the permeability is less, the final upward break of the curve is not so sharp, and the point at which the permeability of the

mixture becomes equal to that of the small spheres is intermediate in position between those of the other curves.

These three curves illustrate the changes in permeability caused by increasing the size of the large spheres in a binary mixture. They also show that if the coefficient of permeability of mixtures of different grain sizes, but in the same proportions, be plotted against the diameter ratio of the spheres, the resulting curve would quickly rise to a maximum for a diameter ratio of about 2:1 and would then quickly descend below its starting-point. Experimental determinations, made on a mixture of spheres with a diameter ratio of approximately 20:1, indicate the maximum depression of the permeability curve to be not much greater than for the ratio 6:1. Consequently it appears that the permeability/diameter-ratio curve would soon tend to flatten as the size of the large spheres increased.

The cause for such a behavior of the curve has already been partly discussed. If the diameter ratio is small, the blocking effect of the larger spheres is small, as is likewise their relative ability to increase the permeability by means of the permeable zone around their surface. But loose packing must occur because the diameter ratio is below the critical ratio. The net result is an increase in permeability.

The maximum disturbance in packing of any large sphere in any plane is equivalent to a free opening slightly less than the diameter of the small spheres. Consequently, with increasingly larger spheres, the disturbance of packing in any plane begins at zero, increases to a maximum, drops to zero, and increases again. But the other two factors—decreased permeability, due to blocking, and increased permeability, due to the permeable surface zone of the large spheres—increase as the large spheres become greater.

If any given volume is equally divided among a number of spheres, then, as the radius of the spheres slowly increases, the total number will decrease very rapidly (a cube ratio). Under like conditions, the total cross-sectional area of the spheres, the sum of the cross sections through the center of the individual spheres, decreases relatively slowly (a direct ratio) as the radius of the spheres increases. Hence, as the size of the large spheres is increased, the total disturbance of packing, which is proportional to the number of spheres present, de-

creases much more rapidly than does the total blocking effect, which is proportional to the total cross section of the spheres.

The effect of the permeable zone around the surface of each sphere increases as the area of the sphere increases, but decreases as the number of spheres decrease. However, the net effect of this permeable zone increases as the size of the spheres increases.

The preceding considerations lead to the conclusion that as the diameter of the larger spheres is increased, the rate of lowering of the minimum permeability point will rapidly decrease.

These conclusions do not agree entirely with those of Furnas[54] for the permeability of beds of broken solids to gases. He obtained a similar rise in permeability when the two materials were of such size that the permeability of one was up to twice that of the other. Beyond this point, as the permeability of the large constituent becomes proportionally greater, the permeability of the mixture decreases; but he represents this decrease as being directly proportional to the ratio of the individual permeabilities of the two constituents. This implies that the greater the size of the coarse material added, the greater will be the decrease in the minimum permeability point, the rate of lowering of this point being directly porportional to the ratio of the permeabilities of the large and small constituents. He further concludes that, for gases, the lowest permeability that can be attained, by the addition of coarse material to fine material, will be $1/3.47$ of the permeability of the smaller constituent. It is interesting to note that in Figure 7, the maximum *reduction* of the permeability is $1/3.43$ that of the smaller constituent.

The point at which the permeability curves stop descending and begin to ascend may be termed the "maximum depression point." In the lower two curves of Figure 7, this point is not at the same percentage composition. As the diameter ratio of the spheres is increased, the point occurs at a mixture containing more large material, which implies that, as the spheres become larger, the number of zones of loose packing must be increased by adding more large spheres before the increased permeability resulting from this can equal the blocking effect due to the larger spheres. On the other hand, as the size of the larger spheres is increased, the slope of the

[54] Furnas, *op. cit.*, p. 70.

curve beyond the maximum depression point will become increasingly steeper, so that changes in the permeability beyond this point will be much more rapid for small changes in the relative proportion of the mixture of sizes.

Multi-component Systems

The curves that have been presented and discussed apply exactly only to a binary mixture. The effect on any binary mixture of substituting a given volume of spheres of intermediate size for an equivalent volume of the larger spheres is to increase the permeability; because, when such a substitution is made, the number of areas of disturbed packing is greatly increased, and the blocking effect is decreased (see Fig. 11). For example, in a mixture of 50 volume per cent small and 50 volume per cent large spheres, let the large spheres have a diameter of 6.23 times that of the small spheres. Now, for two-fifths of the large spheres, substitute an equivalent volume of spheres of a diameter 2.30 times that of the small spheres. From the upper curve it is apparent that the addition of spheres of diameter ratio 2.30 will always raise the permeability. Now, since two-fifths of the large spheres, which lower the permeability, have been replaced in the mixture by spheres of a size which raises the permeability, the net result must be an increase in permeability. In the same way, the substitution of any part of the large spheres by spheres intermediate in size must always raise the permeability, provided the small spheres remain the same size and in the same proportion to the larger components. This conclusion has been checked experimentally and found valid. Similarly, the addition of a fourth and fifth component of intermediate size would likewise raise the permeability.

From the foregoing it follows that in a multi-component system, in which the several constituents are present in approximately the same proportion, the permeability can be approximated by considering the sum of the permeabilities of the individual components, the permeability of each component being computed on the assumption that each constituent occupies a separate layer and affects the total permeability in proportion to its relative abundance in the mixture.[55]

[55] This principle has been found to hold experimentally for gases flowing through clastic beds (Furnas, *op. cit.*, p. 130).

This may appear to be directly opposite to the principle set up for a two-component system, where the permeability of the mixture is less than the permeability of the individual components. However, it has been shown that the addition of an intermediate size will raise the permeability of the entire mixture. Consequently, as the number of components increases, the permeability increases, until, for a mixture of a large number of components, the permeability will be the same, whether the system is stratified or mixed.

Herein lies much of the difficulty incurred when permeability equations based on a uniformly sized aggregate are applied to multi-component mixtures. In a uniformly sized aggregate the effective size is readily determined; and, as previously shown (p. 958), the existing equations may be applied with reasonable accuracy. In two-component mixtures, neither of the sizes present is the effective size, which indeed may be a theoretical size smaller than the smaller size present in the mixture. As the number of components increases, the conditions of permeability slowly approach those existing in a one-component mixture. Consequently, in a multi-component mixture, the conception of an effective size is valid and can be used to express in a single equation the combined effects of the different components. This effective size may be approximated by the equation $(p_1 d_1^2 + p_2 d_2^2 + \ldots . p_n d_n^2)$, in which p_1, p_2, etc., are the proportions by volume of the particles of diameter d_1, d_2, etc. Thus, if the effective diameters of the mixtures in Table VII are calculated in this manner, they are .376, .350, .342 cm., while the effective diameters as calculated from the known coefficient of permeability are .370, .346, and .240 cm. The first two, for mixtures in which the smaller component does not fit perfectly into the void between the larger spheres, show a reasonable agreement in effective diameter; while the effective diameter for the third mixture, in which the smaller component fits easily into the available voids, is very low. The reason for this divergence has been discussed in preceding pages. Moreover, as already pointed out, two-component systems offer the greatest divergence from the permeability conditions in single-sized aggregates; and, as the number of components increases, the more nearly will the foregoing approximation for effective size approach the true effective size.

ARRANGEMENT AND STRATIFICATION

Many of the sedimentary deposits of interest from the standpoint of the present investigation have been accumulated and arranged by moving water. Materials deposited in this manner are characterized by an arrangement of the grains which varies according to size, shape, or specific gravity, and to the process under which they were deposited. The perfection of this sizing, sorting, and stratification varies within wide limits, not only vertically, but horizontally. It is frequently difficult to predict what effect such stratification will have on the permeability of the beds.

Probably the best method of attack on the problem is to consider certain ideal cases of stratification in which the arrangement of the beds is simple, and to determine what influence this has on the coefficient of permeability. These simple ideal cases then delimit the possible variation in permeability; and, from the information thus obtained, it is possible to exterpolate to less ideal conditions.

Consider first two beds of known thickness, situated one above the other, with the flow at right angles to their plane of contact. Further assume that there is no intermixture of material along the contact between the beds. Let the coefficient of permeability of the two beds be k_1 and k_2. The coefficient of permeability, K, of the two beds combined may be determined by considering them as a single bed with a grain-size such as to give the same coefficient of permeability as do the pair of beds under consideration. In other words, the problem is to define K in terms of k_1 and k_2.

From Darcy's equation we have $Q = KFs$, where Q is the quantity transmitted as cm.^{3}/sec., K is the coefficient of permeability, F is the area of the cross section in cm.^{2}, and s is the hydraulic gradient equal to $[P_A - P_B]/L$ in which $[P_A - P_B]$ is the difference in pressure at the ends of the column of conducting material, and L is the thickness of the column. Applying this equation to the two layers, 1 and 2, in which the pressure differences are respectively $P_1 - P_2$ and $P_2 - P_3$, then

$$Q_1 = \frac{k_1 F_1 (P_1 - P_2)}{L_1},$$

where Q_1 is the quantity for the layer having a coefficient of k_1, length l_1, and a pressure difference between the ends of P_1-P_2;

$$Q_1=\frac{k_1F_1(P_1-P_2)}{L_1},$$

where Q_2 is the quantity for the layer having a coefficient of k_2, length l_2, and a pressure difference between the ends of P_2-P_3;

$$Q=\frac{KF(P_1-P_3)}{L},$$

where Q is the quantity for both layers.

Also,

$$(P_1-P_2)+(P_2-P_3)=(P_1-P_3),$$
$$Q_1=Q_2=Q.$$

By substituting the values above for Q, Q_1, and Q_2, and combining, we obtain:

$$K=\frac{k_2L_2k_1(P_1-P_3)}{L_2L_2k_1+L_1k_2}\times\frac{L_1+L_2}{P_1-P_3}=\frac{k_1k_2(L_1+L_2)}{L_1k_2+L_2k_1}. \tag{1}$$

From this it is apparent that if $L_1=L_2=1$ and $k_1=k_2=1$, then $K=k_1=k_2=1$.

Now consider that the flow is parallel to the contact between the two beds: the other conditions are the same as before. F is the total area of cross section, f_1 is the area of cross section of bed with permeability k_1, and f_2 the area of bed with permeability k_2. Then it is known that

$$F=f_1+f_2,$$
$$Q=q_1+q_2,$$
$$q_1=k_1f_1\frac{(P_1-P_2)}{L},$$
$$q_2=k_2f_2\frac{(P_1-P_2)}{L},$$
$$Q=K(f_1+f_2)\frac{(P_1-P_2)}{L}=k_1f_1+k_2f_2\frac{(P_1-P_2)}{L}.$$

Therefore

$$K=\frac{k_1f_1+k_2f_2}{f_1+f_2}. \tag{2}$$

It is now possible to determine the ratio between k_p and K_A, where K_p is the coefficient of permeability for flow parallel to the contact between layers, and K_A is the coefficient of permeability for flow at right angles to the contact.

Assuming that each column is a cube of edge L, then

$$F=L\times L, \qquad f_1=L\times l_1, \qquad f_2=L\times l_2.$$

Substituting these values in (2),

$$K_p=\frac{k_1l_1L+k_2l_2L}{l_1L+l_2L}=\frac{k_1l_1+k_2l_2}{l_1+l_2}. \tag{3}$$

Dividing (1) by (3), and simplifying,

$$\frac{K_A}{K_p}=\frac{L^2k_1k_2}{(l_1k_2+l_2k_1)(l_1k_1+l_2k_2)}.$$

If $l_1=l_2=1$, then

$$\frac{K_A}{K_p}=\frac{4k_1k_2}{(k_1+k_2)^2}.$$

Also, if $l_1=l_2$, then $K_pK_A=k_1k_2$, and therefore $K_p=k_1k_2/K_A$. From the foregoing equations it can be seen that K can be proved to be always intermediate in value between k_1 and k_2 and also that K_p can be proved always greater than K_A. The ratio between K_pK_A for different values of k_1 and k_2, where the beds are of equal thickness and thus $l_1=l_2$, is given as follows:

Ratio k_1/k_2	Ratio K_A/K_p	Ratio k_1/k_2	Ratio K_A/K_p
1	1:1.00	5	1: 1.80
2	1:1.12	10	1: 2.63
3	1:1.33	100	1:25.50
4	1:1.56		

Following is a comparison between values determined experimentally and those computed by the formulas above:

Ratio Diameters	k_1/k_2	Computed		Measured		K of Mixture Measured
		K_p	K_A	K_p	K_A	
6.2:1.....	19.8	14.5	2.6	14.5	2.2	.88
3.8:1.....	11.2	15.1	4.4	13.6	4.9	2.63
1.9:1.....	3.5	6.0	4.2	5.4	4.2	2.66
2.0:1.....	2.9	18.6	14.0	18.4	14.7	9.84
1.3:1.....	1.4	7.4	6.7	7.8	6.3	5.91

If the permeability of a mixture is compared with the permeability obtained when the mass is separated into its two components, it becomes evident that stratification always increases permeability in binary mixtures. The amount of this increase is dependent on the variance between the coefficients of the two constituents, and is greatest when this difference is greatest. Stratification might result in a decrease of permeability in columns composed of very restricted ratios of grain sizes, which could be more closely packed when separated into layers than when all packed together, but such a combination of grain sizes is improbable.

Flow parallel to the stratification is always greater than flow at right angles to it; the relation between the two is expressed by the equation $K_p = k_1 k_2 / K_A$ when equal volumes of material are involved. With flow parallel to the stratification, the layer with the greatest coefficient of permeability has the greatest influence on the quantity transmitted; whereas, if flow is at right angles to the stratification, the bed having the lower coefficient determines the magnitude of the flow. When flow is at some angle less than 90° to the plane of contact between beds, then K will be a value intermediate between K_p and K_A.

The formulas given above are derived from a consideration of only two beds; but if more than two are involved, the resulting coefficient of permeability can be readily computed by substitution in the equations given. Intermixing at the contacts of adjacent beds will tend to lower the average permeability, and this lowering will be greatest when the flow is at right angles to the contact.

Natural deposits may be stratified into distinct beds, as assumed above; or they may consist of one major type of material with interbedded irregular lenses of another type whose permeability may be either equal, greater, or less than that of the surrounding material. In such cases the computation of the average coefficient of permeability becomes laborious and less precise. However, it is apparent from the foregoing equations that the interbedded lenses will raise or lower the coefficient of permeability of the entire section whenever their own permeability is greater or smaller than the permeability of the intervening material.

In the preceding discussion it is assumed that permeability is uniform in all directions within a given bed. In natural deposits such is not necessarily true. Sedimentary features such as imbrication, orientation of elongated, tabular, or flat-sided units—whether pebbles or tiny grains—perfection and distribution of void filling, and minor stratification will create directions of maximum and minimum permeability. Whether the features mentioned above will augment or decrease the total permeability will depend on the orientation of these features with respect to the stratification and the direction of flow. But in the great majority of places where a directional difference of permeability exists, the greatest permeability will be parallel to bedding.

DEPOSITIONAL EFFECTS ON LOCAL PERMEABILITY

GENERAL STATEMENT

Sedimentary deposits are uniform in grain-size and texture only under exceptional circumstances. Consequently, the permeability of sediments is rarely uniform; much more commonly it fluctuates widely and abruptly from place to place, owing to variations in grain-size and texture caused by the varying conditions under which the sediment was deposited.

Certain features of the activity of wave and stream deposition are considered in relation to the variations which they develop in sediments, and an attempt is made to point out the origin of certain textural variations and their significance with regard to permeability. The only depositional processes here considered are those of waves and rivers.

WAVE DEPOSITION

Transportation of Material

Wave action naturally divides itself into two phases: the in-wash (wave proper), and the back-wash. The incoming wave approaches the beach with a maximum velocity which gradually approaches zero as the wave moves toward the land. The out-wash begins with a zero velocity and gradually increases its speed as it moves back toward the sea. These two phases each have a separate and important action on the shape and general structure of the beach material.

Pebbles of diverse shapes are transported differently; the particular method is usually that which requires the least expenditure of energy, but depends in part on the velocity of the water and, in part, on whether the pebbles are essentially roller-shaped, tabular, or round.

Roller-shaped pebbles are those ellipsoidal forms which have more or less equi-dimensional cross section. They are moved either by rolling on the bottom or in suspension in the water, depending upon the velocity of the wave. Irrespective of the position in which such a roller-shaped pebble is lying when reached by a wave, it immediately swings (if the wave is strong enough) so that its longest axis is normal to the direction in which the wave is traveling. If the wave can do no more than thus swing the pebble, the pebble is left in the orientation just noted. If, however, the wave is strong enough to transport the pebble, the pebble will roll with its axis still in this position, i.e., across the line of movement. Still stronger waves will carry the pebble in suspension; but as soon as it again touches the bottom, it immediately resumes an attitude parallel to the front of the wave, retains this as long as motion lasts, and generally comes to rest in that orientation. If the back-wash is sufficiently strong, the pebble is rolled back toward the sea, still keeping the axis perpendicular to the direction of motion. Thus, such a pebble may be repeatedly rolled alternately landward and seaward, keeping its long axis parallel to the general shore line. There appear to be only two exceptions to this orientation of the pebble: (*a*) If the back-wash is not strong enough to transport the pebble seaward, yet is strong enough to swing it, the pebble will commonly be swung into the line of flow of the back-wash, i.e., at right angles to the general shore

line; but this happens only to a relatively small proportion of the pebbles, and even those that take such alignment hold it only till the next incoming wave swings them into the normal attitude of parallelism to the general shore line. (*b*) Where the "floor" on which the given pebble is deposited is rough and irregular, the orientation in which the pebble comes to final rest thereon may be influenced by the irregularities, and a fraction of the pebbles may thus take up haphazard or abnormal orientations. Both these exceptions, however, are distinctly subordinate in effect to that major tendency for the pebbles to line up with their long axes parallel to the general shore line. The foregoing phenomena apply also to pebbles whose mean and smallest axes are notably different—as much as 2:11.

This type of orientation has already been described by Graton,[56] who used it to deduce the probable direction of transportation of the pebbles that compose the gold-bearing conglomerates of the Rand. Motion pictures of groups of such pebbles moved by the waves on a beach leave absolutely no question as to the invariable dominance of the tendency toward this particular manner and direction of orientation. The same orientation relative to direction of water movement was carefully observed and recorded by Hunzicker[57] as a result of investigations on a "stream table." And Hunzicker and Kransdorff,[58] working in collaboration, further confirmed the vigor and constancy of this orientation phenomenon. See later, under "River Deposition," for further discussion of this subject.

The degree of orientation of moderately elongated beach pebbles was determined by counting the number of such pebbles in various positions in four different areas on a sand beach at Revere, Massachusetts, where pebbles were not especially numerous whatever the shape. The areas on which the counts were made is indicated in Table VIII. The count for a 9×9 foot area refers not to pebbles but to a group of bricks that had been subjected for some time to wave action on a pebble beach and were thus considerably rounded; the general agreement of their orientation with that of the natural pebbles is noteworthy.

[56] *Op. cit.*, p. 39.

[57] See W. H. Twenhofel, *A Treatise on Sedimentation* (2d ed.; 1932), p. 36.

[58] Personal communication to L. C. Graton, 1932.

Roller-shaped pebbles are among those easiest to move; in their preferred orientation across the direction of wave motion, they expose a relatively large section to the on-coming water, yet they roll very easily on their more nearly circular cross section.

Pebbles of tabular or disk shape normally lie as nearly flat as possible. Only rarely are they moved by being pushed along the bottom. They generally are transported by saltation. As the velocity of the incoming wave increases to some adequate value, the seaward edge of the pebble is suddenly lifted until the pebble has assumed a

TABLE VIII

ORIENTATION OF ELONGATED PEBBLES ON A BEACH

Orientation of Long Axis	Size of Area (in Feet)										Percentage Average for Five Areas	
	6×6		6×6		1.5×1.5		1.5×1.5		9×9			
	No.	Percentage	No.	Percentage	No.	Percentage	No.	Percentage	No.	Percentage		
Essentially parallel to water line	22	84.6	15	50.0	30	83.3	28	71.8	29	76.3	73.2	83.5
Less than 45° from parallelism	2	7.7	6	20.0	2	5.5	3	7.7	4	10.5	10.3	
More than 45° from parallelism	2	7.7	9	30.0	4	11.2	8	20.5	5	13.2	16.5	

position with its plane of greatest surface approximately at right angles to the direction in which the water is moving, and then the pebble is moved bodily; sometimes it is carried in this position, but commonly it is turned over and over in the water in such manner that its longest axis (which of course lies in its plane of greatest extension) keeps at right angles to the direction of movement. When the motion slackens, the pebble is deposited as nearly flat as circumstances permit, but commonly the final position is such that the plane of the pebble dips toward the direction of major wave action, i.e., ordinarily toward the sea; and if its two major axes are unequal, the longest axis lies across the direction of movement and the mean axis lies with that direction. Tabular pebbles are among the most difficult to transport. Once started moving, they move with rela-

tively high velocity and are carried far, for they expose a large surface-weight ratio; but they get started with difficulty and, when dropped, tend to hold their place tenaciously.

The manner of transportation of well-rounded pebbles is more variable. If small enough, they are carried in suspension. If larger, they are rolled; and the tendency is always for the longest axis to parallel the front of the wave. As a rule, they do not move as readily as do the roller pebbles, for, while having but little advantage in circularity of cross section, they have the disadvantage of exposing a smaller "spread of sail" to the wave. Pebbles that are very close to spherical in shape of course roll easily.

Pebbles of such shapes that they do not fall definitely within any one of the three classes mentioned are transported in accordance with some intermediate treatment; e.g., a pebble of flattish ellipsoidal outline, but not thin enough to be classed frankly as tabular, will exhibit to some degree the behavior of a tabular pebble and to some degree that of a roller-shaped one.

Sorting as to Shape

Since roller pebbles are most easily moved, round pebbles are moved with more difficulty, and tabular pebbles offer the greatest difficulty in transportation, it might well be expected that this would lead to a segregation of the pebbles according to shape. As a matter of fact, wave action is highly selective and such shape-sorting does actually take place. Tabular pebbles are often tossed high up on a beach and remain there until the next storm. Roller and round pebbles may also be carried high up on the beach by storm waves, but commonly the velocity of the accompanying back-wash is sufficient to return them to the active wave line again. Because of the greater mobility of roller and round pebbles, any offshore concentration might well be expected to consist of examples of these shapes which have most nearly circular cross sections.

Local Packing of Beach Material

In a deposit composed mostly of pebbles, there is a tendency for medium-sized pebbles to be heaped on the landward side of a boulder and for larger pebbles and smaller material to be on the seaward side. This is less evident where there are only a few large pebbles; here the

tendency is for the medium-sized material to collect on both seaward and landward sides of the boulder, with the greater amount on the landward side. This general arrangement of sizes is due to the two-fold nature of wave action. As the wave moves shoreward over the beach, it transports everything it can; then, as the velocity of the wave diminishes, the large pebbles are deposited first, and increasingly smaller sizes are laid down in an order determined by their size, shape, and difficulty of transportation. The next incoming wave repeats the same process until it reaches the place where it lacks sufficient force to lift its large pebbles over what was previously deposited, and consequently leaves them leaning against the earlier pebbles. Smaller material, however, is carried past the barrier and deposited farther ashore. The back-wash picks up sizes in the reverse order to that in which the main wave deposited them. The back-wash generally has less transporting power; and consequently, as soon as it meets any considerable obstruction, it drops first that material which it has been carrying with most difficulty; this material is deposited on the landward side of the obstruction. The finer sand is washed out, carried around or over the boulder, and much of it dropped in the eddy on the seaward side. Thus waves deposit the medium-sized material on the landward side of large boulders and the coarse and fine material on the seaward side. Relatively little coarse material remains on the lateral sides.

The material just underneath a boulder is also usually coarser than elsewhere. This relation is especially frequent where a considerable variety of sizes are deposited together.

The filling of fine material is not equally distributed around a boulder. The area along the under side, and especially the under landward side, is often poorly filled, while around the upper part of the boulder, and especially on the seaward side, the filling-in by sand is much more complete. The action of the back-wash probably explains this distribution. Before the boulder is well buried, there is a strong flushing action on the landward side by the back-wash, which removes the finer material and piles it on the seaward side. As the boulder becomes more deeply buried, its value as a riffle decreases and it fails to collect the coarser material. However, by this time, the sand is usually unable to filter down into the voids along the

lower sides, and accordingly they remain incompletely filled. The deposition of sand in gravel and around boulders is largely a process of filling-in from above. As a result, whenever a pebble is deposited with voids underneath it, there is little chance of later filling these voids with sand.

Any angularity in the form of a pebble results in a looser packing of the matrix at that point. Where angular or faceted fragments are packed together, the flat sides tend to lie face to face wherever possible, probably because this is the most stable position. This arrangement is especially noticeable where a boulder is surrounded by smaller grains which always tend to have their flattest side, or one of their flattest sides, against the boulder. This packing makes it difficult for sand to sift in and fill the voids underlying a larger unit (cobble or boulder) because the connecting openings are apt to be smaller than the diameter of the sand grains. Boulders that have a flat side are usually deposited with that side down; this further helps to explain why, as already shown (p. 930), there is often a very porous zone immediately around a boulder, and the greatest porosity is very commonly on its under side (p. 940). Note especially the loose packing below the "boulders" in Figures 3–6.

The porosity in the vicinity of a boulder, then, is usually high along the landward side, still higher on the bottom side, and highest of all at the landward side of the bottom. The presence of a large boulder in a sand bed will denote a higher local porosity, not only because of actually looser packing of whatever surrounds the boulder (because of interference), but also because of the incomplete sand-filling of the interstices of large pebbles in the vicinity of the boulder.

Effects on Permeability

Variations in the packing of beach material, as described above, are sufficiently great to exert a master control on the local permeability. The Great Conglomerate of the Keweenaw Peninsula, Michigan, offers especially clear illustrations of the preservation, in a consolidated sediment, of the arrangements of sand and pebbles observed in recent deposits. In this conglomerate the greater porosity of the under side of boulders is made clearly evident by the amount of introduced material, while the orientation of the pebbles

and the arrangement of the smaller grain sizes around large boulders conform to the generalizations given just above.

In the Calumet conglomerate, in the same district, striking proof of the greater permeability of the under side of large pebbles and cobbles is shown by the distribution of calcite and other minerals introduced into the conglomerate. In localities which were not too heavily mineralized, calcite is invariably concentrated along the bottom of these larger units and also shows a pronounced tendency to occur around one end of the cobble in preference to the other. In the great majority of places examined in this region, the nature of the packing indicates that the source of the conglomerate was to the west (down dip).

RIVER DEPOSITION

A river offers conditions of transportation and deposition differing from those of a wave in that the action is continuous over a longer period and acts always in one general direction. River gravels commonly are not well stratified. As a flood is required to cause extensive transportation, the periods during which river gravels may be moved, sized, and sorted are usually brief and are followed by much longer quiescent periods during which infiltration and arrangement of the finer sizes may be accomplished. Rivers and waves are comparable not only in the general manner in which they transport their load but also in the general arrangement of their deposits, which, however, differ in some details discussed later.

Rivers are required to transport the same three general shapes of clastics as waves, and observations have shown that in general the mode of transportation is essentially the same. Thus, the tendency for the medium-sized clastic fragments to be accumulated on the upstream side of a boulder is a criterion that may be used, as in wave action, to determine the direction in which the water was moving. Orientation of the roller-shaped pebbles and imbrication of the flatter or tabular pebbles are two phenomena which can be readily used to determine the direction in which the water in a river was flowing at the time of deposition. The majority of elongated pebbles lie with their longest axis approximately at right angles to the direction of water movement; tabular pebbles dip toward the direction from which the main flow came, i.e., upstream.

Much confusion seems to have resulted from earlier misstatements[59] regarding the orientation of imbricated pebbles and from the attempt of W. A. Johnston[60] to state the case correctly. The erroneous statement had been that the pebbles are "inclined downstream." This was the fallacy to which Johnston addressed himself; and he gave what is unquestionably the true picture in his major statement: "The important points are that the stones tend to dip upstream . . . . and the upstream stones tend to overlap on the downstream stones."[61] But in descriptions intended to support these conclusions, the critical point on which controversy has later turned is stated in terms that are ambiguous: "The longer axes of the stones usually lie in the direction of the current."[62] From this statement and the context, one is left uncertain as to the exact orientation of the pebbles. Why does he use "longer axes" in the plural? Does he refer to the single longest axis of each of the many pebbles? Or could he possibly have intended to say that both of the longer axes of each pebble, that is, both the longest axis and the mean axis, "lie in the direction of the current"? It seems clear that Johnson did not intend to convey this second idea, since two axes of a triaxial form cannot both lie in the single "direction of the current." Therefore, it might be natural for one who had not taken occasion to make first-hand observation of this phenomenon to conclude that the first alternative given above represents his meaning. And so, at the hands of several writers, we find that Johnston's statement has been paraphrased from "the *longer* axes of the pebbles . . . ." to "the *long* axes of the pebbles lie in the direction of the current," which latter phrasing, though still ambiguous, surely implies, and evidently is intended to mean, that the unique, *longest* axis of the individual pebble lies with the line of the current.

Observations by Graton,[63] Hunzicker,[64] Twenhofel,[65] and Hun-

[59] E.g., G. F. Becker, *Geol. Soc. Amer. Bull. 4* (1893), pp. 53–54.

[60] "Imbricated Structure in River Gravels," *Amer. Jour. Sci.*, 5th Ser., Vol. IV (1922), pp. 387–90.

[61] *Ibid.*, p. 387.

[62] *Ibid.*, p. 388.

[63] Personal communication, 1928.

[64] Reported in Twenhofel, *op. cit.*, p. 36.

[65] Personal communication to L. C. Graton, 1931.

zicker and Kransdorff,[66] and the present writer are not in accord with the foregoing interpretation of Johnston's statement. The situation is put clearly and correctly by Hunzicker, as cited by Twenhofel: the pebbles have "their longest axis perpendicular to the current, the median axis at the same time acquiring a position that dips up-current." That is to say, of the two pebble axes that lie in the (vertical) *plane* which parallels and contains also the *line* of (say, north to south) flow, the longer of these two axes points obliquely (and more or less gently) downward to the north or upstream, while the shorter of these two axes points obliquely (and more or less steeply) downward to the south or downstream. But these two axes are, respectively, the *mean* and the *shortest* axes of the pebble; the *longest* axis lies east-west, across the line of flow. This is exactly analogous to the state of affairs in wave orientation on a beach, as already discussed on pages 978–80. This manner of orientation is readily observable as preponderant in streams ranging from mere rills to great rivers, and in material from small pebbles to huge boulders. One repeatedly finds that the "stepping-stones" by which he crosses a shallow stream are dominantly lined up so that the longer dimension of their unsubmerged portion lies crosswise of the stream.

Transportation of Material

Streams transport the solid portion of their load by two methods —traction and suspension.[67] The relation between the weight of the particle and the velocity of the stream roughly determines by which method the load shall be carried. The coarser constituents of a river deposit are transported mainly during periods of flood. Any individual flood consists of a continuous series of changing discharges which increase until the flood reaches its maximum, and then decrease. At each instant the stream contains systems of currents which are in process of continuous change; the variations in each current depending not only on the shape of the channelway left by the preceding current but also on the total discharge. As long as discharge goes on, variations in the current continue to change the form

[66] Personal communication of D. Kransdorff to L. C. Graton, 1932.

[67] G. K. Gilbert, "The Transportation of Débris by Running Water," *U.S.Geol. Surv. Prof. Paper 86* (1914), p. 26.

of the channel. This results in continuous aggradation and deposition at different localities, so that the load which the stream carries is constantly changing, as does the mode of transportation of that load. McMath[68] observes that "a rising river scours from the deeps to deposit in the shoals, and a falling river scours from the shoals to deposit in the deeps."

Gilbert[69] shows that a river can transport a greater load when mixed sizes are present than when only one size is available, because the force necessary to move large pebbles is also adequate to carry a large amount of smaller material. However, it is difficult to understand how a river capable of carrying all sizes up to large boulders can deposit them at the same time. At any instant, the largest particle that may be moved varies as the sixth power of the current velocity.[70] Small changes in velocity will thus result in great variations in the size of material moved. A 7 per cent increase in velocity would increase the size of particles moved by 50 per cent; a 12.25 increase in velocity, 100 per cent; and a 30.75 per cent increase, 400 per cent. In a gravel which varies from pebbles 10 inches in diameter to sand $\frac{1}{25}$ inch or 1 mm. in diameter, the actual variation is 250 times or 25,000 per cent. Accordingly, the velocity in a stream carrying such pebbles would necessarily be decreased 60 per cent before the 1 mm. sand could be deposited.

Such a sudden decrease in velocity might occur on a beach, or where a river spreads over a flood plain, with consequent dumping of material; but it seems unlikely that such violent changes in the current velocity always occur when coarse material is deposited. Hence it seems improbable that large pebbles and fine sand are necessarily deposited simultaneously. Many gravels are virtually free of sand. If rivers deposit large pebbles and fine sand simultaneously, then sand-free gravels may be explained only by assuming that pebbles were the only material available to the river at that time. Such an assumption would certainly not be admissible for gravel lenses in a sand bed; yet these lenses are commonly remarkably free from small-sized particles. It is easier and more logical to

[68] McMath, *Miss. River Comm. Rept. for 1881*, p. 252.

[69] Gilbert, *op. cit.*, p. 184.

[70] For a discussion of the origin of this statement see Gilbert, *op. cit.*, p. 16.

assume that at any given instant a river usually deposits only a very limited size range of material and that the finer sizes present in gravels are the result of later infiltration. The time at which infiltration occurs depends on the current velocity in the river. As long as a high velocity is maintained, only coarse material can be deposited, except for the minor amounts of fines dropped in eddies and local areas of slack water. If, however, the velocity fluctuates widely, then the infilling of smaller sizes may follow close after the deposition of the coarse material; infiltration would still control the arrangement, although the deposit would possess little uniformity. The situation prevailing during the deposition of gravels is essentially different from that existing during the deposition of fine sands, silts, and clays, which are deposited mainly by slow settling from slack water. Under these circumstances, at each level in the water, there is a heterogeneous mixture of sizes settling toward the bottom; consequently, as long as a supply is maintained at the top of the column, the materials will settle as a heterogeneous accumulation, unless there are present conditions such as a periodic supply, or special factors such as those operative during varve formation.

Arrangement of Material

The methods of transportation and deposition outlined above control the arrangement of the deposit and, therefore, vitally affect the permeability.

The section underlying a shoal in a river consists of a lower, somewhat heterogeneous, mass deposited during the flood and an upper layer of coarser material left by erosion as the current slackened. The thickness of the lower layer depends on the discharge and amount of material available during the flood. The thickness of the upper layer, as also its relative coarseness, depends on the duration of erosion and the velocity of the eroding stream. Usually this upper layer is relatively thin.

The deposit formed in a deep of the river consists of a lower layer of coarse, poorly packed, and relatively homogeneous material which grades upward into finer material. The variations in size and perfection of packing depend on the velocity of discharge and on the material available during the flood. With the recurrence of a flood,

much of the arrangement accomplished by gentler currents is obliterated. However, the same processes working according to the same principles are in operation elsewhere along the channel and tend to produce the arrangement described.

Lenses of coarse material are of common occurrence as the result of fluctuations in the river currents. These lenses may exert a great influence on the total permeability of a deposit because they commonly consist of sized material, loosely packed, and only imperfectly filled with fines. Although these lenses are frequently zones of high permeability, their total effect is usually lessened by their lack of horizontal or vertical continuity.

In regions where streams are subject to floods sufficiently great to cause them to leave their channels and cover the adjacent plains, the distribution of the main channels will greatly affect the permeability of the deposit. During periods of normal stream activity, the water is confined to definite channels in which are laid down well-sized and highly permeable deposits of sand and gravel. With the advent of a flood, the stream overflows its banks and deposits material over the surrounding area. Very frequently in so doing it changes its channel. As a consequence, deposits of this type are apt to develop as accumulations of poorly sized material with interspersed horizontal bands of irregularly distributed, well-sized material. The permeability of such a mass will, on the whole, be low; but locally, along the bands of sand and gravel, it may be high. If these bands are connected horizontally and vertically, they may form a very permeable webb throughout the mass; but more commonly they are not so connected, and consequently the mass as a whole has a low permeability.

Imbrication, because it represents an orientation of pebbles and accompanying voids, will locally tend to give a greater permeability parallel to the imbrication than at right angles to it.

Local Packing

Around any pebble or boulder which is considerably larger than its neighbors, there is commonly a fairly definite arrangement of the smaller grain sizes. One or more smaller pebbles will often be accumulated on the upstream side of a boulder, probably because the

velocity of the water was insufficient to lift them over the obstruction. These smaller pebbles commonly lean against the large pebble, thus tending to protect the underlying voids from later filling. Moreover, the flowing water removes much of the finer sizes from the upstream side and deposits them on the downstream side. Accordingly, the packing and filling on the upstream side of a boulder is generally loose and incomplete. Where large pebbles have been lifted over the obstruction, they generally come to rest a short distance downstream and are not leaning against or otherwise in contact with the large unit, as is the case on the upstream side.

The vertical distribution of grain sizes, and therefore of void sizes, around a boulder is not uniform. Commonly there is coarser material near the bottom and especially on the upstream side, although the sand or other débris on the downstream side may also be coarser near the bottom. In general, there is less difference in the vertical distribution of sizes around a boulder in a stream deposit than in a wave deposit.

Just as the local porosity in a river deposit is non-uniform, so is the local permeability. The absolute size of the openings, the variations in their size, and the continuity of the channelways vary widely and rapidly from place to place. The zone around a boulder, especially the bottom and upstream sides, are favored areas of permeability.

In general, the distribution of permeability in river and wave deposits is rather similar; but as rivers are more capricious in their deposition, their deposits probably offer a greater variation of permeability in a shorter horizontal distance. Each phase of river action has a specific effect on the distribution of the permeability.

RELATIVE PERMEABILITY OF UNCONSOLIDATED SEDIMENTS

RATING OF INFLUENCES

The permeability of unconsolidated clastic sediments is a composite resultant of grain-size, uniformity of grain-size, distribution of grain-size, packing, and shape of grain. These factors, which determine what may be termed the "original permeability," are not of equal influence. Clastic sediments which vary from coarse to fine grain sizes are subdivided essentially on the basis of their grain-size,

which is probably the most important single factor affecting permeability, although its importance may be greatly modified by the uniformity of the grain-size. Packing, taken alone (i.e., variation in packing of a single grain-size), influences permeability within limits which are not sufficiently wide, under ordinary circumstances, to make it as important as grain-size or uniformity. The distribution of the grain sizes may be of very great importance under certain circumstances. The shape of grain is relatively unimportant because of its limited variation in any individual sediment.

RATING OF SEDIMENTARY TYPES

A general comparison of the permeability of clastic sediments is difficult because in any given sediment the general, and also the

TABLE IX

COMMON YIELDS OF VARIOUS SEDIMENTS TO WELLS

Material	Yield in Gallons per Minute
Gravel	1,500–4,000 (commonly)
Sandstone	5–100 (jointed quartzite yielded 140)
Clay	Less than 1
Loess	Less than 1
Till	About $\frac{1}{4}$ from true till (Deep wells in till tapping sandy layers yield around 5) (Deep wells in till tapping gravel layers yield around 100)
Alluvium	About 1

local, permeability may vary so greatly, owing to changes in grain-size, uniformity, and stratification, brought about by variations in the conditions prevailing during the deposition of the sediment. A measure of the relative permeability of the different clastic sediments may be obtained from Table IX. This is based on the relative water-yielding capacity of clastic sediments, which may be taken as a rough quantitative measure of their permeability.[71]

Gravel

Gravel (unconsolidated) and conglomerate (consolidated), because of their large grain-size and resulting coarse texture, are pre-

[71] Meinzer, *op. cit.*, *passim*.

eminently the most permeable of clastic sediments. Since permeability varies as the square of the diameter of the grains, gravel of an average pebble size of 2 inches (50 mm.) and with voids unfilled has a permeability over 600 times as great as a clean sand of an average grain-size of 2 mm. The superiority of gravels (i.e., mixed pebbles and sand) in supplying the water requirements in regions where surface waters are not available and recourse must be made to underground storage may best be judged from the following quotation from Meinzer: "Gravel is the best kind of formation to yield water. In the United States it supplies most of the strong wells and furnishes more water to wells than all other materials taken together."[72]

As porosity is theoretically independent of grain-size, the porosity of gravel should not necessarily be different from that of other coarse clastic sediments. A uniformly sized gravel may have porosities up to about 40 per cent. A poorly sized gravel may have porosities as low as 20 per cent. The average porosity of gravels is around 25–30 per cent.

Gravels vary widely in their uniformity of grain-size. In contrast to gravels (conglomerates) such as the Main Reef Leader in the Rand, which possess a truly remarkable uniformity, alluvial fan deposits of the Western United States contain all sizes and proportions of material from silt to boulders 1 foot or more in diameter. Gravels commonly appear less uniform in contrast to finer sediments, but this may not always be the true situation. Thus, the change from 5 to 10 cm. in a gravel is immediately evident, whereas the change in grain-size of a sand from 1 to 2 mm. is not striking. Sands commonly contain particles ranging in size from 2 mm. to 0.005 mm. (clay), a diameter variation of 400 times. Assuming that in a gravel the smallest grain-size present in important quantities is 1 mm., then a 400-times variation in diameter would require the largest size to be about 40 cm. (16 in.); such variations in gravel are decidedly uncommon.

Marine and lacustrine gravels are usually moderately uniform. Gregory[73] considers that, on the whole, marine gravels are more uniform than fluviatile gravels. Glacial gravels in morainal deposits

[72] Meinzer, *ibid.*, p. 117.

[73] H. E. Gregory, *Amer. Jour. Sci.*, Vol. XXXIX (1915), pp. 487–508.

may contain a great variety of sizes; in eskers they may be remarkably free from fine material.

Below a certain limit of grain-size, the openings between grains are too small to permit free flow of water. From the standpoint of important ground-water movement, anything below a coarse silt (about 0.02 mm.) is practically impermeable. A gravel having no material smaller than 0.02 mm. is permeable throughout. A sand possessing the same degree of size uniformity necessarily contains much material smaller than 0.02 mm.; and these smaller grain sizes lower the permeability very greatly, or render certain portions relatively impermeable. While lack of uniformity in gravels usually lowers the permeability, the chances of rendering any portion impermeable are much less than in sand.

The distribution of the finer material in gravel interstices is difficult to predict. Most deposits seem to have formed by intermittent accumulation, and consequently the fine material is randomly scattered throughout the deposit. The distribution of the well-sized layers or lenses is also difficult to predict, because they are entirely controlled by local conditions of supply and current velocity. An examination of the average gravel deposit usually shows lenses of well-sized material intermixed with streaks of poorly sized gravels and bands of sand, or even of clay. It follows, then, that although the average permeability of gravel will be high, the distribution of the local areas of high and low permeability will be very erratic and absolutely unpredictable unless abundant local detailed data are obtained concerning that particular deposit.

Sand

Sands and sandstones rank next to gravel as water-bearers (Table IX). Sands show a considerable range in their grain-size; few contain grains larger than 5 mm. Most often the coarser material does not exceed 2 mm. in diameter. Many sands contain a considerable amount of fine material in which it is not unusual to find up to 3 per cent of clay. Table X shows the grain-size and uniformity necessary to make a good aquifer.[74] These rather fine-grained but clean sandstones, widely regarded as good aquifers, are believed to yield their

[74] *U.S. Geol. Surv. Water Supply Paper 489*, p. 119.

water from the interstices rather than from joints. They are well sized: only a few per cent of the grains present are larger than 0.5 mm. or smaller than 0.1 mm. The presence of a small proportion of clay, amounting to even a few per cent, would render them unsuitable as aquifers.

The velocities necessary to transport sands are notably less than those required to move gravels; moreover, they fall within the velocities ordinarily present in rivers and waves, whereas more erratic or storm velocities are required to move pebbles. Consequently, sands should show a greater average uniformity than gravels, both in texture and lateral distribution. It is believed that they do. Wind-blown sands commonly have over 50 per cent of their material concentrated in one grade, and commonly over 90 per cent in two or three adjacent grades.[75] Water-laid sands may occasionally approach the same degree of uniformity, but wide variations in uniformity generally occur; and although the greater part of the material falls into three or four adjacent grades, the maximum variations in grain-size are not infrequently as great as in gravels.

TABLE X

MECHANICAL COMPOSITION OF PERMEABLE SANDS

Material	Analyses Number	Uniformity Coefficient	Effective Size (Mm.)
Potsdam sandstone (Wisconsin)	1	1.6	0.16
	1	1.9	0.17
Jordan sandstone (Ohio)	1	2.5	0.21
St. Peter sandstone (Missouri)	17	1.4–2.6	0.12–0.24
Average	.....	1.9	0.17

The lateral distribution of sands is ordinarily much greater than that of gravels, and the grain-size over large areas may be remarkably uniform. The moderate velocities necessary to transport and sort sands are much more likely to exist uniformly over large areas than the higher velocities necessary for the sorting and transporta-

[75] J. A. Udden, *The Mechanical Composition of Wind Deposits*, "Augustana Library Publications," No. 1 (1898), p. 22.

tion of gravel. However, there is equal opportunity for variations in these velocities; hence, sands may occasionally show as sudden and as great textural variations as gravels. Altogether, sands are much better stratified and more uniformly sized, both locally and regionally. Consequently, the general permeability of any given layer of sand is more likely to be uniform throughout and to be comparable with that of a similar area in some other locality than is true of gravel. Furthermore, especially in the finer-grained sands, local variations are less apt to have an important bearing on permeability, because, if the pores are already too small for easy transfer of fluids, small changes in their size or shape are not likely to be sufficient to render them significant channelways.

Clay

Clay is characterized by very small particles and hence very minute pores, together with an enormous interstitial surface area to which water and other fluids are firmly held by adhesion. The aggregate surface area of 1 cubic foot of fine clay is more than 200,000 square feet, as compared with 1,700 square feet in 1 cubic foot of sand 1 mm. in diameter. Clays, of course, are relatively impermeable.

Loess

Loess commonly occurs as a fine-grained, homogeneous deposit of particles somewhat resembling clay, but with the grain-size of silt. Most of its particles range between 0.05 and 0.005 mm. in diameter. The 10 per cent size ("effective size") of the Mississippi Valley loess is approximately 0.005 mm.[76] Because of its small grain-size, loess has a low permeability and yields water to wells but very slowly.

Till

Unmodified glacial till is commonly composed of an unsorted mixture of materials characterized by wide variations in grain-size. Because of this wide variation, the combination has a lower porosity than any component size would have if occurring alone; and the permeability of the mixture is always low. However, the texture and grain-size of glacial deposits which are dependent on the nature of the material supplied to the glacier and on the manner of deposition

[76] Meinzer, *op. cit.*, p. 123.

may be notably discontinuous horizontally. Abrupt changes exist from an impermeable heterogeneous mixture of till to a coarse-grained water-laid glacial sand or gravel of relatively uniform texture and high permeability. The varied composition and abrupt textural changes of glacial material render predictions of permeability highly unreliable, unless they are based on a careful and detailed study of the particular locality.

Unassorted Alluvium

Unassorted alluvium resembles glacial till, but is much more common in the geologic column and far more widespread geographically. Erratic conditions of stream flow which have rendered impossible the segregation of gravel, sand, and clay result in deposits which vary in composition and texture but have the common characteristic of poor sizing. Such deposits are most likely to form on intermontane plains, especially of arid regions, where stream flow is subject to sudden and wide variations and where erosional processes are active and vigorous. During periods of regular flow, well-sized channel deposits of sand or gravel may be formed; but in flood times, great volumes of material are rapidly moved and deposited in utter chaos, not only in the stream channels, but also over adjacent areas.

The permeability of such a mass is generally low; but locally, along lenses of sand and gravel, it may be very high, and especially so if these bands are connected horizontally and vertically. Where they are not so connected, the mass as a whole will have a moderate permeability.

Every possible gradation exists between the extremely chaotic type of deposit just described and the well-sorted and uniform type formed under ordinary conditions by fluviatile or marine processes. Alluvium deposits resemble till, and it is only by the most careful and painstaking studies that accurate estimates of the permeability may be made.

EFFECT OF CONSOLIDATION

GENERAL STATEMENT

Particular attention has just been given to the factors which affect and control the permeability of unconsolidated clastic sediments. Nearly all sediments not still submerged in water bodies have undergone some degree of post-depositional consolidation; and

practically all of those dealt with in connection with ore deposits have been notably consolidated. Accordingly, before a direct application of studies on the permeability of unconsolidated sediments can be made to problems involving consolidated sediments, it is necessary to ascertain how, and to what extent, the consolidation of a sediment will affect its original permeability. Unfortunately, little useful information exists on this amazingly difficult problem.

Sediments, subsequent to deposition, undergo changes both varied and complex, which depend not only on the original character of the sediment but also on its environment since deposition. These changes may be the result of one or more of the general processes of pressure compaction, cementation, and recrystallization. To some extent these processes are interdependent and may be regarded as phases of the same process; but in a general way, and for purposes of discussion, they can be considered separately. Each process may vary greatly in the nature, duration, and extent of its activity. The number of times and the degree to which each process affects sediments depend on the nature, origin, age, and history of the sediment.

COMPACTION

Compaction takes place to an important degree only in fine-grained sediments, wherein it is accomplished by partial destruction of the cohesion between grains, with consequent rearrangement and closer packing. Where bridging is not present, no important compaction can take place until the pressures exceed the crushing strength of the minerals. Accordingly, compaction is of greatest importance in clay, silt, and loess; of less importance in sand; and of least importance in gravel. In coarser sediments, the effect of compaction will, in large part, depend on the grain-size and proportion of the matrix material. The general importance of compaction in lowering permeability is small, because compaction is best developed in fine-grained sediments which have, in any case, an inherent low original permeability.

CEMENTATION

The three general processes whereby cementation may be developed in a clastic sediment are as follows:

1. Cementation by the deposition of cementing material during a period approximately contemporaneous with the deposition of the

detrital sedimentary grains. Recrystallization of this cement may occur later.

2. Cementation with material introduced by solutions which circulate through the sediment subsequent to its deposition and which derive their load from sources outside of the sediment.

3. Cementation developed by local and partial solution and reprecipitation of the sediment itself. No new or extra material is added during this process, which, in general, acts subsequent to the deposition of the sediment and probably at a considerably later date.

These three processes are not of equal importance among themselves, nor do they equally affect different types of sediments. They differ greatly, in the time during which they are effective, the total amount of cementation possible, and the manner in which the cement is distributed. The author is of the opinion that the major amount of cementation is the result either of cementation essentially contemporaneous with deposition or of cementation by local solution and reprecipitation, and that cementation by circulating solutions is of lesser importance in most places. This statement, of course, applies only to sediments which have not been subjected to hydrothermal activity.

Predictions of the influence of cementation on permeability are difficult. Extensive cementation will greatly decrease, if not destroy, the original permeability. Moderate cementation may even increase the original permeability by holding in place the finer constituents and thus prevent their blocking the pores when the fluid moves. To some extent the effect of cementation depends on the process by which it is accomplished. Both by circulating solutions and by local solution and reprecipitation, cementation tends to fill the small pores first, and consequently, by these two processes, may proceed to a considerable degree before producing a marked lowering of the permeability.

If the deposition of the cement is assumed to be contemporaneous with that of the sediment, then the distribution and uniformity of cementation should, in a general way, correspond with the uniformity and distribution of the sediment. The uniformity of cementation should in general be greater horizontally in any given bed, than vertically across beds; hence abrupt changes in permeability from one

bed to another might be expected. The degree of cementation would depend on the process of deposition of the grains and on the chemical conditions during deposition. As the distribution of cementation of this kind is largely independent of the original permeability of the sediment, textural characteristics of the usual sorts cannot be safely used as guides to the distribution of permeability in the cemented sediment. Local variations of permeability are unpredictable unless an adequate knowledge is possessed of the local conditions of cementation during deposition.

The distribution of cementation by circulating solutions is also difficult to predict. Because of the strictly limited pressures under which the usual circulating ground-water solutions move, the mechanical problem of passing large volumes of dilute solutions through the sediments, and of maintaining this flow until cementation is complete, would appear so great that it seems highly improbable that widespread, but uniform and thorough, cementation can be accomplished by circulating ground waters. Besides the difficulty of limited pressure is the chance that many of the voids will become filled with a gas that will dislodge only with difficulty. Still more serious is the probability that cementation by this process would choke itself off by filling the entrances or throats to the pores long before the pores themselves could be completely filled. It seems probable that the distribution of permeability in sediments cemented in this manner will be highly irregular and will not necessarily be in accordance with the distribution of the cementation.

The extent of cementation by local solution and reprecipitation would depend on the intensity of the factors causing these processes and on the nature and distribution of the minerals present. This cementation begins at grain contacts. It should fill the pores in the order of their increasing size. Such cementation should be moderately uniform over considerable areas, and, accordingly, so should the permeability.

Clay does not permit the ready circulation of cementing solutions because of its low original permeability. Any extensive cementation that occurs must take place either at the time of deposition or by recrystallization subsequent to deposition. In any case, the effect will be to lower the already low permeability.

The outstanding difference between sand and gravel is the greater grain-size and consequently larger interstices of gravel. The area of the pores between uniform spheres is proportional to the square of the diameter of the spheres. As between spherical sand grains 2 mm. in diameter and spherical pebbles 5 cm. in diameter, the voids of the latter would be 625 times as great in cross-sectional area and approximately 15,600 times as great in volume as those of the former, but would be much less numerous per unit volume of material. Consequently the problem of filling the pores in these two sediments is quite different.

If cementation is contemporaneous with deposition, the opportunities for either sand or gravel to become cemented are equal, except in so far as their respective methods of deposition may affect the cementation. Gravels are usually deposited as the result of brief but intense conditions, and consequently there is less opportunity for cementation than under the more protracted and quiescent conditions during which sands are accumulated. If cementation is by circulating solutions, it is obvious that solutions can circulate more readily through gravels; but the total surface exposed for attack, reaction, and deposition is much less than in sand. Furthermore, a longer period of deposition is needed to fill the interstices of gravel than to fill the smaller but more numerous voids of sand, provided the rate of forming a different thickness of deposit is the same in all pores. The interstices in gravel will still retain a large measure of their original permeability after the deposition in each of a thickness of cement sufficient to block completely the pores of sand.

Local solution and reprecipitation, which is really a type of recrystallization, produces somewhat the same results. Clays recrystallize readily, partly because of their mineralogical composition and partly because of their small grain-size, which renders them especially susceptible to local solution. Sands probably recrystallize more slowly than clays, and much more recrystallization is required to render them impermeable. The rate of recrystallization in gravels is probably slower than that in sands; moreover, because of their much larger pores, gravels require most recrystallization of all to render them impermeable. Consequently, gravel always retains a greater proportion of its original permeability than sand, up to the stage where both are completely cemented.

RECRYSTALLIZATION

Recrystallization may take place at any period during the existence of the sediment, but it is generally assumed that considerable time is required for thorough recrystallization. It seems probable that most recrystallization has developed by local solution and reprecipitation, except where true hydrothermal processes have been active.

As practically all well-recrystallized sediments are characterized by a very low porosity, recrystallization must have been accompanied by a very considerable diminution in volume, unless we assume (*a*) that the pores were filled previous to recrystallization; (*b*) that, prior to the beginning of recrystallization, the sediment had been compressed until all its pores were closed; or (*c*) that the products of recrystallization are of lighter density, and hence of greater volume, than the materials from which they were derived. Usually there is no way of determining to what degree the first assumption might be true; the second assumption is physically impossible; the third assumption is probably very rarely realized, since recrystallization is most favored at depth, where pressure would tend to make denser rather than lighter new minerals.

Different minerals differ greatly in the ease with which they undergo recrystallization. The soluble minerals recrystallize most readily; and many of the more soluble sediments, such as limestone, gypsum, and salt, show notable recrystallization apparently developed under moderate pressure. Calcite apparently recrystallizes the most readily of the minerals commonly present in detrital sediments; but, probably because of its greater solubility, it usually appears to have been deposited later; or, what is more probable, as it is the least stable mineral present, it is recrystallized many times and its form changes with each change of equilibrium so that in its final state it represents the last stage of equilibrium that has existed. Original argillaceous material is readily recrystallized to mica. Feldspar recrystallizes with more difficulty, so that many grains still preserve traces of their former character. Quartz, the most difficult of the ordinary sedimentary minerals to recrystallize, commonly forms the framework in and around which are arranged the other recrystallization products.

Recrystallization is the most intense of the processes which pro-

duce consolidation in sediments. Thoroughly recrystallized sediments are tight, compact, more or less uniform in grain-size, and possess a low porosity. Original zones of weakness are usually obliterated, and the rock has a low permeability. The porosity and permeability are no longer determined by original sedimentary characteristics but are essentially of a secondary nature, controlled by the mineralogical composition and by fracturing. Consequently, the criteria established for the permeability of unconsolidated sediments are no longer applicable.

The texture and permeability of recrystallized sediments originally rich in quartz are comparable to the corresponding properties of a granite. Many sediments, however, contain impurities which on recrystallization yield either micaceous minerals or carbonates. These impurities are often in bands which commonly persist through the recrystallization and which may, because of their crystallographic properties, have a permeability greater than that of the remainder of the rock.

Sediments of different composition behave differently under differential stress. Quartzites and quartz-rich metamorphic rocks are readily fractured, shattered, and granulated. Rocks rich in micaceous minerals yield to stresses by shearing, but the shears are tight and impermeable. Under these conditions, derivatives of the sand type of sediment have a greater permeability than those derived from the clay type. When, however, the sediments have been subjected to more moderate stresses, so that they have not been fractured, the permeability conditions may be somewhat different. Quartzites, unshattered, are very impermeable. Slates which have not been shattered may possess a very considerable potential permeability due to the channels existing along the cleavages of the micaceous minerals. This difference is emphasized when these two types are subjected to solutions under high pressure, such as hydrothermal liquids. Quartz when strained remains impermeable; while mica, under the same pressure, apparently cleaves readily and allows the passage of solutions. Under these limited conditions, derivatives of the clay sediments may have a greater permeability than those derived from sandy sediments. The original composition of a sediment may thus affect the permeability of its recrystallization product to

whatever extent it has enabled the development of layers of well-oriented mica or similar minerals which may later serve as channelways.

MODIFYING CONCEPTS OF PERMEABILITY

What may be termed the "ideal concept of permeability" is given expression in Darcy's law, according to which all channels, above the size in which flow is prevented by molecular attraction with the walls, will carry flow proportional to the capacity of the channels and the rate of flow will be determined by the size and nature of the channels and the pressures on the fluid; and according to which also, the greatest volume of fluid will always flow along the channel in which there is the least total resistance between source and point of exit, irrespective of the direction in which the channel leads, or of its total length. Certain departures from this ideal concept are, however, both possible and necessary. Such departures may modify, but in no sense do they invalidate the simplified or ideal concept.

Permeability may be regarded from a certain standpoint (which must not be carried too far) as either the capacity to permit flow or the utilization of that capacity. Permeability may therefore be thought of as inherent or potential permeability on the one hand, and as actual or effective permeability on the other hand. This distinction may perhaps be clarified by a subdivision into local permeability and through-going permeability, in which local permeability represents the inherent or potential permeability of any given mass of material under consideration regardless of the permeability conditions of its surroundings, while through-going permeability relates to the potential permeability of the entire system, taken as a whole, from source to outlet. The through-going permeability is thus some appropriate integration of the local, inherent permeability of each of its parts. And the effective or utilized permeability of any part of a system cannot be greater than is permitted by the through-going permeability for the system as a whole.[77] In an extensive and complex permeability system, the whole matter of effective or utilized permeability is, in the last analysis, dependent on the manner in which pressure of the fluid is affected from point to point throughout the entire system.

[77] Cf. Graton, *op. cit.*, pp. 50–53.

The fact that permeability is relative to pressure must, indeed, always be kept in mind. All rocks are permeable if the pressure on the solution is sufficiently great. The choice by a solution of the path which it will follow through a given zone of rocks is limited to the rocks with which it comes in contact. Solutions cannot take advantage of a nearby, more permeable channel unless they reach that channel while sufficient pressure still remains to cause flow. In other words, potential permeability of a given mass is of no avail as regards continuous flow unless that mass becomes part of a through-going system. Any "coefficient of permeability," therefore, is applicable only for a condition of through-going flow.

For a somewhat different, though fundamentally related, reason the inherent permeability of a rock and its actual resistance to the penetration of a given fluid may be quite different. For example, if the pores of a rock are filled with a more viscous liquid which the pressure brought by the fluid seeking entrance cannot dislodge or can drive ahead only slowly, then, although that rock may be of relatively high inherent permeability, penetration by the less viscous liquid will be denied or proportionately reduced.

A somewhat similar state of affairs exists where a gas is liberated or generated (either by evaporation from the entering liquid or by some local reaction occasioned by or in the liquid) at such rate and in such volume that it cannot escape through the permeability system rapidly enough to prevent developing a "back pressure," which will retard penetration and flow of the liquid and may suffice to arrest or even reverse the flow.[78]

Resistance to advance of a liquid because of a special manifestation of the "Jamin effect" in the (capillary) voids lying ahead (see "Systematic Packing of Spheres," p. 908) is another manifestation of the possible disparity between inherent and realized permeability. But the case in which a liquid can no longer advance because the

[78] Professor Graton points out that this is, in effect, the principle of the Kipp generator so commonly employed in chemistry. He also suggests that an analogous explanation may apply to that strikingly abrupt boundary often found between intensely contact-metamorphosed material and the unaltered limestone, the supposed mechanism being that the back pressure of the CO_2 gas generated by the metamorphic reaction controls the advance of the active liquid reagent and may hold its front for a long time relatively static at the position of what is really a liquid-gas interface.

voids of the region lying ahead of it had already been occupied by an independent gas (or by a liquid of lower viscosity) which the liquid cannot dislodge or can no further compress is only a special case of failure of a through-going permeability system.

PENETRATION BY ORE-BEARING SOLUTIONS

THE GENERAL MECHANISM

According to the generally accepted modern theory, most ore-forming solutions possess a low viscosity and are injected under high pressure. These solutions originate from the magma and travel toward the point of outlet along all available channelways. Where fractures or fissures, either open or openable,[79] are available, they furnish an easy means of ascent. Where the definite fractures close or die out, the solutions are forced either to travel through the rock itself or to cease flowing. As ore-forming solutions travel from a region of high pressure toward one of low pressure, they form a hydraulic system and may be expected to behave in accordance with the established laws of hydraulics or hydrodynamics. However, the laws of hydraulics can be applied only when a condition of free flow has been established, and this condition exists only when solutions fill the channel from their point of entrance to their point of outlet.

From the standpoint here pertinent, the period of mineralization may be regarded as consisting of two important phases: the first exists during the time that the front of the solution is penetrating out from its source and working its way to the point or points of discharge; the second phase begins when the solution has thus established a condition of continuous (or potentially continuous) flow between source and outlet. The first phase is largely concerned with the distribution of the mineralizing solutions; the second phase deals with the volumes of solution transmitted along the different channels and with changes in the flow as the channels become filled by deposition.

[79] No attempt is made in this paper to treat that special case of porosity and permeability in which a fracture is widened by the pressure exerted on its walls by the solution, nor that somewhat related case in which enormous (magmatic) pressures transmitted by the solution may rend solid rock and thus produce wholly new openings and a new permeability system.

The mechanical composition of a sediment, as exhibited by its texture, affects both phases but in somewhat different ways. When the solutions move through the rock itself rather than through fractures, the ease of initial penetration by the liquid is dependent on the texture of the material being encountered at the penetration front and on the permeability conditions from source to this front, but may be said to be independent of permeability conditions that may lie farther ahead. When, however, flow has been established throughout the system, by completion of the initial penetration, the quantity of solution transmitted is affected by the permeability conditions *throughout the entire system*, i.e., from source to final outlet.

When ore-forming solutions are liberated at their source, they are driven outward with equal pressures in all directions; the major volume of solution travels in the direction and along the channels offering the least resistance. Channels with which the solution has not established contact can have no influence on the direction of advance. Consequently, the first major penetration will not necessarily be toward the ultimate direction of relief of pressure except in so far as easier channels are offered in that direction. The distance to which solutions penetrate in any given direction is inversely proportional to the resistance which they encounter; therefore the driving pressure is equal at all points along the penetration front of the solution. Because the direction of maximum relief is usually upward, and because all solutions penetrating downward must encounter greater and greater resistance to their flow, the maximum flow is eventually concentrated in a vertical direction, although the solution may travel long distances horizontally in order to seek the point of relief.

The initial penetration of an ore-forming solution may be pictured as follows: For simplicity, consider the solution as advancing from the source in a lateral or vertical direction. As no single continuous fissure may exist to span the entire distance between the source of an ore-forming solution and the surface, assume that the solution must reach the surface, if at all, by utilizing a succession of discontinuous fissures. Suppose that the advancing solution encounters an open fissure or other zone of low resistance. Flow is locally concentrated, and the solution rapidly moves along the fissure. After the solution

has penetrated the length of the fissure, it comes in contact with a more resistant area. The advance slackens, perhaps almost to a standstill. Conditions in the solution change from hydraulic or hydrodynamic to nearly or quite hydrostatic, and equal pressure is restored in all directions. The solution therefore penetrates outward in all directions, the rate of advance in any given direction being inversely proportional to the resistance. If, in the course of this outward groping, another fissure is encountered, the reduced resistance in that direction causes flow to be locally concentrated in the new fissure. This will continue until increased resistance is again encountered, at which time conditions again become quasi-hydrostatic and penetration is again resumed in all directions. This cycle may be repeated indefinitely so long as the available supply of solution and the driving pressures hold out.

In the last analysis, those routes will afford least resistance which eventually link up and lead to the master outlet, the surface. Flow will be concentrated along such a composite pathway and will continue until the pressure at the source becomes insufficient to overcome the entire hydrostatic head and the total resistance encountered along the complete channelway, at which time the flow will, of necessity, cease. Nevertheless, during the entire time that flow is occurring in this through-going channel, pressure will still be forcing the solution out into the rock in all directions, not only in the region immediately around the source, but to some extent outward from the walls of the channelway also. If, in so doing, a second channelway is discovered and established, the relative volume of flow in the two channels will be inversely proportional to the total resistance of each along its course.

The pressure tending to drive the solutions out into the walls of the channel at any given instant, at any point along the channel, will be proportional to the pressure at the source minus the loss of pressure incurred by the solution in its passage to that point. If flow should cease because of a stoppage in any channel, the pressure at all points along that channel below the point of stoppage will become equal to the pressure at the source, minus the hydrostatic head.

The extent to which solutions penetrate the walls is dependent on the ease of penetration (permeability) of the walls, the pressure on

the solution at the source, the resistance or ease of flow along the channel, and the time during which the solutions continue to flow. If a fissure were originally closed and the solutions had to force it open, they would penetrate farther into the walls than if they had found little original resistance to flow. When open fissures are available, flow will be concentrated along them. No important flow will take place through normal, unfractured rock until or unless permeable fissures are not available, either because they do not exist or because they have been in some manner shut off.

EFFECT OF SEDIMENTARY TEXTURES

The original permeability of a sediment, as determined by the original texture but modified by the consolidation and metamorphism which the rock has undergone, may be greater than the secondary permeability produced by fracturing, alteration, or decomposition. In any given rock, the two permeabilities are not usually of equal importance because, when the original permeability is great, the rock has not undergone sufficient consolidation to permit it to fracture readily and to preserve open, continuous fractures. The original permeability of sediments cannot be considered in any way as constant; while the gross permeability of the entire unit may be approximately constant, the permeability of local units may, and generally do, vary both widely and abruptly. Some simple examples of the effect of these textural variations are discussed below.

Where two beds of different permeability, but of the same cross section and length, are fed from a common source, both beds will transmit the solution in proportion to their respective coefficients of permeability, provided a condition of through-going flow is established in each bed. If the coefficient of permeability of one bed is ten times as great as that of the other, it will transmit ten times the volume of solution, other things being equal.

The effect of more permeable lenses in an otherwise homogeneous sedimentary bed is as follows. The solution will advance equally along a front at right angles to the plane of the bed until it comes in contact with the first lens. Through this it will advance much more rapidly. When the end of this lens is reached, the solution will further advance through the main bed, radiating out from the lens, not

only from its end but also from its sides. Eventually, connections will be made with the next lens, and the foregoing process repeated. Finally, a condition of continuous flow will be established through the bed to some adequate outlet. The main volume of solution will then pass along the path of least total resistance, or maximum permeability, which will be along the lenses wherever possible. Although flow will be concentrated through the lenses, there will also be subordinate flow throughout the surrounding portions of the main bed for a distance determined by the frictional resistance and for so long as the interstices remain open.

If a highly permeable bed, such as a gravel or conglomerate, lenses out up dip, solutions ascending along the bed will come to its end and will then spread out in all directions seeking another favorable channel. If a second channel, permeable, but of lesser permeability than the first, is located along the same or a different stratigraphic horizon, but in any event closer to the surface, the solutions will follow the first conglomerate as far as possible before branching to the path afforded by the second. If, however, the upper channel is more permeable than the first, then the main volume of solution will largely desert the first conglomerate at the place that affords easiest travel to the second channel.

CRITICAL PERMEABILITY

"Original permeability" has been defined as the permeability possessed by a clastic sediment before it has undergone consolidation or metamorphism. The volume of an ore-forming solution that passes through any given mass of rock is influenced by the permeability of that mass. The permeability thus existing may be called the "effective initial permeability"[80] of that mass with respect to the ore solution. The distribution of minerals deposited from the solution solely by open-space filling would be governed by the distribution of the effective initial permeability if the solution were uniform in composition and if the factors causing such deposition of each of the minerals were identical and operated uniformly throughout the rock. However, in many ore deposits the minerals have not been deposited simultaneously; rather, they have been precipitated in a definite

[80] Graton, *op. cit.*, p. 61.

order so that we speak of certain minerals as "early" and others as "late." The earliest-formed minerals are commonly of little or no economic importance. They are likely to be deposited in a manner of distribution that closely conforms with the distribution of the initial permeability. But there is no reason to expect that the distribution of the late minerals (generally including those of value) will be equally controlled by the effective initial permeability, which certainly has been altered and may have been, at local places, completely destroyed by the deposition of early minerals. Accordingly, the distribution of any particular mineral will be controlled by the distribution of the permeability at the time of that mineral's precipitation. This permeability may be termed the "critical permeability"[81] for that mineral. The distribution of the critical permeability may vary for each mineral that is deposited. Although the distribution of the critical permeability may differ greatly from that of the initial permeability, it is most likely to be in general accordance with the distribution of the areas of *greatest* initial permeability.

It follows that, in examination of the manner in which the mechanical composition and texture of a sediment have controlled the distribution of ore minerals, it is necessary to consider, not only the initial permeability and the way in which the channelways were distributed throughout the rock, but how this initial permeability was progressively modified by the deposition of worthless material preceding the precipitation of the valuable minerals. This information can be obtained only by detailed local study.

[81] Graton, *ibid.*, p. 62.

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SOME CARBONIFEROUS STRATIGRAPHY IN MONTANA AND NORTHWESTERN WYOMING[1]

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Montana School of Mines, Butte, Montana

ABSTRACT

The name "Quadrant" has been applied to rock strata of Mississippian, Pennsylvanian, and Permian age. However, the true Quadrant is basal Pennsylvanian. Other rocks of central Montana commonly referred to this formation constitute the Big Snowy group and are Mississippian in age. The type section of the Quadrant shows rocks of the Amsden formation occurring above the Madison limestone and below the Quadrant quartzite. The stratigraphic relations, character, and paleontology of the beds commonly called "Quadrant formation" are discussed.

INTRODUCTION

In various parts of the western half of Montana there occurs a series of strata above the Madison limestone and below rocks of Mesozoic age which are commonly referred to as the "Quadrant formation." In central Montana, in the vicinity of the Big Snowy and Little Belt mountains, these strata consist of 1,500 feet of fossiliferous shales, sandstones, and limestones; in south-central Montana they consist of 400 feet of quartzite overlying about 160 feet of limestone and red sandy shale, and in southwestern Montana as much as 1,000 feet of quartzite intercalated with limestone.

The "Quadrant formation" has been a name without exact meaning and has been applied to rocks ranging in age from the Middle

[1] Published by permission of the Director of the Montana Bureau of Mines and Geology.

TABLE I

EVOLUTION OF NOMENCLATURE OF THE QUADRANT FORMATION

	Iddings and Weed 1883–96	Peale 1893	Weed 1894	Darton 1904	Branson and Greger 1918	Bevan 1921	Knappen and Moulton 1930	Weed 1891–99	Hammer and Lloyd 1926	Reeves 1930
	Northwest Wyoming	Southern Montana	Southern Montana	North-Central Wyoming	Central Wyoming	West Montana	North-Central Wyoming and South-Central Montana	Central Montana		
Permian								Jurassic	Jurassic	Jurassic
Pennsylvanian	Quadrant	Quadrant	Quadrant	Tensleep	Tensleep	Quadrant	Tensleep		Quadrant	
				Amsden						
Mississippian	Madison	Madison	Madison	Madison	Amsden Madison	Madison	Amsden Madison	Quadrant: Otter Quadrant: Kibbey Madison	Madison	Quadrant ? Madison

Mississippian to the Permian. No one has known its true limits and, consequently, considerable uncertainty has existed in regard to the stratigraphy of the Upper Paleozoic in Montana, northwestern Wyoming, and certain parts of Idaho. Lack of knowledge of the meaning of the term "Quadrant" has led many geologists to consider the rocks of this formation as extremely variable in nature; yet certain thin zones of limestone and shale are persistent under many hundred square miles in central Montana. The confusion has been due to an inadequate definition of the rocks in the type section, incomplete fossil collections, and to the fact that no one has traced the Quadrant quartzite north in order to observe its relation to other beds.

The purpose of this paper is to define the Quadrant formation, to establish its stratigraphic relationship with other Carboniferous rock strata in Montana and Wyoming, and to discuss briefly certain Mississippian formations of central Montana.

QUADRANT FORMATION

The Quadrant formation was named by Iddings and Weed[2] while working out the geology of the Gallatin Range (1883–93), from its exposure on the southeast corner of Quadrant Mountain in the northwest part of Yellowstone National Park. Peale[3] used the name, after a field conference with Iddings and Weed, prior to the publication of the type section in Yellowstone Park, for rock strata in southern Montana which he thought occupied the same stratigraphic position as the quartzites on Quadrant Mountain; however, rocks of Amsden age were included in the formation, though they were not a part of the Quadrant in the type section.

In 1904 Darton[4] gave the names Amsden and Tensleep to a group of strata occurring between the Madison and the Chugwater in the Big Horn Mountains. His Tensleep occupied the same stratigraphic position as the true Quadrant at Quadrant Mountain, and the Ams-

[2] I. P. Iddings and W. H. Weed, "Descriptive Geology of the Gallatin Mountains," *U.S. Geol. Surv. Mono. 32*, Part 2 (1899), pp. 1–59.

[3] A. C. Peale, "The Paleozoic Section in the Vicinity of Three Forks, Montana," *U.S. Geol. Surv. Bull. 110* (1893), pp. 32–43.

[4] N. H. Darton, "Comparison of the Stratigraphy of the Black Hills, Big Horn Mountains, and Rocky Mountain Front Range," *Geol. Soc. Amer. Bull. 15* (1904), pp. 396–97.

den occupied the zone called "talus . . . . 100 feet" (zone 13 in Table II).

While working out the geology of the Little Belt Mountains in 1891, Weed[5] classified a group of shales intercalated with thin lime-

TABLE II

TYPE SECTION MEASURED BY IDDINGS AND WEED*

	Feet
Teton (Phosphoria)	
Quadrant:	
1. Limestone, dense, white, weathering same color	10
2. Sandstone and limestone in alternating bands	15
3. Limestone, light gray and dense and pure	12
4. Quartzite, pure white, thin bedded, forms a persistent band along face of mountain	6
5. Quartzite, more thickly bedded, white	65
6. Limestone, very light gray, somewhat dense, containing very small fragments of light-colored chert	10
7. Quartzite, white, calcareous, with intercalated bands of limestone carrying quartzite fragments	30
8. Limestone, light gray, with angular fragments of sandstone	5
9. Quartzite, white, weathering pink and rust color but appearing black when seen from a distance, owing to the growth of lichens upon it	130
10. Limestone, light gray, dense	2
11. Sandstone, well banded, white, saccharoidal	6
12. Sandstone, very calcareous, saccharoidal, white and rust color	10
13. Talus slope of sandstone blocks; also of cherty limestone	100

* Op. cit.

stones, occurring along the flanks of the mountains, as the Otter formation, but called them Quadrant in age because of their stratigraphic position above the Madison; however, he considered the beds as Mississippian. Since then all strata in central Montana occurring above the Madison and below the Ellis (Upper Jurassic) have been called Quadrant. In 1899[6] Weed called the red sands and gypsum layers below the Otter the Kibbey formation.

[5] W. H. Weed, "Two Montana Coal Fields," *Geol. Soc. Amer. Bull. 3* (1891), pp. 308–9.

[6] W. H. Weed, "Geology of the Little Belt Mountains, Montana," *U.S. Geol. Surv. 20th Ann. Rept.* (1899), p. 295.

In 1926 Hammer and Lloyd,[7] in a study of the "Quadrant" formation of central Montana, recognized the Mississippian-Pennsylvanian age of this formation. They believed that the red zone (Kibbey) immediately above the Madison was equivalent to the red zone of the Amsden and that certain sandstones near the top of the group were to be correlated with the Tensleep.

Reeves,[8] in 1930, tentatively classified the rocks above the Madison and below the Ellis (Upper Jurassic) in the Big Snowy Mountains as Quadrant, but considered all of them Mississippian in age. He suggested that the limestone beds at the top of the series were probably Chester. Stratigraphic evidence will be presented in this paper which verifies Reeves' suggestion that the upper limestone is Amsden—which, of course, indicates that the Quadrant is not present in central Montana.

Definition.—Under the heading "The Quadrant Quartzite,"[9] the Quadrant formation was originally described as consisting of "white, yellowish, and occasionally pink beds of quartzite, with intercalated beds of drab saccharoidal limestone." The type section at Quadrant Mountain consists of those rocks that lie between the top of the Amsden and the bottom of the Phosphoria formation.

A redescription of the type section is given in Table III.

Distribution and thickness.—The Quadrant quartzite was traced northward from Yellowstone Park to a point 5 miles northeast of Lombard, Montana, where it completely wedged out. The hills in the vicinity of Lombard and Toston, where the Missouri River cuts across the Lombard overthrust, may be considered physiographic dependencies of the southern end of the Belt Mountains. The Quadrant formation is not found north or northeast of these hills and is unknown in central Montana, but it does occur to the northwest of Lombard in the vicinity of Helena. It occurs, also, throughout the extreme southwestern and west-central part of Montana. Good exposures are known in the vicinity of Philipsburg, Melrose, and south and southeast of Dillon.

[7] A. A. Hammer and A. M. Lloyd, "Notes on the Quadrant Formation of East-central Montana," *Amer. Assoc. Petrol. Geol. Bull. 10*, No. 10 (1926), pp. 986–96.

[8] Frank Reeves, "Geology of the Big Snowy Mountains," *U.S. Geol. Surv. Prof. Paper 165* (1931), pp. 135–49.

[9] Iddings and Weed, *op. cit.*

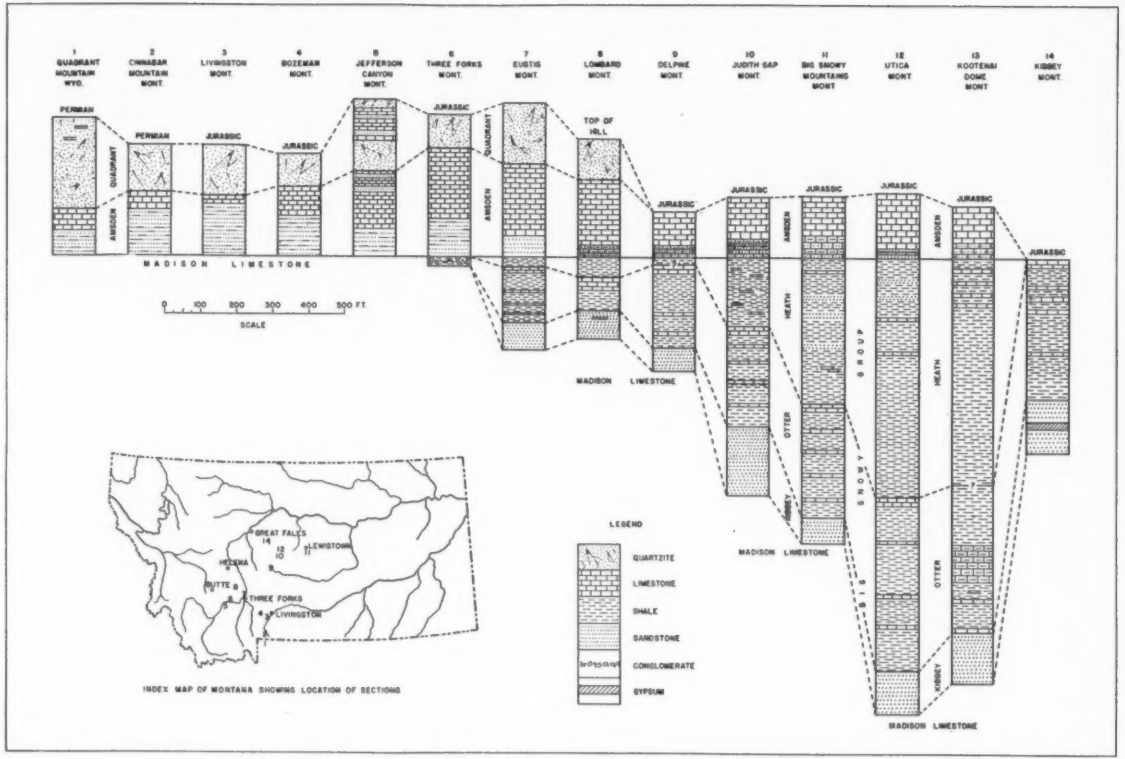

FIG. 1.—Generalized columnar sections showing stratigraphic relationship of the beds between Madison limestone and Jurassic and Permian strata in central and south-central Montana and Yellowstone Park. Formerly all rock strata shown in these sections were called Quadrant.

In the type section the Quadrant is about 230 feet thick. Twenty miles north, at Cinnabar Mountain, it is 125 feet; at Livingston 140

TABLE III

SECTION MEASURED ON SOUTHEAST CORNER OF QUADRANT MOUNTAIN, JULY, 1934

	Feet
Phosphoria formation:	
Dark, coarse-grained, phosphatic rocks with elongate siliceous tubes perpendicular to bedding in basal portion	
Quadrant formation:	
21. Gray brecciated limestone	7
20. Quartzitic sandstone	2
19. Light-gray, cherty, siliceous limestone with sandstone lenses, brachiopod fragments and foraminifera	14
18. Massive, saccharoidal, tan-colored sandstone	15
17. Brown to white quartzite	2
16. Cherty, compact limestone	6
15. Cross-bedded, ripple-marked quartzite, black on surface	11
14. Calcareous quartzite	8
13. Cross-bedded, coarse-grained, tan-colored sandstone; erosional surface at base	6
12. Light-gray, cherty, and fossiliferous limestone	8
11. Hard sandstone, like quartzite	29
10. White to brown quartzite	10
9. Gray limestone	2
8. Cross-bedded gray quartzite, sandy at base, calcareous at top	116
Amsden formation:	
7. Dolomitic limestone	10
6. Hard sandstone	3
5. Gray limestone	5
4. Calcareous sandstone	4
3. Light-gray limestone with a two-inch shale parting near base	21
2. Yellow to brown shale	5
1. Red to purple sandy shale, tends to be covered but well exposed on south side of mountain	61
Madison limestone	

feet; near Bozeman 88 feet; west of Eustis 160 feet; at Lombard about 100 feet; and west of Toston less than 75 feet thick. In the

Snow Crest Range the Quadrant is reported[10] to be as much as 1,000 feet thick.

Character.—All sections of the Quadrant in south-central Montana are very uniform in lithological characteristics. The formation consists primarily of well-bedded, white to pink, fine- to medium-grained quartzite which is occasionally more sandy than quartzitic. Some sections, such as the one at Quadrant Mountain, contain thin beds of limestone in the upper part; however, the limestone is always siliceous and in some cases is classed as a calcareous sandstone. Other sections, such as those at Cinnabar Mountain and Livingston, are composed entirely of quartzite. The calcareous zones become more and more important west of Quadrant Mountain toward the Idaho-Montana boundary line, and less important eastward into the Big Horn Mountains, where the sandstone beds are called Tensleep. In the latter locality the formation is a "buff to cream-colored sandstone, at some places slightly calcareous and argillaceous." The east-west changes in lithology are unquestionably due to the environmental conditions of sedimentation. Knappen and Moulton[11] believe that the uniform size of sand grains and the steeply inclined cross beds of the Tensleep in the Big Horn Mountains indicate an eolian origin for at least the lower part of the formation; whereas the upper calcareous part represents marine deposited or re-worked material.

The sandstone of the Tensleep grades westward into Quadrant quartzite. Similarly, upper calcareous zones in the Tensleep grade into siliceous limestone and finally into nearly pure limestone beds in the upper part of the Quadrant. Furthermore, the eolian characteristics in the Big Horn Mountains gradually give way to true marine characters, and at Quadrant Mountain at least 95 per cent of the formation represents marine deposited or re-worked material. That a marine environment existed at the time of deposition of the Quad-

[10] D. D. Condit, "Relations of the Late Paleozoic and Early Mesozoic Formations of Southwestern Montana and Adjacent Parts of Wyoming," *U.S. Geol. Surv. Prof. Paper 120*, p. 111.

[11] R. S. Knappen and G. F. Moulton, "Geology and Mineral Resources of Parts of Carbon, Big Horn, Yellowstone, and Stillwater Counties, Montana," *U.S. Geol. Surv. Bull. 822A* (1930).

rant rocks is clearly indicated by excellent bedding, marine cross bedding, siliceous limestone, and marine fossils.

Stratigraphic relations.—The Quadrant quartzite is unquestionably a westward extension of the Tensleep sandstone. Geologists working westward from the type locality of the Tensleep invariably classify the quartzite in south-central Montana as Tensleep; whereas those working northward and eastward from Quadrant Mountain identify the same zone in the same section as the Quadrant formation.

The Quadrant quartzite lies parallel to beds of the Phosphoria formation (Permian) in south-central and southwestern Montana and in northwestern Wyoming. Neither an angular nor erosional unconformity has been reported in these areas. However, Thomas[12] recognized an erosional unconformity between the Tensleep and Phosphoria in the Wind River Mountains of Wyoming, where the latter formation rests upon an irregular surface.

In most areas an erosional unconformity is present between the top of the Amsden and the base of the Quadrant-Tensleep; nevertheless, in the vicinity of Three Forks there is evidence which would support a theory of continuous deposition, but in no instance is there known evidence to indicate the existence of an angular unconformity. West and southwest of Three Forks, in the north end of the Tobacco Root Mountains, there is no sharp line of demarcation between the Amsden limestone and the overlying Quadrant quartzite. A few sections show the upper part of the Amsden limestone interbedded with quartzite, which becomes more and more dominant until massive quartzites of the Quadrant are reached.

The stratigraphic relationships of the Quadrant to the so-called "Quadrant" of central Montana are strikingly shown in a number of excellent sections in the Lombard Hills, north of Three Forks. Here the true Quadrant quartzite rests upon as much as 200 feet of Amsden limestone, which in turn conformably overlies a series of shales, limestones, and impure sandstones that overlie the Madison forma-

[12] H. D. Thomas, "Phosphoria and Dinwoody Tongues in Lower Chugwater of Central and Southeastern Wyoming," *Amer. Assoc. Petrol. Geol. Bull. 18*, No. 12 (1934), p. 1661.

tion. The last series of rock strata constitute the so-called "Quadrant" of central Montana. For the stratigraphic relations of these rock strata see Figure 1. It should be pointed out that all the strata shown in these sections have always been classified as Quadrant.

Age.—The determination of the age of the Quadrant formation is a paleontologic problem. Heretofore, fossils from the quartzite have not been reported. The writer recently found a number of microfossils and fragments of megascopic fossils in calcareous zones of the type section. Some of the microfossils, such as foraminifera, ostrocods, and gastropods, are composed of glauconite.

About 40 different varieties of siliceous sponge spicules, which were found in calcareous concretions in a bed of quartzite from the

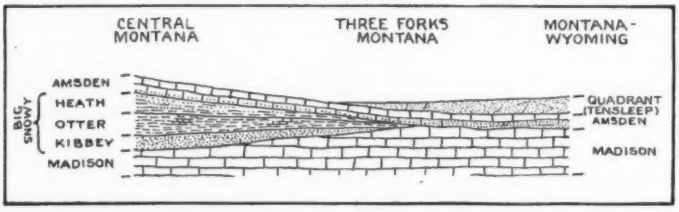

FIG. 2.—Diagrammatic north-south section showing the stratigraphic relations of the Quadrant to the Amsden formation and the Big Snowy group.

middle of the formation, were obtained from a section in Jefferson Canyon, in the north end of the Tobacco Root Mountains. These spicules are analogous to those described by Weller[13] from the Pennsylvanian of Illinois and Indiana.

A few Pennsylvanian fossils are known to occur in the Tensleep, and in consonance with present information the Quadrant is also considered to be Pennsylvanian in age. Its exact position within this period must await the completion of a study now in progress on the fauna. This fauna consists primarily of microfossils.

THE AMSDEN FORMATION

In order to clear up the Quadrant problem of central Montana it is necessary to discuss the Amsden formation. Darton[14] named the

[13] J. M. Weller, "Siliceous Sponge Spicules of Pennsylvanian Age from Illinois and Indiana," *Jour. Paleon.*, Vol. IV, No. 3 (1930), pp. 233–51.

[14] Darton, *op. cit.*, p. 396.

Amsden in 1904 from its exposures along a branch of Tongue River west of Dayton, Wyoming. In the type locality the Amsden consists of those beds that occur between the top of the Madison limestone and the base of the Tensleep formation. It was originally considered as Pennsylvanian in age, but Branson and Greger[15] have shown the Amsden of the Wind River Mountains to be Mississippian.

In south-central Montana and north-central Wyoming the Amsden formation consists of two zones, a lower red magnesium shale or sandstone and an upper limestone member. In these areas the lower member is predominantly red in color and averages about 100 feet in thickness. At Cinnabar Mountain, north of Gardiner, Montana, the lower member is 130 feet thick, but decreases in a northerly direction so that at Eustis, 5 miles north of Three Forks, it is only 25 feet thick and is composed entirely of coarse red sandstone. North and northeast of the Lombard Hills it is impossible to recognize the lower member of the Amsden as a distinct unit. Many of the sections show three distinct lithologic phases of this lower member: a lower phase consisting of dark-maroon, shaly sandstones which are mostly thinly bedded; a middle zone of purple to red calcareous and argillaceous sandstone; and an upper zone of yellow to brown sandy shale. The yellow and purple zones are commonly thin or may be entirely absent. Conglomerates, made of limestone pebbles of the underlying Madison, are not uncommon in the base of the Amsden, but are usually thin and lenticular.

In south-central Montana the limestone member of the Amsden formation is blue gray, thinly bedded, and ranges from 30 feet in thickness near the Montana-Wyoming boundary line to as much as 200 feet in the Lombard Hills north of Three Forks. In central Montana this member averages 160 feet. The increase in thickness of the limestone is in almost exact proportion to the decrease in thickness of the underlying sandy member. Furthermore, there is a lateral gradation in the lithology of the limestone. In conjunction with this lateral gradation in thickness, the limestone becomes arenaceous and pink in color in the basal part. Where the sandstone

[15] E. B. Branson and D. K. Greger, "Amsden Formation of the East Slope of the Wind River Mountains of Wyoming and Its Fauna," *Geol. Soc. Amer. Bull. 29* (1918), 309–26.

member disappears as a recognizable unit, the basal part of the limestone is interbedded with thin red shales, some of which are varvelike, with as many as 60 bands per inch.

In south-central Montana the Amsden rests upon an erosional surface developed on the Madison limestone. If the various units of the latter formation could be correlated, it is probable that the Amsden would be found to rest upon each of these units, indicating a more pronounced erosional surface than has been recognized. Angular relations of the Amsden and Madison formations have not been recognized in any locality except in the Big Horn Mountains, where Thom[16] reports a slight flexing of the Madison prior to the deposition of the overlying Amsden. The magnitude of the erosional unconformity in Wyoming and south-central Montana is made more apparent when one studies the area between Three Forks and Townsend. Here the Amsden is separated from the Madison by a series of fossiliferous sandstones, shales, and limestones that attain a thickness as great as 1,200 feet in central Montana.

In central Montana the Amsden underlies the Ellis formation (Upper Jurassic). Excellent exposures showing this contact can be seen west of Judith Gap in the Little Belt Mountains and in the canyon of the Musselshell River east of Delpine. In neither locality is there any apparent discordance of an angular or erosional nature. The existence of a major time break is made evident, however, by the marked change in the fauna.

A study of the fossils collected from the Amsden of central Montana is now in progress. The following forms listed have been identified:

Composita trinuclea
Composita subquadrata
Productus ovatus
Cleiothyridina sp?
Spiriferina aff. *S. spinosa*
Spirifer aff. *S. menardensis*
Productus aff. *P. inflatus*
Orthotetes sp?
Pustula sp?
Nucula sp?
Cytherella sp?

[16] W. T. Thom, Jr., personal communication to R. S. Knappen and G. F. Moulton, *op. cit.*, p. 12.

In addition to the foregoing list, Girty[17] has identified the following genera:

Camarotoechia	*Bucanopsis*
Sphenotus	*Naticopsis*
Schizodus	*Sphaerodoma*
Deltopecten	*Euomphalus*
Leptodesma	

This fauna appears to be more closely related to the Chester fauna than any other, although certain forms are unquestionably older than Chester and others would appear to be slightly younger. When this fauna is considered in relation to that of the underlying Big Snowy group, and when stratigraphic relationships are taken into consideration, the Amsden of central Montana must be middle or upper Chester in age.

THE BIG SNOWY GROUP

The group of strata that occurs between the Madison limestone and the Amsden formation of central Montana has been designated

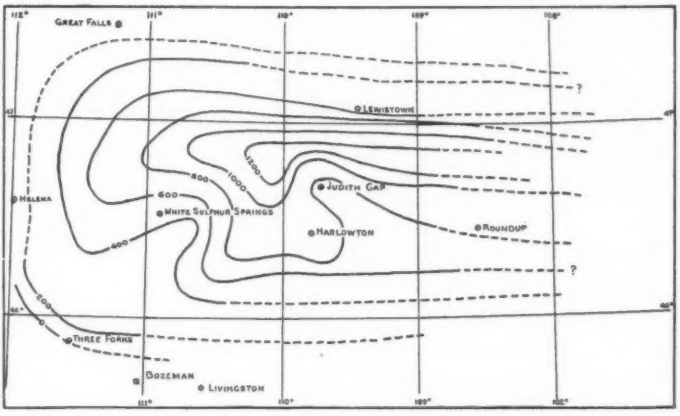

FIG. 3.—Isopach map of the Big Snowy group of central Montana

the Big Snowy group, because of its extensive distribution and excellent exposures in the Big Snowy Mountains. The group is also ex-

[17] Reeves, *op. cit.*

TABLE IV

SECTION MEASURED ON THE NORTH FLANK OF THE BIG SNOWY MOUNTAINS, IN SEC. 6, T. 12 N., R. 20 E.

	Feet
Top of bluff:	
Amsden formation:	
37. Limestone, ostracods and brachiopods common in various zones. *Composita subquadrata, Composita trinuclea, Productus ovatus*	100
36. Red shale	2
35. Fossiliferous limestone	29
34. Red shale	½
33. Limestone	½
32. Red shale	½
31. Limestone	2
30. Gray shale, 2-inch nodular limestone layer at top	3
29. Gray limestone	3
28. Red shale with nodular limestone at top	10
27. Limestone with maroon shale zones. *Productus ovatus*, foraminifera	24
26. Maroon, calcareous shale, fish scales	20
25. Black fossiliferous shale, fish scales, ostracods	¼
24. Gray limestone, ostracods abundant	4
Big Snowy group:	
Heath formation:	
23. Calcareous black shale, ostracods. *Chonetes, Productus, Composita, Pustula, Trepospira*	5
22. Black, impure limestone; clay and limestone of about equal proportions	1
21. Calcareous shale, black	4
20. Calcareous shale	1
19. Black petroliferous shale, fish scales rare	29
18. Brown sandstone	12
17. Black carbonaceous shale	29
16. Cross bedded sandstone with plant fragments	16
15. Black shale	68
14. Black shale, partly covered	43
13. Massive sandstone	27
12. Covered	54
11. Conglomeratic sandstone	7
10. Dark shale, partly covered	10
9. Gray limestone	12

TABLE IV—*Continued*

	Feet
8. Carbonaceous shale, Conodont zone; calcareous concretions filled with *Leiorhynchus carboniferum*, brachiopods filled with amber oil	65
Otter formation:	
7. Limestone, *Productus* zone of other sections	3
6. Vivid green shale	48
5. Gray limestone	4
4. Gray-green shale with limestone float; limestone is oölitic and ripple marked; algae common	63
3. Gray limestone	12
2. Gray shales with intercalated limestone, partly covered	204
Kibbey formation:	
1. Red shaly sandstone, partly covered	70

posed in the Little Belt Mountains, Castle Mountains, and the Lombard Hills. The beds are known to occur over an area of approximately 20,000 square miles and reach a maximum thickness of 1,200 feet. The group is made up of the Kibbey, Otter, and Heath formations, which were laid down in a broad shallow basin of deposition herein described as the Big Snowy basin.

The Big Snowy basin, as shown on the isopach map in Figure 3, had as its western margin a zone between Helena and Three Forks. The southern shoreline was in the vicinity of Three Forks on the west. Although its exact position to the east is unknown, it was certainly north of Bozeman and Livingston and south of the Crazy Mountains. Due to post-Mississippian and pre-Jurassic erosion in north-central Montana, the northern limit of the basin cannot be accurately determined, although it probably did not extend beyond the North Mocassin Mountains. As shown on the isopach map, the eastern part of the basin is unknown, because it is deeply buried by the gently dipping Cretaceous sediments of east Montana. The most easterly known occurrence of the Big Snowy group is shown in oil-well records from Petroleum County, where all formations of the group are present and attain a thickness of about 1,000 feet.

The type section of the Big Snowy group is given in Table IV.

THE KIBBEY FORMATION

In central Montana, forming the basal formation of the Big Snowy group and disconformably overlying the Madison limestone, occurs a red, thin-bedded, unfossiliferous, marine sandstone that averages about 120 feet in thickness and is remarkable for its persistent lithological characters. In 1899 Weed[18] designated this unit as the Kibbey formation from its exposures along Otter Creek, south of Kibbey, Montana. The sandstone is commonly cemented with calcareous material and contains in the upper portion lenses of gypsum as much as 30 feet thick. Good exposures of the Kibbey are not common, because the streams which develop on the mountain slopes often parallel the strike of the formations, and migrate down dip on the top of the Madison limestone, eroding away the loosely consolidated overlying sandstone. However, good sections do occur along Otter Creek and west of Judith Gap in the Little Belt Mountains, on the north and east slopes of the Big Snowy mountains, and south of Townsend.

The Kibbey sandstone rests upon an erosional surface developed on the Madison limestone. In practically all places where the contact can be studied, the surface of the Madison is silicified and glazed. A relief of 4 to 6 feet in 20 is occasionally observed. Three miles south of Townsend, on the east side of the Missouri River, the erosional surface in the Madison is excellently shown. In this area the Madison forms the general surface and the red Kibbey sandstone occurs in pockets of the limestone. The Kibbey appears to represent residual material obtained from weathering of the Madison limestone. Considerable dolomitic and calcareous material is present in most sections; in fact, some beds in the Kibbey in the Lombard Hills could be called dolomite.

The erosional interval between Madison and Kibbey was probably a time during which the land mass was not very high above sea level. Furthermore, there must have been only a relatively short period of time before the invasion of the Big Snowy sea and the initiation of Kibbey deposition. The Kibbey stage was inimical to the existence of life; no fossils are known to occur in these sandstones.

[18] Weed, "Geology of the Little Belt Mountains, Montana," *op. cit.*, pp. 22, 23.

TABLE V

Heath formation

Otter formation:

		6 Feet
29.	Gray-blue fossiliferous limestone; *Productus* zone	15
28.	Impure limestone, ostracods	23
27.	Gray-green shale	½
26.	Impure limestone	1
25.	Gray-green shale	16
24.	Black shale	14
23.	Gray-green shale	4
22.	Limestone	1
21.	Vivid green shale	10
20.	Oölitic limestone	½
19.	Vivid green shale	30
18.	Impure limestone	1
17.	Black shale	22
16.	Fossiliferous limestone, ostracods	1½
15.	Black shale, ostracods	1
14.	Green to brown shale	5
13.	Black to gray shale	13
12.	Tan shale	8
11.	Limestone, ostracods	½
10.	Gray-green shale	2
9.	Impure limestone, ostracods	¼
8.	Purple and green shale	1
7.	Gray shale	10
6.	Fossiliferous limestone	11
5.	Oölitic limestone	1
4.	Olive-green shale	2
3.	Limestone and shale, mostly covered	90
2.	Dark, calcareous shale	12
1.	Covered, probably shale	36

Kibbey formation

THE OTTER FORMATION

Conformably overlying the Kibbey sandstone occurs a series of gray to green shales intercalated with thin oölitic and fossiliferous limestones. In 1891 these beds were called the Otter formation by Weed[19] from their exposures along Otter Creek in the Little Belt Mountains. The Otter shales are predominantly green in color (due to the presence of ferrous iron) in the Little Belt and Big Snowy mountains. In fact, the upper part of the formation commonly consists of a vivid green shale that can be used with great confidence as

[19] *Ibid.*

a horizon marker. The thin limestones interbedded with the shales are very fossiliferous; some zones, a few inches thick, are composed of 75 to 80 per cent ostracods. Other calcareous beds yield great numbers of brachiopods, byrozoans, gastropods, and pelecypods. Many of the limestone beds are ripple marked or oölitic. The uppermost member of the Otter in many localities consists of a limestone bed which ranges in thickness from 2 to 30 feet and carries great numbers of *Productus*. It is a convenient marker for field correlation, because in some sections the contact between the Otter and overlying Heath formation is difficult to distinguish.

A section typical of the Otter is one found in the Little Belt Mountains, 4 miles west of Judith Gap (Table V).

The following fossils have been identified from the Otter formation:

Productus altonensis
Productus ovatus
Productus richardsi
Productus aff. *P. parva*
Spirifer brazierensis
Cleiothyridina sublamellosa
Composita trinuclea
Composita subquadrata
Chonetes (several)
Reticularia sp?
Eumetria aff. *E. verneuiliana*
Rhipodomella sp?
Girtyella (several)
Leda sp?
Allorisma sp?
Aviculipecten sp?
Sulcatopinna missouriensis
Rhombopora sp?
Fenestella (several)

THE HEATH FORMATION

Conformably overlying the Otter formation occur the black petroliferous shales and sandstones of the Heath formation which form the uppermost beds of the Big Snowy group. The type section of the Heath formation is situated on the north flank of the Big Snowy Mountains in Sec. 6, T. 12 N., R. 20 E. The formation consists primarily of black shales, which may attain a thickness of 500 feet. In most sections, three sandstone beds occur in the upper half. On the southeast flank of the Big Snowy Mountains they have been grouped under the name Van Dusen,[20] whereas on the northeast flank of the

[20] The Van Dusen sand is a name used by oil geologists for an oil sand in the Devils Basin field. It was named from the discovery well. This sand should be considered a member of the Heath formation.

mountains Freeman grouped them under the name Tyler.[21] The base of the formation is marked in the Big Snowy Mountains and in the Little Belt Mountains west of Utica by a black, petroliferous, and fossiliferous shale which is readily recognized in the field or in oil-well cuttings by the presence of great numbers of conodonts. This basal zone of the Heath is the only stratigraphic horizon in Montana known to contain conodonts. Furthermore, the horizon is easily identified because of the ubiquitous *Leiorhynchus carboniferum*, which occurs in great numbers in calcareous concretions of the conodont zone.

The Heath formation conformably underlies the Amsden in all areas where it occurs except in a few localities where the latter formation was removed by post-Mississippian and pre-Jurassic erosion. At the Heath gypsum plant, south of Lewistown, fossiliferous beds of the Ellis (Upper Jurassic) rest upon the conodont zone of the Heath formation.

The type section of the Heath is given in Table IV.

The following fossils have been identified from the Heath:

Productus ovatus
Productus inflatus
Echinochoncus sp?
Spirifer sp?
Chonetes chesterensis
Chonetes (several)
Leiorhynchus carboniferum
Composita subquadrata
Lingula sp?
Orbiculoidea sp?
Cypricardella sp?
Trepospira sp?
Aviculipecten sp?
Cytherella sp?
Conodonts (many assemblages)

AGE OF THE BIG SNOWY GROUP

Certain zones of the Otter and Heath formations are exceedingly rich in fossils, and a large number of collections were taken from several localities. All of the collections show a remarkable relationship and indicate environmental conditions of similar character throughout the Big Snowy basin. In fact, certain faunal zones are

[21] O. W. Freeman, "Oil in the Quadrant Formation in Montana," *Eng. and Min. Jour.*, Vol. CXIII (1922), p. 827. The Tyler sand was named by Freeman from exposures near Tyler, Montana. However, these sands are neither a lithologic, paleontologic, or mapable unit over broad areas, and at the present must be considered a member of the Heath formation. The Tyler and Van Dusen members occupy the same stratigraphic zone at the top of the Heath.

persistent over 10,000 square miles. Taken as a whole, the Big Snowy fauna is peculiarly rich in brachiopods and microfossils. One of the most interesting of the brachiopods is the common *Leiorhynchus carboniferum*, which occurs at the base of the dark Heath shales in the Big Snowy Mountains. In so far as known, this fossil makes its appearance in the Upper Mississippian. Girty[22] describes

TABLE VI

CORRELATION CHART OF CARBONIFEROUS ROCKS IN MONTANA AND NORTHERN WYOMING

	North-Central Wyoming	Yellowstone Park, Wyoming	Southern Montana	Central Montana	
Pennsylvanian	Tensleep sandstone	Quadrant quartzite	Quadrant quartzite		
Mississippian	Amsden limestone and shale	Amsden limestone and shale	Amsden limestone and shale	Amsden limestone	
	Absent	Absent	Absent	Big Snowy group	Heath shale and sandstone
					Otter shale and limestone
					Kibbey shale and sandstone
	Madison limestone	Madison limestone	Madison limestone	Madison limestone	

it from the Moorefield shales of Arkansas and also reports its occurrence in the Brazer limestone of Utah. Croneis[23] found it to be a common form in the Moorefield. Its known geographic range is from Alabama on the south to Alaska on the north and it appears to be restricted to this general western zone. *Leiorhynchus carboni-*

[22] G. H. Girty, "The Fauna of the Moorefield Shale of Arkansas," *U.S. Geol. Surv. Bull. 439* (1911).

[23] C. G. Croneis, "Geology of the Arkansas Paleozoic Area," *Ark. Geol. Surv. Bull. 3* (1930).

ferum is abundantly represented in central Montana by young, mature, and old forms.

The presence of *Productus altonensis*, *P. ovatus*, *Spiriferina transversa*, *Cleiothyridina sublamellosa*, *Composita trinuclea*, *C. subquadrata*, *Leiorhynchus carboniferum*, *Chonetes chesterensis*, *Sulcatopinna missouriensis*, *Spirifer brazierensis*, *Pustula*, *Eumetria*, *Girtyella*, and *Camarotoechia* is considered sufficient to establish the Mississippian age of the Big Snowy group. However, it must be pointed out that certain gastropods which occur near the top of the group, and which are commonly considered Lower Pennsylvanian in age, apparently had their initial development in the Upper Mississippian. This gastropod fauna is not peculiar to the Big Snowy group, because it appears to be related to those of the Brazer and certain Upper Mississippian strata of Arkansas.

A detailed report on the Big Snowy fauna is near completion and, until it is finished, considerable caution must be used in correlating any particular part of the Big Snowy group with other well-known rock strata. It is probable that this group will be found to range from Middle Valmeyer to Middle Chester. It is certain that the age is not older than Warsaw nor younger than Upper Chester.

CONCLUSIONS

Conclusions to this study of the Carboniferous of Montana can be grouped under three heads:

The Quadrant formation:

1. The Quadrant formation does not occur in central Montana.
2. The basal 100 feet of the type section of the Quadrant were originally listed as "talus" and cannot be considered a part of the formation. This lower zone is the Amsden formation.
3. The Quadrant is the westward equivalent of the Tensleep.
4. It is a marine formation. Limestone beds and marine fossils are common.
5. Stratigraphic and paleontologic evidence indicates that the Quadrant is basal Pennsylvanian in age.
6. The red zone above the Madison in south-central Montana which is commonly classified as the base of the Quadrant is in reality the Amsden. The Quadrant consists of the overlying quartzites.

The Amsden formation:

1. The Amsden occurs at Quadrant Mountain below the true Quadrant formation and above the Madison limestone.
2. The limestone which occurs below rocks of Jurassic age and above the Big Snowy group in central Montana is a northward extension of the Amsden.
3. The lithological characteristics of the Amsden grade from dominantly red shaly sandstone overlain by a thin gray-blue limestone in south-central Montana to thinly bedded limestone that weathers pink and is arenaceous in the basal part in central Montana.
4. Sections in the Lombard Hills, north of Three Forks, show the Big Snowy group, the Amsden, and the Quadrant formation in their true stratigraphic relationships.
5. Stratigraphic and paleontologic evidence shows the Amsden to be Mississippian in age, probably Chester.

The Big Snowy group:

1. The Kibbey, Otter, and Heath formations make up the Big Snowy group.
2. They were deposited in a long, shallow basin with an east-west axis.
3. The fauna of the Heath is closely related to those of the Brazer limestone of Idaho and the Moorefield formation of Arkansas.
4. The Big Snowy group is not older than Warsaw nor younger than Upper Chester. Its exact age cannot be determined until a study on the fauna is completed. This study is now well advanced.

SOME HEAVY MINERALS OF THE FRONT RANGE GRANITES

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ABSTRACT

The pre-Cambrian granites of the Front Range of Colorado and southern Wyoming outcrop in ten batholiths and two stocks. The granites have been classified according to relative age and mineral content. (1) The granites of the oldest group of batholiths generally carry 15 kinds of heavy minerals which average from 5 to 40 per cent of the weight of the samples. (2) The intermediate group includes 17 to 19 heavy minerals which comprise from 3 to 25 per cent of the rock. (3) The youngest group generally carries 21 heavy minerals, the amounts of which range from 2 to 10 per cent per sample.

The characteristics and important variations of each diagnostic mineral are discussed in relation to the three groups.

INTRODUCTION

A study of the heavy minerals of the granites of the Front Range of Colorado, in progress since 1931, is part of a comprehensive investigation of the pre-Cambrian rocks of that area.[1] This paper is a report of progress and a summary of information on areas where the field work has been detailed; it includes reports on some regions in which reconnaissance work only has been done.[2]

TECHNIQUE

Two hundred and fifty samples, 2 by 4 by 4 inches, were chosen from among two thousand specimens of granite and granitic rocks and crushed to pass a 65-mesh screen. Fifty grams of each crushed sample were washed by decantation four times and dried. Ten to fifteen grams of the finest rock flour were removed by washing. Remaining fines were separated by bromoform (s.g. 2.8), washed in either benzol or alcohol, and dried. The heavy-mineral residues were separated by an electromagnet and the magnetic and non-magnetic

[1] M. F. Boos and C. M. Boos, "Granites of the Front Range," *Geol. Soc. Amer. Bull. 43* (1932), p. 170.

[2] The work was assisted in 1932 and 1933 by grants-in-aid from the National Research Council and by use of the laboratory facilities of the Colorado School of Mines, the University of Wisconsin, and the U.S. Bureau of Mines, Bartlesville, Oklahoma, Station.

portions weighed and examined dry. Permanent mounts were made in Canada balsam and studied under the petrographic microscope. Heavy minerals in the residues were checked, in most cases, against thin sections from the same rock samples.

THE THREE GROUPS OF PRE-CAMBRIAN GRANITES

On the basis of field and laboratory studies of ten batholiths and two stocks of granite in the Front Range the rocks may be classified into three groups according to relative age, general petrography and petrology, and heavy-mineral content.[3] The heavy-mineral suites show features related to age, texture, quartz-feldspar content, and mode of occurrence of the individual granites. In general, the oldest granitic rocks carry the least variety, but the greatest abundance of heavy minerals. Those of intermediate age have a greater number of minerals but these occur in lesser abundance. The youngest group of pre-Cambrian batholiths is featured by a profusion of crystals of smaller size than those in the older groups, but the total amount of heavy minerals is less per unit of rock.

The oldest group of granitic rocks.—The oldest granitic rocks outcrop in the Boulder Creek and Mount Evans batholiths and the Tabernash and Stove Prairie stocks (Fig. 1). The granite, granite-gneiss, and quartz-monzonite of these areas vary in texture, color, and content, and the variations are reflected locally in the content and amount of heavy minerals present.

In general, fifteen kinds of heavy minerals occur in large-sized, strongly colored, and locally clustered crystals in most phases of the oldest granitic rocks (Tables I and II). Five to 40 per cent of the rock consists of dark-brown biotite, titaniferous magnetite and ilmenite, brown to yellow titanite, yellow monazite and brown orthite, large, clear, colorless apatite, and blue-green hornblende. Zircon, allanite, pyrite, and tourmaline occur in very tiny crystals.[4]

Granites of intermediate age.—The Pikes Peak and Sherman granites are of intermediate age and are much alike in field relations,

[3] For an account of the general geology of the Front Range pre-Cambrian, see M. F. Boos and C. M. Boos, "Granites of the Front Range: the Longs Peak–St. Vrain Batholith," *Geol. Soc. Amer. Bull.* 45 (1934), pp. 303–32.

[4] Estimated amounts of each mineral are based on modified Rosival counts of the grains, imbedded in Canada balsam, and in some cases, on grain counts.

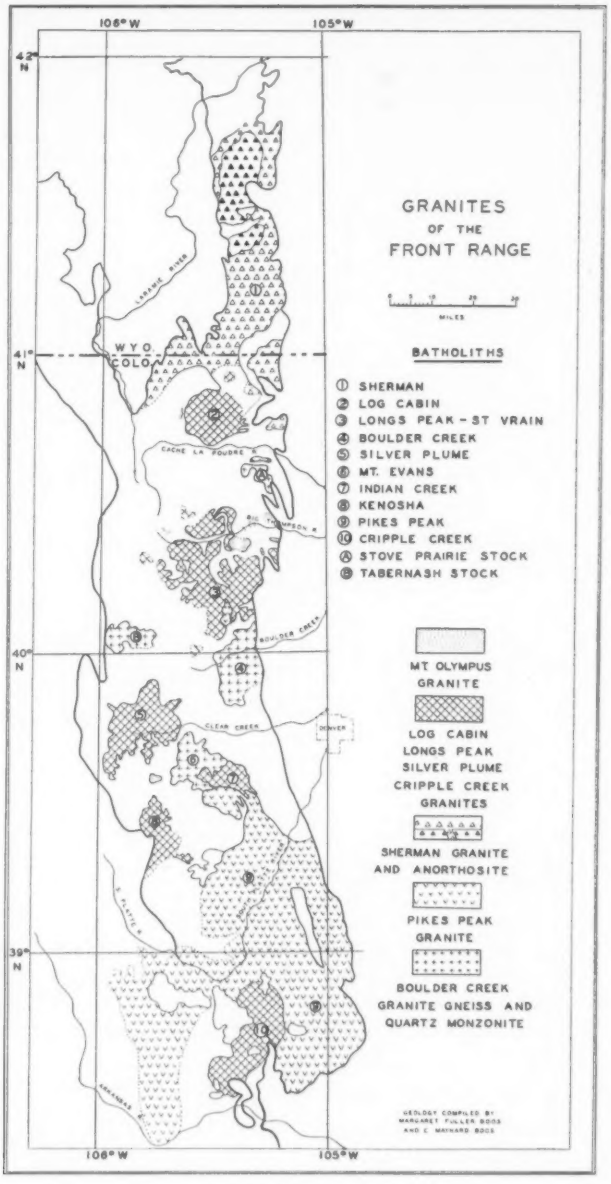
106° W
105° W
42° N
41° N
40° N
39° N
WYO.
COLO.
LARAMIE RIVER
CACHE LA POUDRE
BIG THOMPSON R.
BOULDER CREEK
CLEAR CREEK
DENVER
S. PLATTE R.
ARKANSAS R.
GRANITES
OF THE
FRONT RANGE
0 5 10 20 30
MILES
BATHOLITHS
① SHERMAN
② LOG CABIN
③ LONGS PEAK - ST VRAIN
④ BOULDER CREEK
⑤ SILVER PLUME
⑥ MT. EVANS
⑦ INDIAN CREEK
⑧ KENOSHA
⑨ PIKES PEAK
⑩ CRIPPLE CREEK
Ⓐ STOVE PRAIRIE STOCK
Ⓑ TABERNASH STOCK
MT OLYMPUS GRANITE
LOG CABIN
LONGS PEAK
SILVER PLUME
CRIPPLE CREEK
GRANITES
SHERMAN GRANITE
AND ANORTHOSITE
PIKES PEAK
GRANITE
BOULDER CREEK
GRANITE GNEISS AND
QUARTZ MONZONITE
GEOLOGY COMPILED BY
MARGARET FULLER BOOS
AND E. MAYNARD BOOS

FIG. 1

general appearance, texture, color, and mineral content. The Pikes Peak batholith occupies a large area at the southern end of the Front

TABLE I

BOULDER CREEK BATHOLITH

		SAMPLE NO.	BIOTITE	MAGNETITE	MUSCOVITE	APATITE	ZIRCON	SILLIMANITE	TITANITE	ALLANITE	PYRITE	TOURMALINE	FLUORITE	HEMATITE	LEUCOXENE	HORNBLENDE	BERYL	MONAZITE	NON MAGNETIC	MAGNETIC	TOTAL PERCENT
CENTRAL PART	DARK	13	VA	VS		S	R		VS		VR	VS				VS			0.80	10.40	11.20
		318	VA	VS	R	S	VS		VR		VR			VR		R		S	0.80	12.80	13.60
		293	A	VS		C	S			VR								R	.12	22.00	22.12
		510	A	VS		C	VS		R	VR						C	R?		0.80	14.80	15.60
		332	-	-		-	-							-					0.80	5.60	6.40
		383	-	-		-	-		-	?						-			2.60	13.80	16.40
	LIGHT	573	VA	S		S	R		R	R	R		R				S	R	11.00	32.20	43.20
		298	VA	VS	R	C	R		S	VR	S	VR		VR		VR				23.40	
		521	A	C	VS	S	VS		R	VR								R	0.40	5.86	6.26
		537	A	VS		VS	R		VR		VS		R					R	0.40	2.20	2.60
		381	-	-		-	-		-							-			1.40	10.60	12.00
		617	-	-		-	-		-												
NORTH EDGE		117	VA	VR	S	VC	VS	VR	VR	R	VR	VR	R			VR			0.80	18.80	19.40
		504	VA	VS		R	R		R	VR				R		R			0.20	10.80	10.80
		648	-			-	-		-							-		-			
EAST EDGE		508	A	S		-	-		-	-										4.20	4.20
		590	A	R	C	VS	VS		R	R				R					0.02	9.20	9.22
		597	A	VS	R	S	R		C			VR			R	R	R	VS	0.68	10.60	11.28
		601	A	VS	-	C	VS		VS	VR						C			0.32	1.40	1.72
WESTERN MARGIN		87	VA	R		VS	VS		S		VR		R			S		R	0.74	22.00	22.74
		517	F	VS		R	VR		VR		VR						R	R	0.24	3.20	3.44
		518	R	VR	VR	S	VS		VR	VR									0.04	12.80	12.84
		530	VA	?		VA	R		VR		VR			R				R	0.02	5.40	5.42
		526	VA	VR		VC	R		C	R	VR					R			2.00	19.80	21.80
		647	-	-		-	-		-							-					
		513																	6.20	28.20	34.40
SOUTH MARGIN		547	-	-		-	-		-										1.00	7.40	8.40
		549	F	VS	VR	VS	R											VR	0.02	6.20	6.22
		550	VA	VS	VR	S	VS		VS	R							R		0.30	10.60	10.90
		592	A		S	C	VS		R			R		R			S		0.80	10.60	11.60
		637	VA	VS	VS	S	R		VR										1.18	12.40	13.58
		640				-	-		-	-							-				
COAL CREEK		540	VC	S	S	R	VR		VR	VR	VR			R			R?	S	0.30	1.70	2.00
		612	VA	S	VS	S	S		VR	VR									0.22	6.80	7.02
		674A	VS	S	C	R	R		R	VS	VR			VS					2.20	2.40	4.60
		674	-	-		-	-			?											
DIKES		276	-	-	-	-	-	-			-			-							
		565	VA	VS	R	VR	VR		VR	R									0.08	3.20	3.28
		572	VC	S		VS	VS		VR		VS							C	0.48	8.40	8.88
		647	-	-		-			-												

-OBSERVED IN THIN SECTION
F-FLOOD VA-VERY ABUNDANT A-ABUNDANT VC-VERY COMMON
C-COMMON S-SCARCE VS-VERY SCARCE R-RARE VR-VERY RARE

Range and the Sherman granite outcrops widely over the northern part of the range.

In the Pikes Peak granite there are 19 heavy minerals which comprise 3 to 15 per cent of the rock and include green to brown biotite, titaniferous and non-titaniferous magnetite, yellowish-green to pale-brown titanite, muscovite, zircon, apatite, fluorite, allanite, and pyrite. Blue-green hornblende, beryl, and riebeckite are of rare local-

TABLE II

MOUNT EVANS BATHOLITH, TABERNASH STOCK AND STOVE PRAIRIE STOCK

	SAMPLE NO	BIOTITE	MAGNETITE	MUSCOVITE	APATITE	ZIRCON	HORNBLENDE	TITANITE	ALLANITE	PYRITE	TOURMALINE	FLUORITE	HEMATITE	LEUCOXENE	BERYL	MONAZITE	NON MAGNETIC	MAGNETIC	TOTAL PERCENT
MT. EVANS BATHOLITH	353	VC	C	R	S	S	VR	VS					R	S		S	2.60	7.20	9.80
	650	F	VS		R	R		VS								VS	4.20	17.80	22.00
	657	VA	VS		S	R		VS		R					VS	R	7.00	11.40	18.40
	659	VA	S		VS	VR		VC	R	S				VS		VS	0.40	13.60	14.00
	662	-	-		-	-		-											
	663	A	VS	VR	VS	R		VS				VR				VR	1.00	10.20	11.20
	664	-	-		-	-		-								-			
	669	F	VS	VS	VS	VS		VR	R	VR				R			0.80	8.20	9.00
TABERNASH STOCK	17	A	VS	R	VS	R		VR		VR	VR	S					1.60	7.20	8.80
	106	A	S		S	VS	S	R	?	VR							1.20	8.40	9.60
	158	VA	VS		S	VS				R							0.40	20.00	20.40
	629	VA	VS	VR	C	R	C	VS					R		VS	VS	0.20	26.00	26.20
	630	VA	VS	R	S	S		R	R	R					VS		0.80	20.00	20.80
	628	-			-	-	-	-	?										
	618	-			-	-							-						
	633	-	-		-	-	-	-	?										
	636	-	-		-	-			-								2.40	19.40	21.80
STOVE PRAIRIE	7	A	VS	VS	S	R		R	R	VR			VS				0.20	4.40	4.60
	248	VA	R	S	VS	R		VR		R			VS						
	369	R	VC	S	R	VR		VS	R	R									
	437	VC	R	R	VS	VR		R		VR			VR		C	R?	0.60	38.80	37.40
	233	-		-	-			-	-										
	104	-	-		-	-		-	-										

-OBSERVED IN THIN SECTION

F-FLOOD VA-VERY ABUNDANT A-ABUNDANT VC-VERY COMMON
C-COMMON S-SCARCE VS-VERY SCARCE R-RARE VR-VERY RARE

ized occurrence (Table III). All minerals are less strongly colored than those in the oldest granitic rocks.

Seventeen heavy minerals constitute 3 to 13 per cent of the Sherman granite. The residues consist of green to brown biotite, titaniferous and non-titaniferous magnetite, yellowish to pale-brown titanite, rare muscovite, zircon, apatite, allanite, blue-green horn-

TABLE III

Sherman and Pikes Peak Batholiths

		SAMPLE NO.	BIOTITE	MAGNETITE	MUSCOVITE	APATITE	ZIRCON	TITANITE	ALLANITE	PYRITE	TOURMALINE	GARNET	FLUORITE	ILMENITE	HEMATITE	LIMONITE	LEUCOXENE	HORNBLENDE	BERYL	MONAZITE	RIEBECKITE	NON MAGNETIC	MAGNETIC	TOTAL PERCENT
SHERMAN BATHOLITH	N	422	S	VS	?	VR	R	S	R								S	R	S	?				
	CENTRAL	1	S	R		S	R		R		R		R					VA		VS		0.62	7.00	7.62
		430		VS		R	S											VA				0.20	20.00	20.20
		432	A	S	VR		R	R	VR													0.12	4.00	4.12
		435	A	VS	VR	VS	VR		VR				VR		R			?						9.04
	E.	414	A	VS		VR	VR	VR	R						VS			VR	VR					5.74
		415	VS	C		C	VS	VR	R						S			VC				1.00	11.20	12.20
	SOUTH	#152	VC	VS	VS	VS	VS				R							?	C			0.26	2.20	2.46
		385	A	VS	VR	R	VR	C									C	C	?			?	18.00	18.00
		388	A	S	VR	VS		C	VR								C		R					10.00
		421	A	VR		C	C	VS	VR								S	S	VR			1.08	24.80	25.88
	SOUTH WEST	392		VC		VS	VS	R	R	R	VR							S				2.72	12.40	15.12
		#384A	R	VS		VR	VR	VR	VR									A				13.72	9.80	22.52
		394B	C	VS		VS	R	VS	R	VS	VR			S				VS	VS					12.04
		396		S		R	VS		R						R			A	R					2.80
		397	R	C		R	R	VS	R	VR	?				VS			VS	VC	?		5.48	19.60	25.08
PIKES PEAK BATHOLITH	N.W.	322	S	VS	R	VS	R	R	R	VR			?	C				VS	VS			0.52	4.00	4.52
		744	VC	C	S	R	R	VR	VS	VR			VR		R			VS				3.6	9.60	13.20
	PLATTE CANYON	137	VA	R	VR	S		R		VR				VR						VS		0.36	4.00	4.36
		251	VA	VR	R	VS	S	R	VS	VR		R	C		R			C		VS		0.32	4.40	4.72
		354	A	VS	VR	R	VS				VR		S					S				0.80	2.20	3.00
		355	VA	S	R	R	R	VS					C					VS				0.42	2.20	2.62
		#300	R	R	VS			R	R				R					F	R		VR			10.40
	CRIPPLE CREEK	159	VA	VR	R	R	R	R	VS	VS	VR											0.08	1.40	1.48
		715	A	R		VS	R	VR	R	VR			VS					C				2.00	7.20	9.20
		'716	R	VC		R	VS	VR	R					S	R							1.36	12.80	14.16
		717	VC	VS		VS	VS	VR	R				C					VS				0.68	27.20	27.88
	PIKES PEAK	110	VA	VS		C	VS	R	R	VR			R		R							0.68	2.80	3.48
		256	VA	S		R	R	VS		R	R		VC	R				?	C	?		1.02	3.60	4.62
		329	VA	VS	S	R	R	VS	R	VS	R		?					VR				0.04	2.80	2.84
		741	A	S		S	VS	VS	VR	VR	VR		R					VS				4.00	10.00	14.00
	WEST MARGIN	309	A	VS	R	VS	S	VR	VS				R		R	R						0.60	3.40	4.00
		708	R	S	C	VS	VR								R			?				2.20	1.60	3.80
		721	VA	VS	VR	C	VS	R	R				VR						R			0.30	12.00	12.30
		736	R	VR		C	VS	C									S	A				5.60	16.80	22.40
		"739	A	VS	VR	VS	VS	VR		VR		S						VS				5.60	12.80	18.40
		'725	A	VS	S	S	VS	VR	R						R							0.60	8.00	8.60
		'721	A	C	VR	S	VS	C		R	R	VR					S					4.20	17.20	21.40
		'735	A	R		C	VS	C				VR					S	VS	R			2.24	30.00	32.24

#DIKE "SPRING CREEK GRANITE 'GNEISS

F-FLOOD VA-VERY ABUNDANT A-ABUNDANT VC-VERY COMMON
C-COMMON S-SCARCE VS-VERY SCARCE R-RARE VR-VERY RARE

blende, and hematite. The crystals are a little larger than those of the same minerals in the Pikes Peak granite, but are mostly smaller than those in the oldest group of pre-Cambrian granites.

The youngest pre-Cambrian granites.—The youngest granites outcrop over more than 1,200 square miles from north to south throughout the Front Range. They are widely spaced in six batholiths: the Log Cabin, Longs Peak–St. Vrain, Silver Plume, Kenosha, Indian Creek, and Cripple Creek. Twenty-one heavy minerals, in an abundance of small crystals, constitute 2 to 10 per cent of the rock. There are lilac-colored zircon crystals, clear, colorless apatite prisms, muscovite flakes, allanite grains, sillimanite needles, green to olive-green biotite masses, dull to glossy magnetite grains, pyrite cubes, very rare titanite and tourmaline grains, and hematite granules (Tables IV, V, VI, and VII).

MINERAL CHARACTERISTICS

Zircon.—Two types of zircon, represented by very rare crystals, occur in the Boulder Creek and Mount Evans batholiths: (1) large, deep rose to orange-colored prisms with rod-like inclusions; (2) small, clear, colorless oval grains and slender rods which are locally pitted and corroded. The first type occurs in the coarse-grained, granitoid masses, and the second predominates in the gneissoid and dike rocks. The Tabernash stock on the west side of the range carries both types. The granite of the Stove Prairie stock on the eastern side of the range has a few large orange-colored prisms and small, very rare, colorless grains surrounded by wide pleochroic halos in biotite.

Even in the coarse phases of the Pikes Peak granite zircons are small and rare. Locally, the rose-red, euhedral prisms are more numerous than the tiny colorless grains and rods. The Sherman granite carries yellow to pinkish zircon prisms which have dark centers made of skeletal crystals, minute oval grains and rods of allanite and smaller zircons. Zircon crystals imbedded in biotite have wide pleochroic halos.

Zircon is a persistent widespread mineral in all the granite batholiths of youngest pre-Cambrian age and also occurs in both types described above from the older granitic rocks. The large euhedral

TABLE IV

MOUNT OLYMPUS AND LONGS PEAK GRANITES

				Sample No.	Biotite	Magnetite	Muscovite	Apatite	Zircon	Sillimanite	Titanite	Allanite	Pyrite	Tourmaline	Garnet	Fluorite	Ilmenite	Rutile	Hematite	Leucoxene	Chlorite	Chalcopyrite	Magnetic Heavy Residue	Non-Magnetic Heavy Residues	Total
MT. OLYMPUS GR.	LONGS PEAK GRANITE	Longs Peak Area	St. Vrain Area	Red Gr.	8	A	VS	C	VS	R	R		VR	VR						R				2.62%	0.24%	2.86%
					10	VA	A	VS	S	R	S		R	R	VR					VS				2.22	0.18	2.40
					192	VA	VS	C	S	VS	C		R	VR						R		R		2.53	0.79	3.32
				Normal Phase	6	VA	C	R	S	R	C	S	VR		VR			?						2.13	0.30	2.43
					12	VA	C	VC	VS	R	VR			VR										2.29	0.30	2.59
					173	VA	R	S	C	S	VS		VS											3.79	0.53	4.32
					174	VA	R	A	R	R	VR		VR							VS				3.02	1.14	4.16
					191	VA	S	S	C	S	VS	R	R	R				?				VR		3.90	0.81	4.71
					192	VC	S	S	VS	VS	S	R	C					?		R	VR			3.14	0.19	3.33
					196	VA	S	VC	S	S	R	VS	S	VR				?		VS	?	R		3.04	0.50	3.54
					248	A	R	VC	S	VS	VS	R	S				R			VS				2.08	0.67	2.75
					272	VA	VS	C	S	S	VS	VR	S							R		R		3.42	0.32	3.74
				East Margin	177	A	S	S	VS	R	C		VS	VR						R				2.36	0.19	2.55
					183	VA	VR	C	S	VS	R	VS								C				3.09	0.52	3.61
					188	VA	R	VC	R	R	VR	VR	R							C				3.49	0.62	4.11
					269	A	VS	S	S	R	VS	R	VS						VR	R		R	VR	1.73	0.39	2.12
				Transition Area	187	A	VS	S	VS	VS		R		R		VR	VR	R			R			2.32	0.26	2.58
					198	VC	VS	S	S	VS	VR	S	VR	VR	VR		VS				VR			3.82	0.21	4.03
					216	VC	C	VC	C	R		R		S	R					R	R	VS	R	1.05	0.19	1.24
					359	VA	VS	C	S	VS	VC			VR										3.57	0.33	3.90
				Continental Divide Area	24	A	VC	S	VS	R	VS	R	C	VR						VR		R		2.40	0.20	2.60
					51	VA	VS	C	R	VS		R	R	VR						VR				1.50	0.18	1.68
					42	VC	C	VR	C	S		VC		R	VR	R	VR	R		VR	R		VR	8.75	1.87	10.62
					73	VC	S	VS	S	VS	A	S	VR	VR	VR									2.54	0.76	3.30
					351	VA	C	R	VS	VS	S			VR				VS						2.79	0.11	2.90
				Estes Park Area	36	A	S	C	VS	S	R	R	VS	R	VR				VS	VS		R	VR	2.72	0.24	2.96
					39	F	VS	VS	VS	C		R	VR	VR										2.73	0.06	2.79
					247	VA	S	R	S	VS	VC		VR	VR										2.53	0.41	2.94
					248	VA	R	VS	VS	S	C	R		VR	VR		VR		R					3.24	0.42	3.66
				*	271	A	R	VS	VS	VS	VS	VR		C	VR					R	R				1.34	0.17	1.51
					180	VA	S	S		VS	VR		VS	VR					R		VR			1.46	0.10	1.56
					9	VA	VS	VA	C	R	VR			R			S				VR			4.75	0.33	5.08
					38	VA	S	VC	S	S	VA	R	R	VR							R				9.45	1.37	10.82
					74	F	VS	VS	VS	S	R		R	VR			R				VR				3.46	0.09	3.54
					190	A	C	VS	C	S	VS	VS	R	VR				R			VS	R	R		5.55	0.81	6.36
					175	A	R	S	C	S	VA	R	VR									R			3.52	0.46	3.98
					198	C	VC	R	VS	S	R	R	C					VS		R	VS	VS			1.97	0.43	2.40
					230	VC	VS	VC	VS	R	VS	VS	C							VR		R			1.85	1.89	3.74
					227	VA	VS	C	VS	C	R	R	S						VS	VS					1.58	0.98	2.56
					282	A	VS	S	VS	R	VS		VR							C		R			0.71	0.07	0.78
					245	VA	VS	S	C	R		R	VR	R	VR		R			VR			VR		2.96	0.49	3.45

* Fine grained granite dike rocks.

F- Flood VA- Very abundant A- Abundant VC- Very common
C- Common S- Scarce VS- Very scarce R- Rare VR- very rare

crystals have a distinct lilac color, are a millimeter long and a third as thick, are delicately and completely zoned, and are thickly spotted with minute cubic inclusions of an unknown mineral, or with small rods of colorless zircon, grains of allanite and apatite. In

TABLE V

SILVER PLUME BATHOLITH

	SAMPLE NO.	BIOTITE	MAGNETITE	MUSCOVITE	APATITE	ZIRCON	SILLIMANITE	TITANITE	ALLANITE	PYRITE	TOURMALINE	GARNET	FLUORITE	ILMENITE	HEMATITE	LEUCOXENE	LIMONITE	CHALCOPYRIT	NON MAGNETIC	MAGNETIC	TOTAL PERCENT
NORTH PART	18	VA	VS	R	VC	VS	R	-	VR	R									0.46	4.17	4.63
	27	VA	VS	C	S	C	-	-	S	VR					VS				0.46	2.97	3.43
	125	VA	VS	S	S	R	C	R	R	-					VR				0.35	2.04	2.39
	133	A	C	VC	VS	R	R	V	VR	VR					VR				0.36	2.13	2.49
	350	VA	R	VS	S	R	VC	-	R	VR					VR				0.40	2.43	2.83
	90	VC	C	VC	VS	VR	VR	VR	-	-		R		VR		VR	VR		0.47	2.49	2.90
SOUTH PART	83	A	C	S	VS	R	R	R	C	VS	VR				R				0.39	5.11	5.49
	92	A	C	VS	C	VS	-		C	S	R	VR			R				0.31	4.91	5.28
	100	F	S	VS	VS	VR	VS	VR	VS	VS					VR				0.25	3.08	3.33
WEST INTERIOR EAST	3	A	VS	C	VS	R	-		R					VR					0.73	5.32	6.05
	20A	A	R	VS	VS	S	-	-	R	VR	VR				VS				0.16	1.18	1.34
	20C	VC	C	S	S	VR	VS	VS	VR	VR	-	VR							1.00	6.33	7.33
	23	VA	S	VC	S	VS	R	R	R	VR	VR	VR	VR			VR		VR	0.65	2.57	3.22
	45	VA	VS	S	C	VR	-	R	-	VS					VS			VR	1.00	4.82	5.82
	87	A	C	VC	S	S	VR	-	-	VS	VR		VR		VR			VR	1.02	5.82	6.84
	88	VC	C	VC	VS	R	-	S	-	VS	VR	R	VR		VS	VR	VR		0.92	1.94	2.86
	89	A	VS	C	S	VC	-	-	-	VR	-	R			VR				0.83	4.18	4.79
	42	VA	S	S	VS	R	-	-	-	VS					VS				0.15	2.84	2.79
	129	A	C	C	VS	R	-	R	VR	R								VR	0.81	1.44	2.05
	103	A	VS	S	VS	S	VR	VR	R	VS	R	VR			R				0.46	2.48	2.94
	344	A	C	S	S	S	VS	-	R	-	VR	VR	VS		R				1.14	3.12	4.20
DIKES	97	F	S	S	C	VS	VR	VR	VR	R	R	-	VR						0.84	4.89	5.23
	33	VC	C	VR	VR	R	-	VR	-	VR					VR				0.16	1.72	1.88
	84	VA	S	R	VS	VR	VC	VR	VS	VR	VR								4.36	5.58	9.94
	208	F	VS	VS	S	S	R	R	VR	VS	VR				VR				0.42	2.01	2.43
	126	VA	VS	S	VS	VS	R	R	VS										0.22	3.77	3.99

-OBSERVED IN THIN SECTIONS

F-FLOOD VA-VERY ABUNDANT A-ABUNDANT VC-VERY COMMON C-COMMON
S-SCARCE VS-VERY SCARCE R-RARE VR-VERY RARE

the coarse-textured granite both large, colored crystals and small, clear, rounded rods and grains of zircon occur in the same sample. In the fine-grained granitic dikes, and in the Mount Olympus granite, which is a differentiate[5] of the Longs Peak and Silver Plume granites,

[5] M. B. Fuller (M. F. Boos), "General Features of Pre-Cambrian Structure along the Big Thompson River, Colorado," *Jour. Geol.*, Vol. XXXII (1924), p. 52.

the small colorless crystals predominate. The granite of the stocks and cupolas of the Longs Peak–St. Vrain batholith is locally crowded with large, lilac-colored zircon prisms.

TABLE VI

INDIAN CREEK, CRIPPLE CREEK, AND KENOSHA BATHOLITHS

	SAMPLE NO	BIOTITE	MAGNETITE	MUSCOVITE	APATITE	ZIRCON	SILLIMANITE	TITANITE	ALLANITE	PYRITE	TOURMALINE	GARNET	FLUORITE	ILMENITE	HEMATITE	HORNBLENDE	BERYL	NON MAGNETIC	MAGNETIC	TOTAL HEAVY
INDIAN CREEK BATHOLITH	675	VA	R	C		R	VR	VR	S		VR			VR				2.00	3.12	5.12
	#677	S	C		S	VS		C		R					#			2.80	4.32	7.12
	683	V	S	VS	S	VS		R	R	R					VR			1.20	3.60	4.80
	688	A	VS	S	R	VS		?	R									0.40	10.00	10.40
	693	C	VS		VS	VS		R	R	VR	S					VR		3.20	10.40	13.60
	576	-	-		-	-			?											
	676	-	-		-	-			-					-	-					
	680	-	-		?	-			?											
	687	-	-	-	-	-			-						-					
CRIPPLE CREEK BATHOLITH	2	VA	VS	R	VS	R		R		VR	VR		VR		VR		R	0.14	2.40	2.54
	##704	VC	R	VR	VS	VS		R	VR		VR		S				R	3.60	8.40	12.00
	709	C	S	C	VR	R			VS						R			1.72	2.80	4.52
	718	VC	S	R	S	VS		VR	VR	R								1.60	4.80	6.40
	720	C	VS	R	VS	R		VS	VR	VR			VR					1.20	4.40	5.60
	726	A	S	VS	S	VS			R		?							1.84	10.00	11.84
	729	A	VS		R	R		VR	R									1.24	1.80	2.84
KENOSHA BATHOLITH	11	A	S	VS	S	R							VR					0.16	0.60	0.76
	243	VS		A	VS	VS		R?	VS	R								0.36	3.20	3.56
	248	A	VS	S	R	VR		VR	R		?							0.08	3.20	3.28
	298	VC	S	VS	VS	S			R	R	VR								3.40	3.40

#CONTAMINATED ##MAY BE PIKES PEAK –OBSERVED IN THIN SECTION

F-FLOOD VA-VERY ABUNDANT A-ABUNDANT VC-VERY COMMON
C-COMMON S-SCARCE VS-VERY SCARCE R-RARE VR-VERY RARE

Apatite.—Apatite is a common constituent of all the pre-Cambrian granitic rocks of the Front Range. There are usually two generations in the Boulder Creek granite and granite-gneiss. The coarser phases of the granite are well supplied with large, stout, corroded and rounded apatite prisms which have cloudy gray centers marked

by minute rounded apatite grains and small rods of zircon. Apatite and titanite parallel each other in abundance in most samples, and the long directions of the crystals parallel the shearing in gneissoid granites. In the Mount Evans batholith the apatite is everywhere associated with biotite. On the western slope of the range the large,

TABLE VII

LOG CABIN BATHOLITH

	SAMPLE NO.	BIOTITE	MAGNETITE	MUSCOVITE	APATITE	ZIRCON	SILLIMAN	TITANITE	ALLANITE	PYRITE	TOURMALINE	GARNET	FLUORITE	ILMENITE	HEMATITE	LEUCOXENO	HORNBLENDE	BERYL	NON MAGNETIC	MAGNETIC	TOTAL HEAVY
CENTRAL PART	297	A	C	R	-	VS	VR			VR								R	0.24	5.84	6.08
	348	A	C		VC	VR		VS		S								VS	0.92	7.32	8.24
	373	-			-	-		-	-						-						
	383	-	-		-	-		-	-												
	#384	VS	VS	R	VS	R		VR	A	VR			R		C				1.32	5.22	6.54
	385	VA	R	R	VR	R		VR	R				R					R	0.76	6.40	7.16
	386	-	-		-	-		-													
	362	VA	S	VR	R	VR		VS							VS			R	0.80	2.02	2.82
	389	A	VS	R	S	VS		R	VS										0.56	9.32	9.88
	407	A	VS	S	VS	VS	R		R						VR				1.08	6.80	7.88
	436	VC		S	VS	VR	R		R		VR				VS			VS			1.10
N.M.	360	A	S	VR	C	VS		VC						S		R		?	1.80	8.02	9.82
	370	A	C		R	VR	VR	VC						S		R		VR	1.16	8.20	9.36
SOUTH MARGIN	357	A	S	R	S	R		VS	VR	VR								VS	0.76	3.72	4.48
	381	-	-		-	-		-	-				-								
	#445	A	S	VC	VR	VS		VR	R		VR				R				1.04	6.20	7.24
	#447	A	VS	VC	VS	R	VR	R	VR	VR									0.04	5.00	5.04
EAST MARGIN	378	A	C		VR	VR			R								S		0.72	6.64	7.36
	403	F	A		R	-													0.48	3.22	3.70
	405	VS	VA	R	R	VR		R	VR						VS				1.98	4.58	6.56
	410	A	VS	VC	VR	R		R	VR	VR										4.40	
WEST MARG.	379	A	VS	VR	VS	R		VR	R						VS			S	1.12	4.56	5.68
	381	A	S	R	S	VS		C		VR		VR			R				1.20	9.28	10.48
	448	A	VS		S	R		VS		VR									0.40	9.68	10.08

#DIKE -OBSERVED IN THIN SECTION

F-FLOOD VA-VERY ABUNDANT A-ABUNDANT VC-VERY COMMON
S-SCARCE VS-VERY SCARCE R-RARE VR-VERY RARE C-COMMON

thick apatite crystals have rounded terminations, etched crystal faces, and abundant inclusions.

The apatite of the Pikes Peak batholith occurs in two generations, an older, composed of clear angular rods and grains with no inclusions, and a younger, consisting of large broken crystals with corroded faces, subdued outlines, and a common association with biotite.

Dissimilarly, the older generation of apatite in the Sherman granite consists of large, stout, rounded, and much fractured crystals which are generally clear and glassy, but those of the younger set are

angular and rod-like. Locally the large euhedral grains have purplish cores and rare mineral inclusions.

Apatite grains are small, rounded, commonly isolated, and not abundant in most of the youngest pre-Cambrian granites. Where any titanite is present the crystals of apatite in the same sample are large, thick, and show pitted and corroded crystal faces, purplish centers, and have numerous skeletal inclusions, particularly in the Log Cabin and Silver Plume batholiths. Generally a local increase in the number of apatite grains parallels a like increase in the amount of zircon.

Allanite.—Allanite (orthite) occurs locally in abundance in the oldest pre-Cambrian granites where it is of deep brown color and much intergrown with brown titanite and monazite.

Clusters of rose-red to greenish allanite grains are evenly scattered through the granite of the Pikes Peak batholith. The color varies, since most crystals have a fresh greenish interior surrounded by a reddish, dusty, altered surface coating. Very rare grains are disseminated through most of the Sherman granite, but are commoner in the coarse red granitic masses of the southern part of the batholith than in the gray granite of the northern and western parts.

In the youngest pre-Cambrian granites there are thin lenses of allanite, much bleached and discolored, which occur as partings between biotite lamellae, in addition to the fairly common reddish to greenish grains. Roof, border, and satellitic areas of the batholiths carry more allanite than do the interiors. Locally there is more of this mineral in the granite from the St. Vrain portion of the Longs Peak–St. Vrain batholith than in that from the Longs Peak area.

Titanite.—Titanite is an abundant and important constituent of the heavy mineral residues from the oldest pre-Cambrian granites. Large, brown, rugose, and broken crystals are a feature of the Boulder Creek batholith. Isolated grains occur throughout the granitic rock, but titanite is mostly intergrown with biotite and apatite. Colors of crystals range from grayish yellow in the fine-grained, light-colored dike rocks through rose red to deep seal brown in large tetrahedrons which are coated with black iron oxide. Chains and clusters of small, pale, colored grains occur in the same specimen with large yellow to brown crystals, suggesting that the two genera-

tions of titanite crystallized under diverse conditions. Large brown titanite crystals are the most common non-magnetic mineral in heavy residues from the Mount Evans batholith. The deep brown to yellow titanite is present but not common in the oldest granitic rocks of the western slope of the Front Range.

Titanite is locally abundant in the granites of intermediate age. Crystals are small and irregular, generally grayish yellow, rarely red or brown. Large brown to rose-colored double tetrahedrons occur in granites of the border phases of the Pikes Peak batholith. Titanite shows in most samples from the Sherman batholith but is generally concentrated in the dikes and in granite near bodies of schist. The grains are generally larger, have a greasier luster and a deeper color than the titanite from the Pikes Peak area.

Titanite is very rare in the youngest pre-Cambrian granites. A few gray to yellowish waxy grains are scattered in dikes and satellitic rocks of the Silver Plume and Longs Peak–St. Vrain batholiths. Along the north margin of the Log Cabin batholith greenish to brown tetrahedrons of titanite are associated with abundant apatite and ilmenite. It is believed that this titanite was developed through assimilation of the adjacent schist and of included masses of Sherman granite.

Hornblende.—Hornblende is a common heavy mineral in the femic phases of the oldest pre-Cambrian granites, particularly in the coarse gneissoid rocks of the Boulder Creek and Mount Evans batholiths. Leucocratic phases and dike rocks have none. In thin sections the crystals are large, ragged, dark blue-green and sieve-like, being poikilitically intergrown with biotite and titanite. Grains in the residues are black, glossy, and have good amphibole cleavage. On the western slope of the Front Range hornblende occurs locally with titanite in the Tabernash stock. Bright green to black hornblende occurs in scattered samples from the Pikes Peak batholith where biotite is lacking or rare. It is disseminated in small amounts through most of the Sherman batholith, but is common only near the southern and eastern margins. Grains are large, very irregular, fresh and glossy in appearance.

No hornblende has been found in any of the granite samples from the youngest pre-Cambrian batholiths.

Biotite.—Biotite is present in all the granites of the Front Range.

Abundant clusters of felted, strongly pleochroic, fresh, dark olive-green to brown flakes make up 90 per cent of the heavy mineral residues from the oldest granitic rocks. In samples which carry hornblende the biotite is usually bright green and the small rare zircons imbedded in the flakes have wide, deeply colored pleochroic halos.

Heavy mineral residues of the Pikes Peak batholith contain 85–90 per cent of fresh, dark-brown to olive-green biotite. Residues high in magnetite have rare flakes of biotite generally altered to green chlorite in the meshes of sagenite rutile webs. Dark-green to olive-green biotite is a common constituent of samples from the Sherman batholith where hornblende is rare or lacking. In general, biotite is slightly commoner and occurs in larger flakes in the Pikes Peak granite than in the Sherman.

Biotite is the commonest and most persistent dark-colored heavy mineral of the youngest granitic batholiths. The fresh, strongly pleochroic flakes flood the heavy residues and pepper the granite specimens with evenly scattered glistening patches. The biotite lamellae are locally separated by thin lenses of allanite and liberally spotted with numerous small zircon prisms which are enclosed by distinct halos. In the Log Cabin batholith the green to olive-green biotite flakes are crowded with inclusions of rutile, zircon, small apatite, allanite, and magnetite crystals.

Muscovite.—Primary muscovite is unknown in the oldest pre-Cambrian batholiths. It occurs sparingly in small flakes, probably of secondary origin, in the Pikes Peak and Sherman granites. It is intergrown with biotite in the youngest pre-Cambrian granites. In the Log Cabin batholith the flakes are so loaded with small crystals of magnetite, zircon, apatite, rutile, and allanite that they sink in bromoform and form a large part of the non-magnetic heavy residues.

Magnetite.—Magnetite is present in most of the pre-Cambrian granites. The large, fresh, glossy, silvery-black, strongly titaniferous crystals in the oldest granitic rocks are intergrown with titanite and biotite and locally altered to leucoxene. There are rare, fresh, black cubes of magnetite in the Pikes Peak granite, but most of the grains are small and dull. The magnetite of the Sherman granite occurs as

myriads of tiny cubes and octahedrons imbedded in quartz, feldspar, and micas.

Magnetite occurs sparingly in small, dull, non-titaniferous grains associated with biotite in the youngest pre-Cambrian granites. In the Log Cabin batholith most grains are altered to leucoxene and are enclosed in flakes of muscovite.

HEAVY MINERALS OF LOCAL DISTRIBUTION

Beryl.—Beryl is a common accessory mineral in the border facies of the Pikes Peak and Sherman batholiths. Glassy, colorless, much broken crystals occur sparingly along the northern border of the Pikes Peak batholith and are of common occurrence in the granite of its western margin. Granite from the southern margin of the Sherman batholith and from xenoliths of Sherman granite enclosed in the Log Cabin batholith is locally crowded with small milky-white to glassy grains of beryl.

Parts of the Log Cabin batholith are contaminated by beryl from partly assimilated Sherman granite. No beryl is known elsewhere in the youngest pre-Cambrian granites.

Tourmaline.—Tourmaline is practically unknown in the oldest granitic rocks, occurs rarely in the Pikes Peak and Sherman granites, and is locally abundant in the diaschistic rocks and pegmatites related to the youngest group of pre-Cambrian batholiths. The crystals are commonly tabular, deep green to black, much fractured and intergrown with milky quartz. Masses of radiating tourmaline needles form selvages and ribbon structures with quartz in the dikes and veins related to the Longs Peak—St. Vrain and Silver Plume batholiths.[6]

Garnet.—Garnet is not known in the oldest pre-Cambrian batholiths and is very rare in those of intermediate age. Dike and satellitic rocks associated with batholiths of youngest pre-Cambrian intrusion are peppered with small almandine crystals ranging from less than a pinhead in size to thickly clustered octahedrons 2 inches across in the coarse granite of the massive dikes on Longs Peak.

Monazite.—Monazite occurs with deep brown titanite in the femic

[6] For details of structure and crystal growth, see M. B. Fuller (M. F. Boos), "Contact Metamorphism in the Big Thompson Schist of Northern Colorado," *Amer. Jour. Sci.*, 5th ser., Vol. XI (1926), pp. 194–200, and M. F. Boos and C. M. Boos, "Granites of the Front Range: the Longs Peak–St. Vrain Batholith," *loc. cit.*, p. 324.

phases of the Boulder Creek and Mount Evans batholiths. The honey-yellow to greenish grains are strongly radioactive. Where biotite is intergrown with monazite the mica is bleached and altered and spotted with green and brown patches. Monazite occurs with beryl along the southern margin of the Sherman batholith. There are rare grains in the Log Cabin granite but none in any of the other youngest pre-Cambrian batholiths.

Riebeckite.—Riebeckite is found only in the Pikes Peak batholith on the eastern slope of Pikes Peak. The dark-blue crystals occur in broken masses with quartz in the normal Pikes Peak granite.

Fluorite.—Fluorite occurs in the pre-Cambrian granites where Tertiary porphyries have invaded the batholiths. Some of the purple to colorless fluorite which is of common occurrence in the Pikes Peak granite may be of primary origin, but most of it is secondary and related to the Tertiary mineralization of the Front Range.

Pyrite.—Pyrite occurs in rare irregular grains in the oldest granitic rocks. It is a little more common in the rocks of intermediate age, and appears sparingly in the youngest batholiths. It shows in nearly every sample of granite from the Silver Plume batholith. Like fluorite, it is related to the Tertiary mineralization of the region.

Sillimanite.—Sillimanite is unknown in the granitic rocks of the oldest batholiths and is very rare in those of intermediate age. Roofs and sidewalls of the youngest batholiths are locally full of felted masses of slender colorless sillimanite needles. The proximity of roofs and sidewalls removed by erosion may be judged from the concentration of sillimanite in exposed granite masses.

CONCLUSIONS

Primary heavy minerals of local distribution, especially those of radioactive nature, occur more commonly in the older pre-Cambrian granitic rocks than in the younger. The oldest granitic rocks are characterized by titanite-apatite-hornblende-monazite assemblages; those of intermediate age by apatite-pale titanite-zircon-allanite groups, and the youngest by a colored zircon-apatite-allanite-tourmaline association.

Many heavy minerals, such as zircon, apatite, and titanite crystallized in two generations. As a rule those of the earlier period are smaller than those formed during the later time of solidification.

ROCK RESISTANCE AND PENEPLAIN EXPRESSION

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ABSTRACT

The surface configuration of eastern Ohio and adjacent West Virginia and Kentucky is governed by varying resistance and attitude of strata which have been differentially sculptured during four cycles of erosion. The occurrence of resistant Mississippian and basal Pennsylvanian rocks between weaker rocks, in combination with the geologic structure, has produced a cuesta and lowland which dominate the landscape of eastern Ohio, and a portion of Kentucky. Because of differences in topographic expression and physiographic history the cuesta and lowland should not be included in the Kanawha section of the Appalachian Plateau.

INTRODUCTION

In a recent article[1] the writer correlated erosion surfaces in eastern and southern Ohio and adjacent areas in Pennsylvania and West Virginia with the recognized surfaces in eastern Pennsylvania and adjoining areas. Projected profiles substantiated the existence of these surfaces but the influence of rock resistance on their development and present configuration was not discussed. The present study was undertaken because these areas offer exceptional opportunity to demonstrate dependence of geomorphic forms upon structural control, through the medium of varying resistance of different strata in an area of comparatively simple geologic structure.[2] While numerous writers[3] on the physiographic forms of this area have been impressed with the rôle which rock resistance has played in the development of the present landscape, their studies have been confined to limited portions rather than the entire region.

[1] W. S. Cole, "Identification of Erosion Surfaces in Eastern and Southern Ohio," *Jour. Geol.*, Vol. XLII (1934), pp. 285–94.

[2] H. S. Sharp, "The Geomorphic Development of Central Ohio," *Denison Univ. Bull. 27* (1932), pp. 8, 9; S. Happ, "Drainage History of Southeastern Ohio and Adjacent West Virginia," *Jour. Geol.*, Vol. XLII (1934), pp. 265–67; A. M. Miller, "The Geology of Kentucky," *Ky. Geol. Sur. Bull. 11* (1919), 6th ser., pp. 229–33.

[3] J. E. Hyde, "Geology of the Camp Sherman Quadrangle," *Ohio Geol. Surv. Bull. 23* (1921), 4th ser., pp. 1–191; D. D. Condit, "Conemaugh Formation in Ohio," *Ohio Geol. Surv. Bull. 17* (1912), 4th ser., p. 156; Sharp, *op. cit.*, p. 9; Happ. *op. cit.*, pp. 273, 275.

The index map (Fig. 1) shows the area over which detailed study was made.

METHODS OF STUDY

Projected profiles (Fig. 4) were constructed across representative areas in order to gain an accurate reconstruction of the erosion surfaces. These give an excellent cross-sectional view of the various elements of the terrain.

A contour map, with an interval of one hundred feet, was constructed by plotting hilltop elevations from the topographic maps of the area (Fig. 2). A sufficient number of elevations was selected from each 5-minute division of the quadrangles to present an average of the highest hilltops. The contours were slightly generalized when drawn through these points. This map was then checked against the projected profiles. Where the projected profiles illustrate the erosion surfaces in cross section, the map indicates their areal distribution and regional relationships.

This method of constructing a maximum elevation contour map has been used by Ochocka[4] in his analysis of the relief of Poland and portions of neighboring countries. Happ[5] constructed a map of a portion of the area under discussion from projected profiles. This map agrees in all essential details with the one constructed by plotting hilltop elevations directly from the topographic sheets.

The completed contour map was then compared with a geologic map, prepared by enlarging a portion of the recent edition of the "Geologic Map of the United States" (Fig. 3). For areas where geologic folios or special state survey reports were available detailed comparison of the contour map and projected profiles with the geologic formations was made.

Three additional profiles are here presented to cover certain critical areas not illustrated in the previous article.[6]

BIDWELL-ARNOLDSBURG SECTION

This section (Fig. 4, top) is the eastward continuation of the Higginsport-Sciotoville section,[7] connecting the latter with the

[4] J. Ochocka, "Krajobraz Polski W Świetle Mapy Wysokości Względnych (Carte des hauteurs relatives de la Pologne), *Travaux Géographiques Publiés sous la Direction de E. Romer* (Lwów and Warsaw, 1931), Livraison XIII, p. 37.

[5] *Op. cit.*, p. 273. [6] Cole, *op. cit.*, pp. 285–94. [7] *Ibid.*, pp. 289–90.

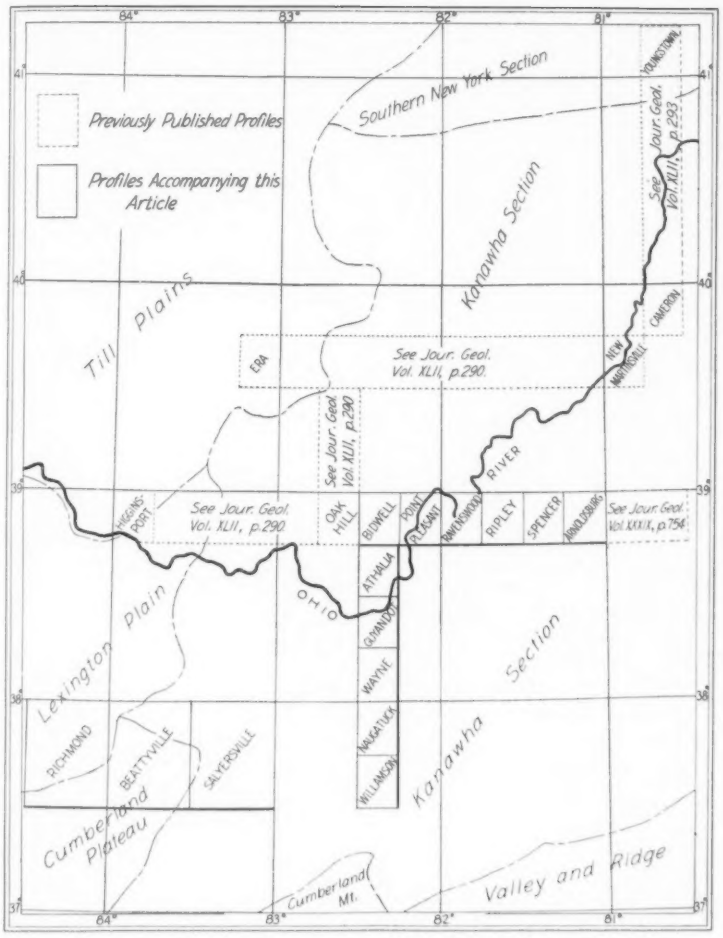

FIG. 1.—Index map. The heavy line in each section indicates the base line

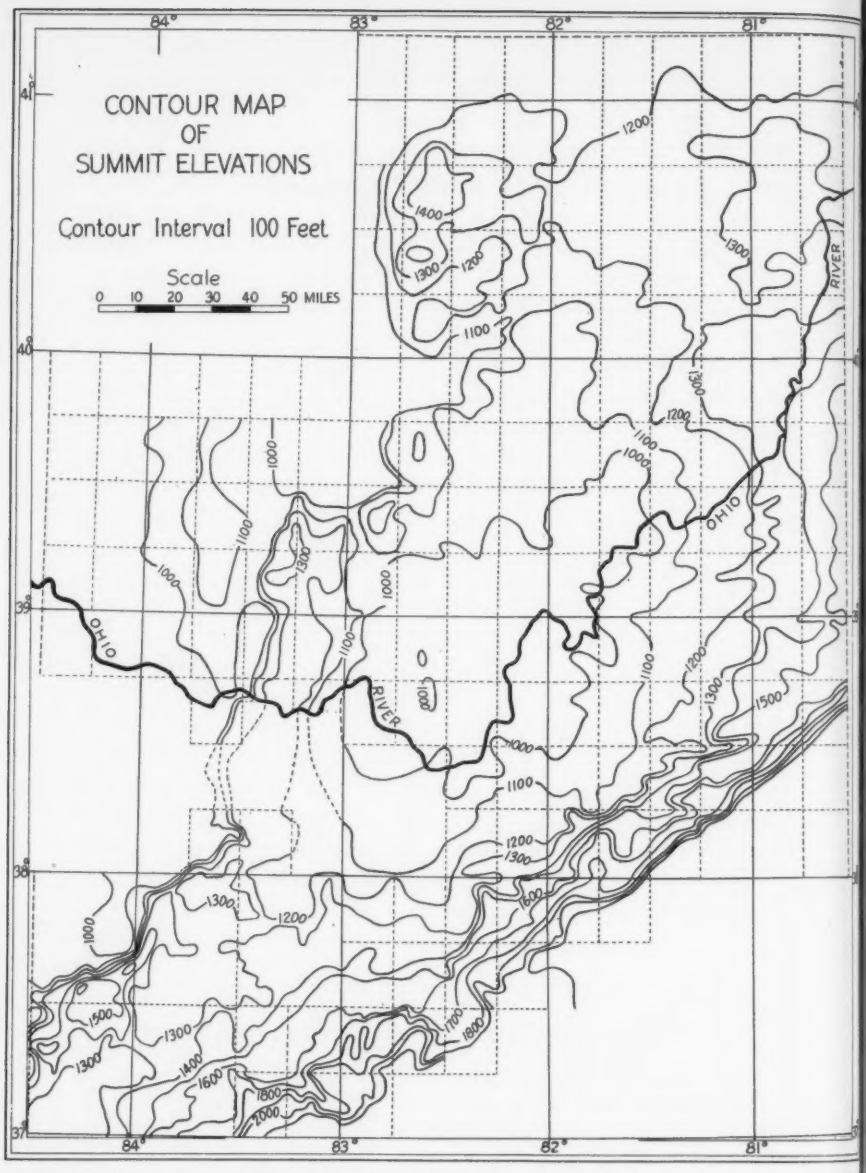

FIG. 2.—Contour map, drawn from hilltop elevations, to show the major features and areal expression of the erosion surfaces. The dotted lines indicate position of the quadrangles.

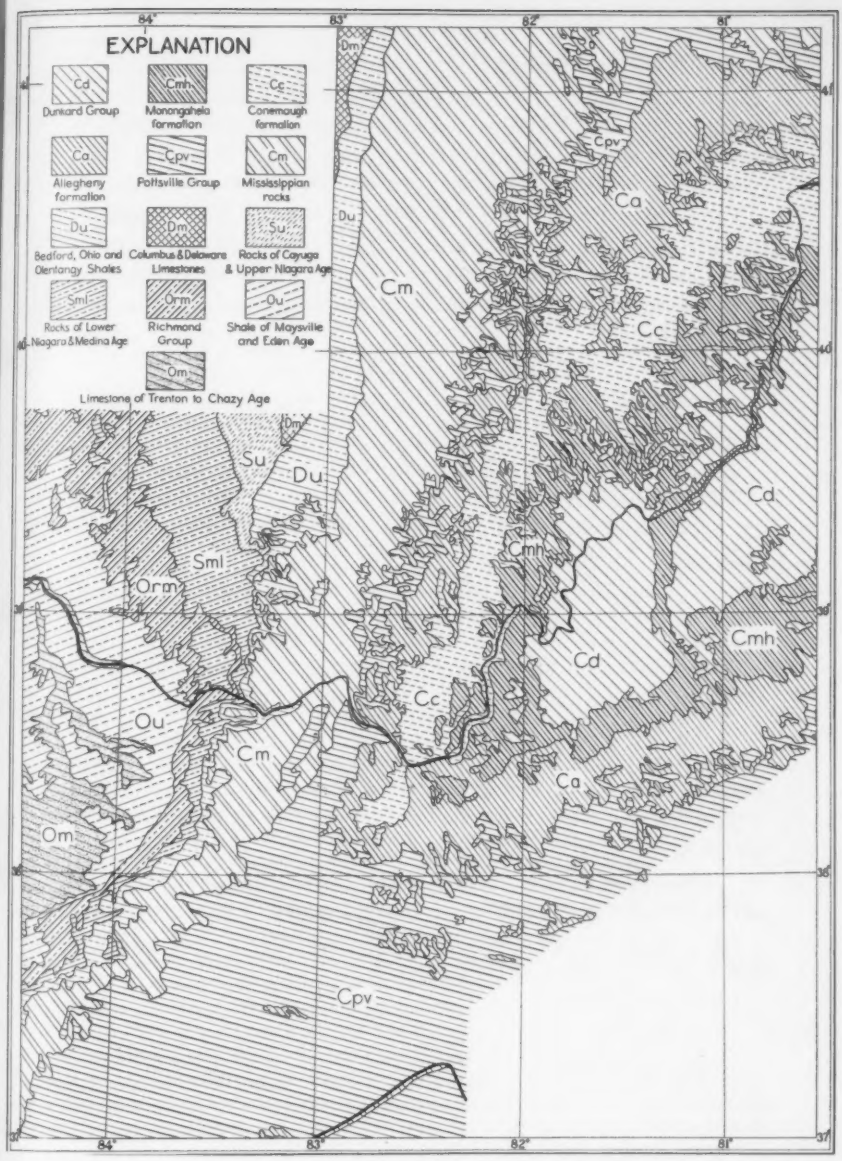

FIG. 3.—Areal geologic map

Sago-Glenville section[8] published by Fridley and Nölting. This section in its entirety, that is, from the Higginsport, Ohio, quadrangle to the Sago, West Virginia, quadrangle essentially duplicates features already discussed in connection with the combined Era–New Martinsville[9] and the New Martinsville–Frostburg sections.[10]

The break between the Allegheny and the Lexington (Worthington) surfaces is shown clearly on the projected profile of the Arnoldsburg quadrangle. This break is of the same type and magnitude as that shown on projected profiles at the western edge of the New Martinsville area.

WILLIAMSON-ATHALIA SECTION

The Lexington (Worthington) surface is present at an elevation of about 1,000 feet over the entire area of the Athalia quadrangle (Fig. 4, center) and extends southward to the Wayne quadrangle.

The Allegheny surface is poorly developed in this area and represents a gradual north-sloping surface interrupted by many monadnock areas. At the southern end of the section, the break between the Allegheny and the Upland[11] surfaces may be observed.

RICHMOND-SALYERSVILLE SECTION

The cuesta form of the topography is illustrated by the profile across the Richmond and Beattyville quadrangles (Fig. 4, bottom). This represents the southern continuation of the same cuesta seen on the Circleville, Lancaster, and Logan quadrangles and on the Peebles, Otway, and Sciotoville[12] quadrangles.

The marked lower elevations seen on the Beattyville quadrangle represent the southern continuation of the level developed at 900 to 1,000 feet on the Bidwell and Point Pleasant quadrangles to the north. On the Beattyville quadrangle these lower elevations repre-

[8] H. M. Fridley and J. P. Nölting, Jr., "Peneplains of the Appalachian Plateau," *Jour. Geol.*, Vol. XXXIX (1931), pp. 754–55.

[9] Cole, *op. cit.*, pp. 290, 291–92.

[10] *Ibid.*, pp. 750–55.

[11] This is the same surface which has been correlated with the Kittatinny by Fridley and Nölting and the writer. Because of the confusion which exists concerning the use of the names "Kittatinny" and "Schooley" it may be best for the time being to refer to this surface above the Allegheny as the Upland surface.

[12] Cole, *op. cit.*, p. 290.

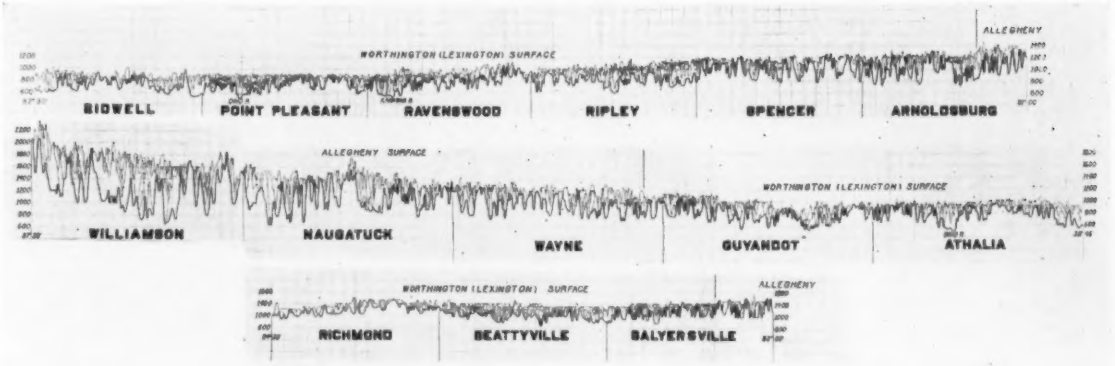

FIG. 4.—Top: projected profiles of the Bidwell-Arnoldsburg section. Base line along the southern boundary of the quadrangles. Center: projected profiles of the Athalia-Williamson section. Base line along the eastern boundary of the quadrangles. Bottom: Projected profiles of the Richmond-Salyersville section. Base line along the southern boundary of the quadrangles.

sent the less perfect development of the Worthington (Lexington surface.

The Allegheny surface is represented by the higher elevations on the eastern portion of the Salyersville quadrangle. Eastward from the Salyersville quadrangle the Allegheny surface rises rapidly toward the upland surface. The Allegheny surface in this area is in reality only a narrow bench and does not show the perfection of development that it exhibits farther north in West Virginia and Pennsylvania.

COMPARISON OF THE CONTOUR AND AREAL GEOLOGIC MAPS

The contour map (Fig. 2) indicates that the area may be divided into five distinct portions. (1) The western part is a plain of low relief. (2) This plain is bordered on the east by a broad, low ridge which separates the plain from a great depression. (3) The depression east of the ridge is best developed in southeastern Ohio where it is the major relief feature. (4) Eastward the basin gradually rises in a long slope which is continued beyond the limits of the map as an undulating upland area.[13] (5) In the southeastern corner a very steep rise to a much higher level can be observed.

A comparison of the contour map with the areal geologic map (Fig. 3) at once brings to focus the importance of structural control. The trend of the contours is identical in many places with the geologic boundaries. The most striking case shown is the correspondence of the western ridge with the area of Mississippian and basal Pennsylvanian rocks. The basin is developed mainly on rocks of Pennsylvanian age while the eastern slope is developed upon Pennsylvanian and Permian rocks.

RESISTANCE OF THE ROCKS

Rocks of Ordovician, Silurian, and Devonian age underlie the major area west of the ridge referred to above. These rocks are almost entirely limestones and shales, which act together as a unit to the forces of denudation. Physiographically, they are weak and in consequence a broad, undulating, nearly featureless plain, the Lexington surface, having about the same elevation, has been developed

[13] G. H. Ashley, "The Scenery of Pennsylvania," *Pa. Top. and Geol. Surv. Bull. G6* (1933), p. 24; H. M. Fridley and J. P. Nölting, Jr., *op. cit.*, p. 754.

on them. This surface is now maturely dissected by Parker strath and more recent valleys in the southern portion and blanketed by glacial drift in the northern.

A resistant series of rocks is encountered in the Mississippian. Such beds as the Berea sandstone, the Buena Vista sandstone, and the Logan sandstone, which is composed of several very resistant members (Byer, Allensville, and Vinton), exert important physiographic control. Their importance in the development of the western ridge in the vicinity of Chillicothe is admirably discussed by Hyde.[14] His map and sections demonstrate clearly that these beds, together with the overlying Pottsville, form the resistant cap rocks which are maintaining this cuesta upland.

With the exception of the resistant sandstones and conglomerates in the basal Pennsylvanian, the succeeding formations are weak, composed dominantly of shales and coals with thin limestones, sandstones, and sandy shales. The resistance of the rocks of the Permian series scarcely exceeds that of the Pennsylvanian, if the basal group of the latter is not considered.

HISTORY OF THE WESTERN CUESTA AND THE INTERVENING VALE

Although the cuesta character of the prominent upland across the western part of the area has been noted by several workers,[15] the map for the first time clearly presents the cuesta in its entirety.

The basin east of this cuesta corresponds to the vale developed under typical coastal plain conditions. However, its present configuration is not the result of a first cycle of denudation, but that inherited from a much later cycle, the Worthington (Lexington). Attention should be called to the fact that the basin becomes higher toward the north and south as well as toward the east and west. These higher areas entirely surrounding the basin are considered to represent the major divides during Worthington (Lexington) time.

Happ has lately verified Tight's restoration of the former drainage in this area. If Happ's map[16] of the former drainage is superimposed

[14] *Op. cit.*, p. 153.

[15] *Ibid.*, pp. 165–66; Sharp, *op. cit.*, pp. 14, 27–28; Happ, *op. cit.*, p. 273.

[16] *Op. cit.*, p. 268.

on the contour map here presented, it will be observed that the main drainage lines are concentrated within the basin area. It is reasonable to assume that the drainage system, representing the Parker stage, was directly inherited by superimposition from a broad alluvial covering on the Worthington (Lexington) surface.

The course of the master stream during Worthington time as well as during the Parker strath stage, was across the Mississippian cuesta in the vicinity of Chillicothe. The resistant rocks of this area acted as a temporary baselevel for the areas east of the cuesta. During Worthington time the wide depression resulted from subaerial denudation assisted by lateral planation of the streams concentrated within this area of weaker rock types.

After the development of the Worthington erosion surface with its broad, alluvial cover in this area, slight uplift caused the rejuvenation of the entire system. This resulted in the incision of valleys from this covering below the Worthington surface. These valley floors form the present Parker strath.

The course of the Teays valley with its long, gentle, incised-meander bends is typical of a stream in advanced maturity superimposed from an alluvial cover on the underlying rocks. This logically explains the apparent lack of structural adjustment which the Teays valley shows in certain portions of its course across the basin.

Present evidence does not warrant the restoration of the drainage system before Worthington (Lexington) time. However, it may be suggested because of the uniform northwest trend of all preglacial valleys throughout Ohio, West Virginia, and Kentucky that their general course is directly inherited from the Allegheny surface. This surface, without doubt, extended across this area before the present sculpture was attained during Worthington and later time.

Happ[17] has suggested that these systems were inherited from a former extensive sedimentary cover in the manner suggested by Johnson[18] for the Atlantic slope drainage. It seems more reasonable to assume that these streams were developed from a system on the Allegheny surface and superimposed from broad, extensive flood

[17] *Ibid.*, pp. 282–83.

[18] D. Johnson, *Stream Sculpture on the Atlantic Slope* (New York: Columbia University Press, 1931).

plains during uplift of this surface with slight arching toward the northwest away from the major axis of uplift. Both concepts, however, should be regarded for the present as working hypotheses.

CORRELATION OF THE SURFACES DEVELOPED

The correlation of the basin surface east of the cuesta has been discussed by Ver Steeg[19] and the writer.[20] It is considered the time equivalent of the typical Lexington surface west of the cuesta. The writer has maintained that the cuesta should be considered a major divide on the Lexington and of the same age as the lowland surfaces east and west of it. This view is substantiated when the resistance of the rocks comprising the ridge is considered. It has remained as a prominent topographic feature long after the less resistant rocks east and west of it had been reduced to much lower elevations.

It should be noted that both north and south the elevations of the cuesta increase until they approach the elevations of the typical Allegheny surface farther east. This increase in altitude apparently is due to the distribution of the streams which carved the Worthington (Lexington) surface. A secondary fact may be the protection afforded by increasing resistance of the cap rock. It is well known that the character of the Mississippian and basal Pennsylvanian rocks changes markedly within short lateral distances.

These higher areas may be considered but slightly reduced outliers of the Allegheny surface. Inasmuch as the more perfectly developed Worthington (Lexington) surface entirely surrounds them, they should be considered generic portions of that surface, i.e., as a portion of a major divide whose elevations are controlled by a set of fortuitous conditions.

The southern extension of the western cuesta into Kentucky illustrates a beautiful relationship between rock resistance and topographic development. The major cuesta scarp is capped either by Pottsville conglomerate and sandstone, or by a thick series of limestone of St. Louis, Ste Genevieve, and Gasper age. Where the Pottsville is absent, there are indications that it has been but recently removed. Such situations may be seen at Pilot Knob and vicinity, 5

[19] K. ver Steeg, "Erosion Surfaces of Ohio," *Pan-Amer. Geol.*, Vol. LV (1931), pp. 188, 199.

[20] *Op. cit.*, p. 289.

miles east of Berea, Kentucky, and on the main escarpment in the region east and north of Irvine, Estill County, Kentucky, where the Pottsville occurs in most instances as outliers some distance back of the main escarpment. Moreover, the limestones of this area are less susceptible to rapid removal than older formations west of the cuesta. The capping limestones of the cuesta are massive, dense, and in places extremely cherty.

In the Morehead and Salt Lick quadrangles there are numerous outlier remnants of a much lower cuesta or cuesta-like bench west of the main cuesta scarp. This lower cuesta owes its origin to the so-called Buena Vista sandstone. In the eastward retreat of the main escarpment, it was resolved into a double cuesta because of the occurrence of these resistant beds below the upper capping series. Farther south, where the Buena Vista beds are absent, only the main scarp was developed.

THE EROSIONAL BREAK BETWEEN THE ALLEGHENY AND WORTHINGTON SURFACES

The projected profiles indicate a distinct break between the Worthington (Lexington) surface and the Allegheny at the eastern margin of the vale. This break is interpreted as an erosional feature, influenced only locally by the varying resistance of the rock. Comparison of the contour map with the areal geologic maps shows that this break in many places cuts directly across rocks of different ages, but of very similar resistance.

Studies in areas of which detailed geologic maps are available emphasize this local structural control. The geologic maps of the Charleston and Huntington[21] areas indicate that the Allegheny surface is influenced by the presence of the Charleston sandstone.

The break between the Allegheny and Worthington (Lexington) surfaces appears more distinct on the projected profiles than in the field. This sharpness of the break is emphasized because the outliers of the Allegheny surface are all projected on one line. In reality, the line of juncture between the two surfaces is a very ragged and interlocking one, as would be expected in an area where a lower surface was etched from a higher surface.

[21] *U.S. Geol. Surv. Folios 69* and *72*.

RELATIONSHIP OF THE AREA TO PHYSIOGRAPHIC DIVISIONS

The area, according to the present system of classification,[22] lies within two major divisions, the Interior Plains and the Appalachian Highlands. These two divisions are subdivided into several provinces and sections. Of these the Kanawha section of the Appalachian Plateaus occupies the major portion of the area under discussion (Fig. 1).

This study has shown that the Kanawha section as now recognized is composed of two distinct parts: (1) the western portion, which comprises the dissected cuesta and lowland east of it; and (2) the eastern portion having a dissected, undulating, upland surface. The relief of the western portion is much more subdued, the elevations are distinctly lower, and the physiographic history is more complex. Thus, areas of unlike characteristics are grouped together under the present classification.

Smith[23] has recently published a map and description showing the relative relief of Ohio, which clearly illustrate the differences in relief in the areas defined above. The broad vale at the present time has a local relief of only 300–400 feet, the result of dissection of the Worthington (Lexington) surface in Parker and more recent times. To the east, the relief becomes distinctly greater at the edge of the Allegheny upland and to the west along the unglaciated sections of the cuesta upland.

While the eastern portion of the Kanawha section, representing the Allegheny erosion surface, should be classified as a section of the Appalachian plateaus, the western part shows characteristics intermediate between the Appalachian plateaus and the Interior Low plateaus.

If this area is to be considered a northern continuation of the Interior Low plateaus, this province must be extended to northeastern Ohio and thence across Pennsylvania into central New York. In New York, the area between the Portage escarpment on the south and the Niagara-Onondaga escarpment on the north would be within this province. The area of this extended province could be divided

[22] Physical Divisions of the United States (prepared by N. M. Fenneman).

[23] Guy-Harold Smith, "Relative Relief of Ohio," *Geog. Rev.*, Vol. XXV (1935), pp. 272–84.

into two sections, namely, a glaciated northern section and a non-glaciated southern section.

If, however, this area is retained in the Appalachian Plateau province, it should be made a distinct section, bordered on the east by the erosional scarp separating the Allegheny surface from the lower Worthington (Lexington) surface, on the north by the glacial boundary, and on the west by the present boundary of the Appalachian plateaus.

The first classification seems to the writer more appropriate in view of the history of this region. However, both suggestions are tentative as the writer feels that others, personally acquainted with this region, should be given the opportunity of expressing their ideas before a final reorganization of the physical divisions of this area is undertaken.

SUMMARY

Eastern Ohio and adjacent areas in West Virginia and Kentucky show a striking relationship of the terrain to the varying resistance of the underlying rocks. The resistant Mississippian rocks have caused the development of the major prominence of this area, a dissected cuesta extending northeast, southwest across Ohio and northern Kentucky. Less resistant rocks east and west of this cuesta have given rise to extensive plains.

The break between the Allegheny and Worthington (Lexington) surfaces is a very irregular one, locally influenced by rock resistance, but in the main cutting across rocks of the same resistance.

The cuesta and eastern depression have characteristics and a physical history which warrant separating them from the Kanawha section of the Appalachian plateaus with which they are now classified. Whether this area should be classified as a separate section of the Appalachian plateaus, or should be considered a portion of the Interior Low plateaus, is still open to debate.

ACKNOWLEDGMENTS.—The writer acknowledges his indebtedness to Dr. Guy-Harold Smith for reading this manuscript and to Dr. Paris B. Stockdale for valuable stratigraphic data.

A MOUNTED SKELETON OF THE MAMMAL-LIKE REPTILE *DICYNODON TIGRICEPS*

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ABSTRACT

A skeleton of *Dicynodon tigriceps* from the Upper Permian of the Murraysburg District, Cape Province, South Africa, has been mounted in the Walker Museum of the University of Chicago. The specimen is briefly described in the present paper. The large dicynodonts, which were prey of contemporary large carnivores, appear to have lived in a region of moderately warm, moist climate.

An almost complete skeleton of *Dicynodon tigriceps* was discovered by Dr. A. S. Romer during a trip to South Africa undertaken for the University of Chicago by him and Mr. P. C. Miller in 1929. This specimen has since been mounted by Mr. Miller and placed on exhibition in the Walker Museum of the University of Chicago. It is the first skeleton of a large dicynodont to be mounted in any museum, although mounts of certain smaller species are to be found in other institutions.

This fine skeleton was found about 20 miles northwest of Murraysburg, in the Murraysburg district of Cape Province, a region which proved to be the richest visited during the trip. The horizon in which the specimen occurred is the Murraysburg division of the Upper Permian. In the same beds fragmentary remains of other dicynodonts, a gorganopsian, and the skull of the rare diapsid reptile *Youngina* were found.

The matrix in which the specimen was embedded is an extremely hard, compact, dark-gray shale. The tail of the animal was weathered out and partially destroyed; a small part of the base of the pelvic girdle was exposed so that the bone on either side of the puboischio symphysis was lost. The rest of the skeleton is almost complete except for the feet which were only partially preserved.

The skull of the animal, although slightly flattened, is very nearly complete. It is marked by a very large, lateral temporal fenestra which is separated from the orbit only by a narrow bar composed of

the postorbital bone. The small pineal opening is completely surrounded by the preparietal bone which forms a slightly raised elliptical area on the dorsal surface of the skull between the temporal openings. The nasal bones are broad and heavy and overhang the nostrils. No teeth were ever developed in the jaws, although in the type specimen of the species two tusk-like canines are present. Broom has cited evidence which indicates that the presence or absence of canine teeth in otherwise identical forms is probably a

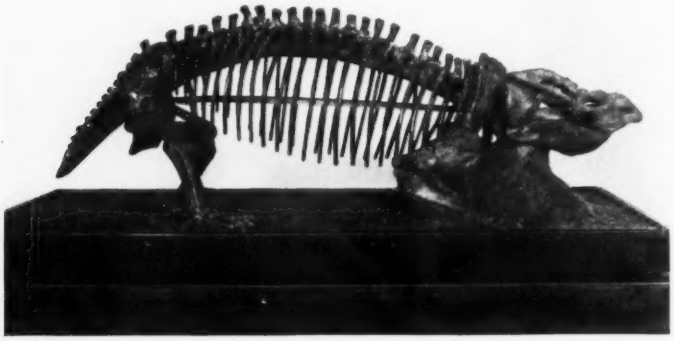

FIG. 1.—Lateral aspect of skeleton of *Dicynodon tigriceps*. Length of original about 6½ feet.

secondary sexual character, the males presumably possessing the tusks.

The vertebral column is composed of 26 presacral vertebrae, 5 sacrals, and about 15 caudals. It is mounted in such a way that the vertebrae midway between the anterior and posterior extremities form the apex of an arch which slopes fore and aft to bring the skull and the tip of the tail close to the ground. This appears to have been the condition which existed in the living animal. No sharp line of demarcation is visible between the dorsal and cervical and dorsal and lumbar vertebrae in the mounted specimen. The dorsal part of the atlas is composed of a pair of elements which forms the arch; the dorsal part of the axis has a narrow arch and a broad but short spine. The central and intercentral elements of these vertebrae can-

not be made out in the mounted specimen. The remaining presacral vertebrae are all very much alike, with the possible exception of the first five or six on which the facets for the articulation of the ribs are not clear. The neural spines of all the dorsal vertebrae are about 2 inches long, being short in comparison to the size of the animal. The 5 sacral vertebrae are similar to the dorsals except for the transverse processes which are large and fused to the ribs. The centra of the first 8 caudal vertebrae are preserved. A rapid decrease in their size posteriorly indicates that the tail was composed of not more than 15 segments. Thirteen are represented in the mounted skeleton, the last 5 being restored.

Several complete ribs and many nearly complete ones were found associated with the spinal column. Free ribs are present on all the presacral vertebrae with the possible exception of the atlas for which no rib was found. They are all relatively long, the maximum length being reached on about the fifteenth vertebra. Even the most anterior cervical and the most posterior dorsal ribs are over one-half the length of the longest dorsal one. The arched curve of the spinal column brings the distal ends of most of the ribs into approximately the same horizontal plane, giving the trunk a barrel-like appearance. The five sacral ribs are very heavy and are indistinguishably fused to the transverse processes of the vertebrae. All the caudal ribs appear to have been fixed.

The limbs have been mounted in the position which they must have assumed when the animal was walking. The shoulder and pelvic girdles and the long elements of the limbs are complete except for the pubo-ischio symphysis of the pelvis. In the shoulder girdle there is a well-developed, spatula-like scapula, a coracoid and precoracoid, a large clavicle, and a moderately large interclavicle. The pelvic girdle has an expanded ilium to which the sacral ribs attach, and a comparatively small pubis and ischium.

The limbs in the dicynodonts are partially rotated from the primitive position, in which they project straight out to either side, so that the knee points forward and the elbow points somewhat backward. In *Dicynodon tigriceps* the rotation is much more pronounced in the hind limb than in the front. The tendency of this rotation is to bring the limbs under the body and to make for efficient locomo-

tion. The limb bones of the large dicynodonts are relatively short and heavy and possess strong processes for the insertions of muscles. Only a few of the carpals, tarsals, and more distal elements of the feet were found associated with the specimen of *Dicynodon tigriceps;* the remaining parts have been restored from other specimens.

The general outlines of the skeleton, the indications of a heavy limb musculature, and the toothless condition of the jaws which almost certainly terminated in a horny beak, suggest that the large dicynodonts were sluggish, herbivorous forms which lived in areas of abundant, soft vegetation. The climate of the region must have been radically different from that which today can support only a meager flora. However, very few plant fossils are found in the beds in which the vertebrate remains occur, and, consequently, it is difficult to picture the precise conditions under which the Upper Permian fauna existed. But it may be confidently inferred that the region was one of moderate to warm temperatures the year round and of sufficient rainfall to support a moderately luxuriant flora.

The dicynodonts must have been entirely defenseless against attacks of the contemporary large carnivorous reptiles such as the gorganopsians, unless, as seems highly improbable, the tusks which were present in the males were a means of defense. They may possibly have sought protection in swamps and other localities in which a heavy growth of vegetation would render them inconspicuous. But it can hardly be doubted that these large, rather clumsy, slow-witted herbivores furnished a major source of food for both the true carnivores and the carrion feeders of their times. It is Broom's opinion that the destruction of these forms by the carnivores accounts for the rare occurrence of complete skeletons which makes the one now mounted in the Walker Museum a particularly valuable addition to the collection of dicynodonts.

PRELIMINARY NOTE ON A PHYSICAL PHENOMENON RESEMBLING MOUNTAIN-BUILDING

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Lima, Peru

INTRODUCTION

The black waterproof covering of a cinema screen, 1.25 m. square, became so sticky that it could not be rolled, and was hung vertically in a simple U-fold with the coating outside. Under its own weight the coating developed a viscous flow and began to drop off the end of the screen. The coating consisted of a brilliant black viscous underlayer and a dry, dull-black outer surface. The flow was in the lower viscous layer, and developed in the outer layer folds and wrinkles that resembled miniature mountain ranges. Owing to differential movement in the substance, horizontal and vertical tears developed in the outer layer simulating faults in the earth's crust and exposing the underlying viscous layer. Locally the effect was of an "ice pack" of black ice in a black sea. The folds and faults were most abundant near the folded bottom of the screen and decreased upward. Figure 1 shows the phenomena near the bottom and the dripping of the viscous material from the bottom of the folded screen.

THE BLACK COATING

The black waterproof covering was examined by Dr. G. Fester, professor of the Faculty of Industrial and Agricultural Chemistry of Santa Fé, Argentina, and was found to consist of linoxine impregnated with lamp black instead of rubber. Linoxine is a product of polimerization and oxidation of linseed oil and is a solid colloid. If the lacquer is blown with air instead of boiled, it subsequently decomposes, forming lighter organic acids that are liquid and sticky. In this viscous medium the lamp black rises to the surface and forms the dry, dull-black coating.

The viscous layer penetrates between the threads of the muslin cloth of the screen, which are horizontal and vertical and number 96 by 96 to the square inch.

[1] Secretary of the Geological Society of Peru.

THE FLOW

Because of the differences in the thickness of the viscous coating and the degree of its decomposition, the rate and direction of flow

FIG. 1.—Structures produced in decomposed coating of cinema screen

varied, but always with a downward component due to gravity. The adherence of the base to the muslin also caused drag. These differential forces gave rise to the folds and fractures illustrated in Figure 1.

The average daily rate of flow of the surface over a period of 67 days in a room of average temperature of 24° C. was 0.26 mm. The rate of flow was determined by measuring from pins stuck in the screen as markers. The folds bent around the pins in semicircular arcs, owing to a retardation of one-third in the velocity of the flow at the pins. The outer layer generally flowed past the pin without fracturing, showing that it was in a state transitional between solid and liquid. The real distinction between the dull outer layer and the underlying layer is that the outer layer has many more lamp-black particles imbedded in the viscous material than the underlying—a purely physical or mechanical difference which makes it less plastic.

FOLDING

The folds are more numerous and accentuated where there is an obstacle to flow, and they decrease in abundance and become flatter with increasing distance above the obstacle. This phenomenon is seen in Figure 1, in which the folds are more abundant and higher as the flow is retarded by the accumulation of the material at the bottom of the screen.

The amplitude of the folds is generally one-third to one-fourth the thickness of the coating, 0.5–0.4 mm., but in some of the very sharp folds amounts to 1.6 mm., simulating true anticlinoria.

The pattern of the folds reflects the flow of the substratum and the obstacles to that flow.

FAULTING

When differential flow gives rise to tension that exceeds the limit of resistance, true dislocations or faults are produced. They are mainly tension or tear faults parallel to the direction of flow or gravity. Compression faults are rare and occur only in the strongly folded zones.

GEOLOGIC APPLICATIONS

There is a striking similarity between these systems of folds and the folds of the great mountain systems of the world, as is evident by comparing Figure 1 with tectonic maps of the Himalayas, Andes, Appalachians, and Alps. Anticlines, synclines, domes, cuvettes, overthrust sheets, amygdaloidal folds, echelon folds, parallel folds, virgations, and many other tectonic features are common to the screen and to the terrestrial crust.

Just as the earth is regarded by some geologists as consisting of three concentric layers—an interior and exterior solid and an intermediate liquid—so the screen consists of an interior solid layer, the muslin; a viscous substratum; and the nearly solid, exterior dull-black layer.

The mandibular theory of Suess exacts an improbable rigidity and competence in crustal blocks of sial. The compressive forces of a "foreland" can operate over only short distances. It is improbable that they can cause uniformly folded zones with a breadth of hundreds, and even thousands, of kilometers when the thickness of the crust is thought to be only a few kilometers.

According to the phenomena of the screen, a theory of different rotational velocities of the telluric spheres appears more probable. Dragging, due to the irregularities of the surfaces of contact between them, disturbs the equilibrium of the solid spheres, resulting in folds and faults in the external or crustal one.[2] The plutonic earthquakes,[3] with foci at depths of hundreds of kilometers, appear to indicate a drag against the surface of the nuclear solid sphere that may correspond to that between the viscous liquid and the muslin base of the screen. Experimental geology may thus throw light on the physics of the globe and clear up many obscure points in the internal structure of the crust.

While making these observations, I received the excellent work of Professor H. S. Summers.[4] In describing recent experiments on materials with the properties of sial and sima he recounts the experiences of J. S. Mann with substances similar to those on the screen and the failure of his experiments. These experiments were more successful. Without resorting to expedients that impaired the similarity of the laboratory phenomena to the terrestrial, we reproduced almost perfectly the natural structures.

[2] Arthur Holmes, "Radioactivity and Earth Movements," *Trans. Geol. Soc. Glasgow*, Vol. XVII, Part III (1928–31), pp. 559–606.

[3] James B. Macelwane, "Plutonic Earthquakes," *Natl. Research Council Bull. 90*, "Physics of the Earth," Part VI (1933), *Seismology*, pp. 32–36; G. J. Brunner, "The Earthquake of September 6, 1933, and Its Bearing on the Problem of the Deep Earthquake," *Trans. Amer. Geophysical Union 15th Ann. Meeting* (1934), pp. 72–77.

[4] H. S. Summers, "Experimental Tectonic Geology," *Rept. 21st Meeting Australian and N.Z. Assoc. for the Advancement of Sci.* (1932), pp. 49–75.

STUDIES FOR STUDENTS

NORMAL AND REVERSE FAULTS

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McGill University

ABSTRACT

Much confusion arises from the current use of "normal" and "reverse" to describe faults. The sources of confusion are discussed in this article. Suggestions are offered which, if generally adopted, will leave no room for doubt as to the meaning intended when these terms are used in descriptions of faults. At the same time their usefulness will not be restricted.

The terms "normal" and "reverse" have been used in describing faults for so long that one would expect their meanings to be thoroughly standardized and clear-cut by now. Many geologists assume that this is the case. Unfortunately it is not. In the following paragraphs an attempt will be made to expose fully the sources of confusion. It is felt that when these are more generally understood there should be no difficulty in gaining adoption for definitions which leave no room for misunderstanding.

If a "plane" rock structure (bedding plane, dike wall, vein wall, etc.) is cut by a fault, it meets the fault in two parallel straight lines, which, unless the movement has been exactly parallel to them, become separated as a result of the fault movement. These two lines will be referred to as the hanging-wall and foot-wall traces of the plane. In Figure 1, *GH* and *EF* are such traces, *EG* is the slip, *KG* the dip slip, *EK* the strike slip. The distance between the two traces *GH* and *EF*, measured along the fault dip will be referred to as the dip separation.[1] In Figure 1 this is *MG*.

By referring to Figures 2, 3, 4, 5, and 6 it will be seen that the dip separation is identical with the dip slip in the following cases: (1) where the disrupted plane is horizontal; (2) where the strike of the fault is parallel to the strike of the disrupted plane (strike fault); (3)

[1] "Dip separation" is used here in place of "apparent slip," which has been used by the Committee on Fault Nomenclature of the Geological Society of America. The latter is not specific. It has been used for a similar measurement in oblique sections. This use of dip separation is quite in accord with the Committee's definition for *separation*.

where the net movement is equivalent to a simple dip slip (dip-slip fault).

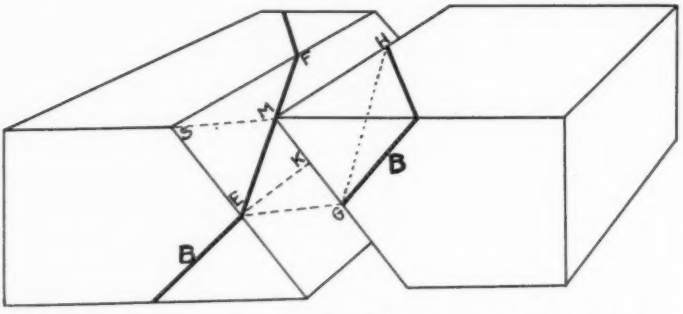

FIG. 1.—An oblique-slip fault with strike oblique to that of the inclined beds cut (oblique-slip oblique fault). *EF* and *GH* are foot-wall and hanging-wall traces of bed *B*. *EG* is the slip; *KG* the dip slip; *EK* the strike slip; *MG* the dip separation. Note the difference between the dip slip and the dip separation.

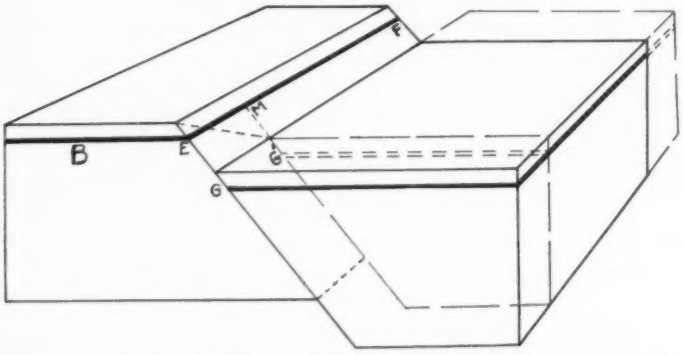

FIG. 2.—A normal dip-slip fault cutting horizontal strata. *EG* is the dip separation and is equal to the slip. A strike-slip component of any magnitude may be added without changing the dip separation. This is illustrated by the block shown in dashed outline. *EG'* is the slip. *MG* is the dip separation, which is equal to the dip component of the slip (i.e., = the dip slip).

Under any other circumstances, the dip separation does not agree in magnitude with the dip slip and it may even happen that the two traces appear in cross sections to have moved with a dip component

in the reverse sense from that of the actual movement between the fault walls (Fig. 5). Confusion in the use of normal and reverse

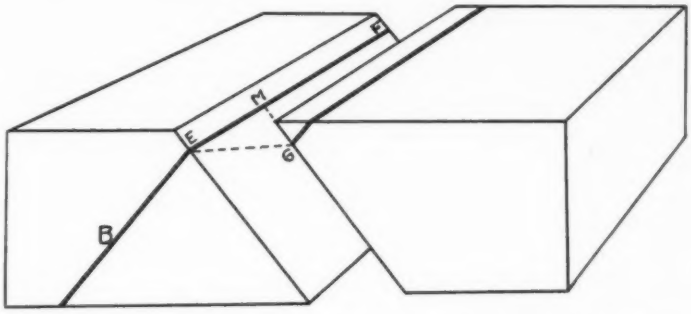

FIG. 3.—An oblique-slip strike fault. As in Figure 2, the dip separation *MG* is equal to the dip slip. *This is true for any plane whose traces on the fault are horizontal.* The dip separation of such a plane, therefore, gives a correct view of the dip component of the fault movement regardless of the dip of the plane or the direction of the slip.

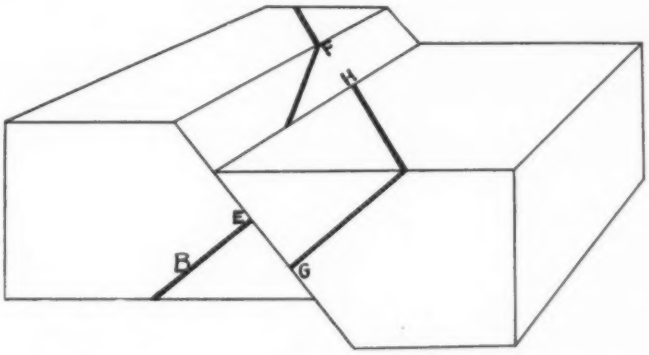

FIG. 4.—A dip-slip oblique fault. The dip separation *EG* is equal to the slip. *The dip separation of any surface cut by a dip-slip fault gives a correct view of the slip which created it.*

arose at first from failure to distinguish clearly between the dip slip and the dip separation.

Ransome called attention to this difficulty in 1907. A committee

of the Geological Society of America considered the problem and, in 1912, recommended that the terms "normal" and "reverse" should

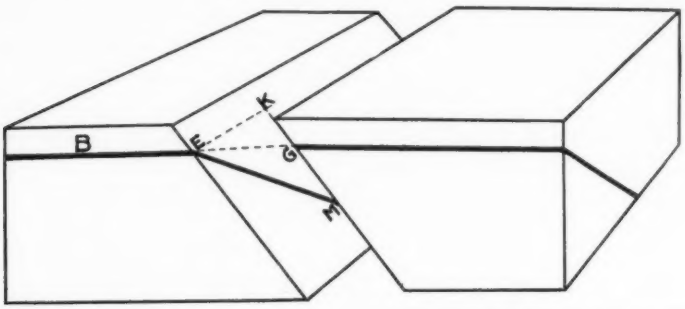

FIG. 5.—An oblique-slip dip fault. *MG*, the dip separation, is not equal to the dip slip *KG*. Note that, while the hanging wall has moved downward relative to the foot wall, the hanging-wall fault trace of plane *B* appears above the foot-wall trace in any dip section. Following the terminology recommended in a later part of this paper, this is a normal fault, but the plane *B* shows a reverse dip separation.

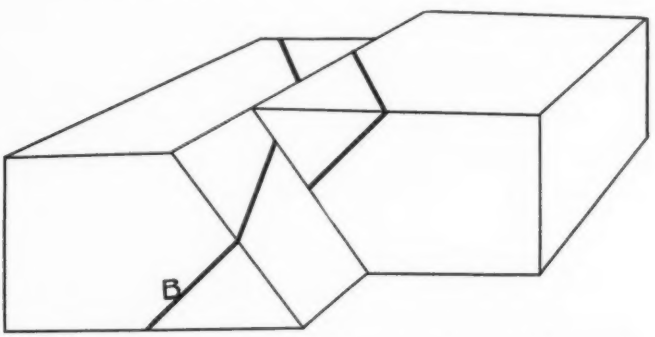

FIG. 6.—A strike-slip oblique fault which has produced a normal dip separation in bed *B*. Following the G.S.A. committee's nomenclature, this would be a normal fault.

be used to refer to the apparent displacement of beds as seen in the dip plane of the fault. Their statement is in part as follows:

Strike faults have usually been treated as if they were also dip-slip faults and classified into

Normal faults, where the hanging wall has been depressed relatively to the footwall.

Reverse faults, where the hanging wall has been raised relatively to the foot wall.

Vertical faults, where the dip is 90°.

The relative displacement has usually been determined by means of a dislocated bed. . . . .

The expression "normal" and "reverse" may be used in connection with oblique and dip faults, even when they are strike-slip or oblique-slip faults, provided that they are applied to designate the apparent relative displacement of the two parts of a dislocated stratum, or other recognized surface, in a vertical plane at right angles to the fault strike.[2]

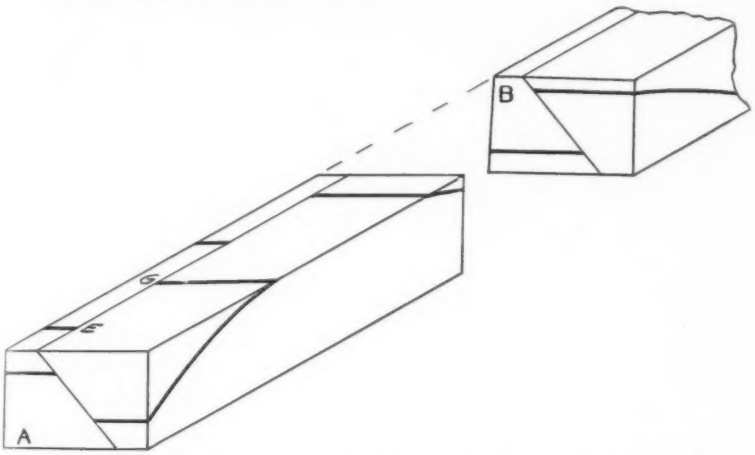

FIG. 7.—A strike-slip dip fault with slip *EG*, cuts a folded series. According to the G.S.A. committee's definition the fault is normal at *A*, reverse at *B*, but the movement is the same throughout.

These recommendations have been followed by writers of some of the more recent textbooks of economic and structural geology.[3]

In the writer's opinion the committee's decision with reference to oblique and dip faults was unfortunate. One important point was

[2] *Geol. Soc. Amer. Bull. 24* (1913), p. 177.

[3] W. Lindgren, *Mineral Deposits* (McGraw-Hill, 1933), p. 140; Ries and Watson, *Engineering Geology* (John Wiley & Son, 1925), p. 182; W. H. Emmons, *Principles of Economic Geology* (McGraw-Hill, 1918), p. 106; F. H. Lahee, *Field Geology* (McGraw-Hill, 1931), p. 204. Similar ideas appear in *Structural Geology* by C. K. Leith (Henry Holt & Co., 1923), p. 68.

overlooked, namely, that changes in the attitudes of rocks cut by a fault occur, not uncommonly, within short distances along the strike of the fault or down its dip. Because of this, two cross sections of the same translational fault, separated by only a few hundred feet, may show opposite apparent displacements of the same bed (Fig. 7). Different formations with different attitudes, in contact because of faulting or the presence of an angular unconformity, may appear to

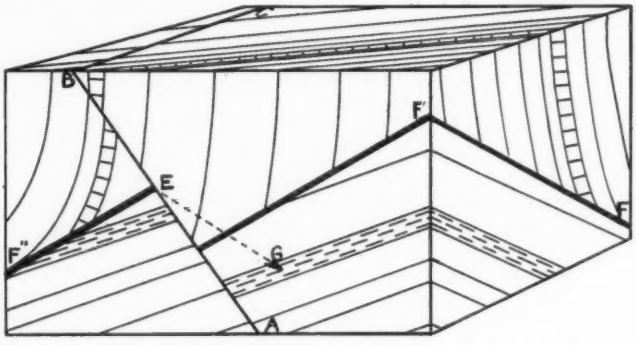

FIG. 8.—An oblique-slip fault *ABC* cutting two series of sediments brought together along a thrust fault *FF'EF"*. Following the committee's recommendations, the fault is normal if one views the apparent displacements in the lower series; it is reverse if one views the apparent displacements in the upper series. The relations along *ABC* are the result of a single movement represented in direction and magnitude by the line *EG* on the fault plane, the hanging wall having moved in the direction of the arrow, relative to the foot wall. According to definitions presented in this paper, *ABC* is a normal fault. Members of the lower series and the fault *FF'F"* show normal dip separations. Members of the upper series show reverse dip separation.

be displaced in opposite directions in the same cross section (Fig. 8). Lastly, where veins or dikes are involved, it is common to find apparent displacements in opposite directions (Fig. 9).

It should be obvious that in any one of the cases illustrated the fault is both normal and reverse, following the committee's definitions. The confusion arises because a feature which may be only of local significance has been used as a basis for a general description of the fault.

If normal and reverse, when applied to a fault in a general sense,

are always used to refer to the true relative movement between the fault walls, no confusion can result.

The usefulness of the terms "normal" and "reverse" may be further extended by using them to qualify the dip separation. Thus, in the case represented by Figure 9, one might say vein *A* shows a normal dip separation, since in the dip plane of the fault its hanging-wall trace lies below its foot-wall trace. Vein *B*, on the other hand, shows a reverse dip separation. These statements are clear and, if the vein attitudes are given, useful information is imparted.

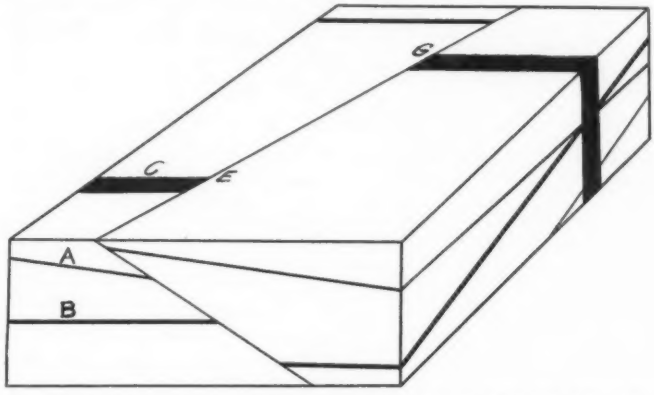

FIG. 9.—*A* and *B* are intersecting veins or dikes cut by a strike-slip oblique fault. *EG* is the slip which produced a normal dip separation in *B* and a reverse dip separation in *A*.

That it is not rare for opposite apparent displacements to result from the same movement can be seen from the following considerations. Where faults traverse areas of deformed rocks the traces on the fault of strata cut are, if anything, more likely to be inclined than horizontal. If the trace is steeply inclined to the fault strike, a normal dip slip (a relative movement of the hanging wall downward) may produce a reverse dip separation (hanging-wall trace above) or vice versa for quite a range of oblique movements. In Figure 10 the foot-wall face of a fault is shown. The foot-wall traces of two plane-like bodies *A* and *B*, cut by the fault, appear as full lines. The corresponding hanging-wall traces are shown by dashed lines. The

arrow OO' represents in magnitude and direction the movement of the hanging wall relative to the foot wall and will be referred to as the hanging-wall slip vector. It will be seen from the diagram that, as a result of the oblique slip represented (hanging wall obliquely downward), any vertical section at right angles to the strike (e.g., through BE) would show B's hanging-wall trace to lie below its foot-wall trace—in other words, B shows a normal dip separation. As a result

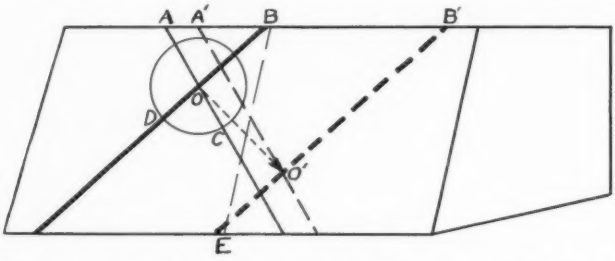

FIG. 10.—A and B are the foot-wall traces of two veins cut by a fault. A' and B' are the corresponding hanging-wall traces. The slip is OO', the arrow being the hanging-wall slip vector. For the case diagrammed, A shows a reverse dip separation, B a normal dip separation, as can be seen by observing the relations between the traces in any dip section across the fault, such as through BE.

of the same movement A shows a reverse dip separation. With veins as shown in Figure 9 the dip separations would be as follows:

If the hanging-wall slip vector from O lies in

1. Angle AOB, dip separations are both reverse
2. Angle DOC, dip separations are both normal
3. Angle AOD, dip separations are A normal, B reverse
4. Angle BOC, dip separations are A reverse, B normal

From the foregoing considerations it should be apparent that the dip separation of a single surface (or a parallel series) can be relied on as evidence of the upward or downward movement of the hanging wall relative to the foot wall only where the traces on the fault of the displaced surface observed are horizontal. How, then, can the true movement be determined in other instances?

If the line of movement between the walls can be determined from striae, fracture cleavage, tension cracks, etc., observation of the apparent displacement of one panel makes it possible to work out the

true movement. This assumes a single movement on the fault. Care should be taken to look for evidence of more than one movement.

If the dip or strike separations of two or more planes with different attitudes are known, the true movement can be worked out.

Unless the true movement is known from evidence of the kinds mentioned above, or from surface features in the case of recent faults, the fault should not be described as normal or reverse.

To summarize, if "normal" or "reverse" are used in accordance with the following definitions, there should be no misunderstanding.

A *normal fault* is one at which the hanging wall has been depressed relative to the foot wall.

A *reverse fault* is one at which the hanging wall has been raised relative to the foot wall.

These definitions are in accord with the ideas held by many geologists as to the meanings of *normal fault* and *reverse fault*, but much greater care should be exercised in applying them to oblique and dip faults, than has been done in the past. The true direction of movement of one wall relative to the other must be known before they can be used correctly to describe such faults in a general sense.[4]

It is suggested that where the true movement cannot be determined, but the apparent displacement of a surface has been observed in a cross section of the fault, the terms "normal" or "reverse" may be used to qualify the dip separation, indicating the relative positions of the two segments according to the following definitions:

A *faulted body* shows a *normal dip separation* when, in the dip plane of the fault, its hanging-wall trace lies below its foot-wall trace.

A *faulted body* shows a *reverse dip separation* when, in the dip plane of the fault, its hanging-wall trace lies above its foot-wall trace.

Finally, the writer would urge that these terms be stripped of all implications as to the type of stress which produced the fault. It should also be emphasized that the movements referred to are entirely relative as between the two fault walls. Nothing is implied as to the movement of either wall in relation to any outside datum of reference.

[4] Even in the case of rotational faults there should be no confusion in using "normal" or "reverse" as recommended here, so long as the rotational character of the fault is noted. The reader will then understand that there is a possibility of a reversal of the relative movement at a distance from the point of observation.

REVIEWS

Metamorphism: A Study of the Transformations of Rock Masses. By ALFRED HARKER. London: Methuen & Co., 1932. Pp. ix+360; figs. 185. $5.90.

A really thorough review of this excellent and timely book would call for many pages. This reviewer is especially impressed with the great strides, as reflected in Dr. Harker's book, made in the study of metamorphism since the appearance of Van Hise's treatise. Van Hise dealt with all kinds of secondary changes in rocks, whereas Harker restricts himself at the outset to deep metamorphism. The approaches also differ: Van Hise, compelled by the need of the pioneer, sought to generalize; Harker is largely empirical. Van Hise's was essentially a field and structural treatise and his chemistry simple and deductive, his conclusions strongly colored by his field observations, and thus necessarily limited; Harker is thoroughly conversant with the newer chemistry, his outlook is primarily petrographic, and his examples are derived in large part from a careful perusal of the works of other petrologists. The excellent *Metamorphic Geology* by Leith and Mead lies midway between the treatises of Van Hise and Harker as to most of these particulars.

The greater part of the book deals with two general types of metamorphism—dynamic and thermal—and thus does not recognize in separate category the "load metamorphism" distinguished by some geologists. Under each class the fundamental process is first discussed; this is followed by a detailed consideration of the characteristic structures and of the products to be expected from each type of sedimentary and igneous rock. In this section numerous examples, especially valuable to those not familiar with the details of British and continental European metamorphic geology, are cited and many thin sections are figured as illustrations.

To speak of Dr. Harker as a petrologist is to indicate one of the few weaknesses in the book—an underemphasis of the concepts now current among structural geologists. His analysis of metamorphosing stresses (pages 137–45) is lucid and practical but perhaps too greatly simplified. Moreover, one looks in vain among his footnotes for such familiar names as Cloos, Born, and Sander, not to mention such older writers as Becker (on cleavage), Dale (on slate), and the like.

When this has been said all remaining adverse criticism refers to details

and can be balanced by commendable features of greater importance. Thus, the absence of photomicrographs (dictated by economy, no doubt) is a shortcoming, for the camera is an honest eye and gives a clue to problems not clarified by the more subjective drawings with a camera lucida. Yet the numerous excellent sketches, well discussed in the text, are invaluable in affording plentiful examples.

The author offers cogent reasons for preferring the term "thermal metamorphism" to "contact metamorphism." His discussion of pressure as an agent in metamorphism is inspiringly fresh in emphasizing, more than is customary in this connection, the fact that great stresses are still vectorial, despite the recognition that rocks undergoing metamorphism, chemically considered, may be regarded as homogeneous liquids. The concept of chemical equilibrium is generally successfully applied, though in some instances with slight errors. Thus the reader may contend that Dr. Harker has slightly overemphasized the practically irreversible nature of metamorphic changes (page 11).

The interpretation of spotted argillites (*Knotenschiefer*) as essentially made up of glass resulting from metamorphism (pages 14–16) is excellent. So, also, is the emphasis upon the localization of metamorphic changes in the rock mass as a whole.

Perhaps the finest part of the entire book is the first chapter. This is somewhat confusingly entitled "Thermal Metamorphism: General Considerations," whereas it should more properly have been placed in an entirely distinct section under a title such as "Metamorphism: General Considerations," since it lays the groundwork for both types of metamorphism and logically prepares for the rest of the text.

From first to last this work is full of ideas, partly entirely new, partly newly applied or illuminatingly expressed. It far transcends a text in content and style, and is such a work as no scientist who wishes to consider the subject beyond the elementary stages can afford to neglect.

CHARLES H. BEHRE, JR.

Deutsches Erdöl, II Folge. By AUGUST MOOS, H. STEINBRECHER, and O. STUTZER. Publication No. 9 of the Gebiet der Brennstoff-Geologie, edited by Otto Stutzer. Stuttgart: Ferdinand Enke, 1934. Pp. 98; figs. 8. Rm. 9.80.

Of the three sections in this publication, the first is by H. Steinbrecher and O. Stutzer. Here are found brief discussions of the chemical and physical properties of various German petroleums, with tabulation of the oils according to source. Comparisons are made by means of graphs.

The second and largest section of the book contains detailed geologic information secured from wells in the Rhine Valley in the vicinity of Bruchsal. The data given here are largely stratigraphic in character and are in part presented in the form of quite detailed well logs. Several pages are given to structural geology and to the character and occurrence of the petroleum.

The last section is a symposium, edited by O. Stutzer, devoted to the question of the source beds of the oil produced in Hanover. Twenty-four European and American geologists present their ideas as to whether the oil now found in Mesozoic sediments originated in beds of that age or in the Zechstein.

EDWARD L. TULLIS

Dip and Strike Problems: Mathematically Surveyed. By KENNETH W. EARLE. London: Thomas Murby & Co., 1934. Pp. x+125; figs. 122; pl. 1. 12*s.* 6*d.* net.

In his Preface, the author states that "in view of the extreme limitation of all practical and 'field' methods of solution of such [dip and strike] problems, it has appeared . . . . that insufficient attention has been devoted in the past to the trigonometrical side of the question. . . . ." This book, therefore, stresses the mathematical solution of problems involving dip and strike, both in surface outcrops and by means of boreholes. Separate chapters are devoted to calculation of true dip, thickness of strata, calculation of the dip and strike of underground seams by means of boreholes, headings and adits, anticlines, faults, faulted anticlines and synclines, and mechanical devices for solving dip and strike problems. A glossary of structural terms and a bibliography are included.

Some of the material in this book is new, and undoubtedly will be of value in the solution of those dip and strike problems which require trigonometrical accuracy. However, the reviewer is inclined to criticize the work chiefly on two grounds. First, the book is too mathematical; some of the problems seem to have been chosen because they lend themselves nicely to trigonometrical solution and not because they represent actual geological situations. Second, it is doubtful if there are sufficient new data presented to warrant publication in book form. Much of the material is superfluous, especially in the chapter dealing with faults, and by careful selection of his data the author could have condensed it all into a single paper. Presentation in such manner would have had the added value of greater accessibility.

G. A. BERG

The Mesozoic Palaeontology of British Somaliland. Part II of *Geology and Palaeontology of British Somaliland.* Government of the Somaliland Protectorate, 1935. Pp. 228; figs. 78; pls. 25. £1, 10*s*.

Eight paleontologists have co-operated in writing this volume on *The Mesozoic Palaeontology of British Somaliland* which forms the second part of a three-volume series upon the geology and paleontology of this territory. They have described in detail the Foraminifera, corals and Hydrozoa, Echinoidea, Crinoidea, Brachiopoda, Gastropoda and Lamellibranchia, and Cephalopoda occurring in the Mesozoic rocks.

The material used in the studies is mainly from collections made by Mr. R. A. Farquharson in 1923–24, by the Somaliland Petroleum Company's expedition under Dr. W. A. Macfadyen in 1928–30, and by Mr. C. Barrington Brown in 1929. The text of the report, though chiefly a description of specific forms, also discusses the general characters of the faunas, their relations to contemporary faunas of other regions, and the stratigraphic succession of the different groups of fossils.

The text is illustrated with figures showing the significant structural details of many of the forms discussed. There are 25 plates containing excellent reproductions of the important members of the groups. These consist of photographs of the specimens except in the case of the Foraminifera, where drawings have been used.

E. C. Olson

"Contributions to the Geology of Northern Mongolia." By R. H. Beckwith in *Bulletin of the American Museum of Natural History.* Vol. LXVII, Art. VII. 1934. Pp. 41; figs. 4.

Much of the scientific exploration of Mongolia has been done by Russians and described in their language. To make available in English a short account of the geology of parts of northern Mongolia, R. H. Beckwith, who has spent considerable time in China as geologist on the Central Asiatic Expeditions of the American Museum of Natural History, has prepared this paper from a series of annotated abstracts which he has made from the original Russian works.

The areas treated are included under three headings: (1) "The Geology of the Eastern Border of the Kharkira Range," (2) "The Geology of Northeastern Mongolia," and (3) "The Jargalante Terraces." The original articles were written by Z. A. Lebedeva, B. M. Kupletsky, and B. B. Polynov and I. M. Krasheninnikov, respectively.

The first two abstracts deal with the topography, lithology, stratigraphy, structure, and geologic history of their respective areas. The third takes up the geologic features of the terraced area along the Jargalante River, with special emphasis on the physiography.

E. C. Olson

"Gravity" by E. A. GLENNIE, in *Geodetic Report 1934*, chap. iii, pp. 42–67. Calcutta: Survey of India. Pls. 8; charts 11.

This valuable report will be read with much interest by students of isostasy.

According to the hypothesis of crustal warping, gravity anomalies are mainly due to the up and down warpings of the lower layers of the crust. These tend to balance out, and over a very wide area equilibrium is reached very closely. This is a necessary result of the workings of the ordinary laws of mechanics and of strength of materials.

In computing the "crustal warp anomaly," Hayford compensation is assumed to be lacking over an area of about 1,600 square miles immediately surrounding the station. This anomaly is then assumed to give a measure of the underlying crustal warping combined with the effect due to local departures of the superficial strata from normal density.

In India there appears to be a general broad warping of the lower crustal layers superimposed on the more local warpings. Two great upwarps flank the coasts of the peninsula and another crosses it in about latitude 25°. "Between these the crust underlying the Indian peninsula has been wrinkled and squeezed up and down." The extreme south of India, and likewise Ceylon, appear to have been squeezed down between upwarps.

The formation of the Maldive Islands is considered in accordance with two alternative hypotheses. "If the isostatic anomalies are due to local, superficial anomalies of density, the Maldive Islands mark the original site of an oceanic ridge, not a continental block, which has since sunk down under isostatic adjustment." The conclusion to be drawn from the crustal warp anomalies is that these islands overlie an area of light rock which has subsided as a result of downwarping of its underpinnings.

Several colored charts contouring the gravity anomalies of the different types and representing the trend lines of upwarps and downwarps in India and its immediate surroundings give the reader an opportunity to make some instructive comparisons.

R. T. C.

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